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A TREATISE
ON
HYDRAULICS.

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PREFACE.

THE present treatise is the outcome of lectures delivered in McGill University during the last ten or twelve years, and although intended primarily for the use and convenience of the student of hydraulics, it is hoped that it may also prove acceptable to the engineer in general practice.

In order to render the treatment of the subject more complete, free reference has been made to standard authors on the subject. The examples introduced to illustrate the text have also been selected in part from the works of such well-known writers as Weisbach, Osborne Reynolds, and Cotterill, but the greater number are such as have occurred in the course of the author's own experience. The tables of coefficients of discharge have been prepared from the results of experiments carried out in the Hydraulic Laboratory of the University. These experiments are still being continued and may probably form the subject of a special paper.

The author desires to acknowledge many suggestions offered by Mr. Bamford, and to express his deep obligation to Professor Chandler for much labor and time given to the revision of proof sheets.

HENRY T. BOVEY.

MONTREAL, November, 1895.

PREFACE TO SECOND EDITION.

THE present edition of the work on "Hydraulics" has been practically rewritten, the various chapters having been rearranged and in some cases completely altered in order to allow of necessary corrections and of the introduction of much new matter.

In Chapter I, articles on the whirling and rotation of fluids have been inserted, the article on "Weirs and Notches" has been completely rewritten, and there has been added a résumé of Bazin's experimental work on weirs, a complete account of which appears in the *Annales des Ponts et Chaussées*.

In Chapter II will be found a large amount of new material, including the results of experiments collaborated and tabulated by Mr. C. W. Tutton of Buffalo, to whom I also owe many thanks for various useful suggestions and for the graphical representation of the results of the pipe-flow experiments.

Chapter III has been considerably changed and lengthened. The results of the experiments by Bazin, Ganguillet and Kutter, and others are given in detail and tables giving the values of the constants in the several standard formulæ, both in English and metrical units, are added at the end of the chapter.

Chapter IV contains new articles on "accumulators, presses, and water-engines."

Chapter V has been completely rewritten and now includes

a discussion of the analysis of the impact, Borda, centrifugal, and other turbines.

Chapters VI, VII, and VIII in the new volume replace Chapter VII of the old volume. Chapter VI deals exclusively with water-wheels; Chapter VII contains new matter and treats of the various classes of turbines which have not been dealt with in Chapter V. Chapter VIII is entirely new and deals with centrifugal pumps. Much of the information incorporated in this chapter has been obtained through the kindness of Mr. A. F. Hall of Boston, who has given valuable hints and suggestions and who has also furnished important practical examples.

It is hoped that the large amount of new material and the various tables which have been added to this volume will indicate the progress which is being made in reducing the subject of Hydraulics to an exact science and that these additions, more especially the tables, will add considerably to the usefulness of the book for the purposes of the practical engineer.

I have now only to express my gratitude to my colleague, Dr. Coker, for suggestions made from time to time and for his great kindness in revising the proof sheets.

HENRY T. BOVEY.

October, 1901.

CONTENTS.

CHAPTER I.

GENERAL PRINCIPLES, FLOW THROUGH ORIFICES, OVER WEIRS, ETC.

	PAGE
Fluid-motion	I
Steady motion.....	I
Permanent régime.....	I
Stream-line motion.....	2
Motion in plane layers.....	2
Laminar motion.....	2
Density	3
Compressibility.....	5
Head ..	7
Continuity.....	7
Bernouilli's theorem.....	8, 108
Applications of Bernouilli's theorem.....	12
Rotation of a fluid.....	17
Whirling fluids	19
Orifice in a thin plate.....	22
Torricelli's theorem.....	24
Flow through orifices in vessels in motion.....	26
Flow in frictionless pipe.....	27
Hydraulic coefficients.....	29
Tables of coefficients of discharge	39
Miner's inch.....	44
Inversion of jet.....	48
Time of emptying or filling a lock.....	50
General equations.....	53
Loss of energy in shock.....	55
Mouthpieces.....	58
Energy and momentum of a jet	69
Radiating current.....	70

	PAGE
Vortex motion.....	74
Large orifices in vertical plane surfaces.....	78
Notches and weirs.....	83
Reservoir sluices.....	97
Bazin's flow over weirs.....	99
Examples	109

CHAPTER II.

FLUID-FRICTION AND PIPE-FLOW.

Fluid-friction.....	121
Laws of fluid-friction.....	123
Surface-friction of pipes.....	126
Darcy's results.....	127
Reynolds' results.....	129
Critical velocity.....	129
Poiseuille's results.....	130
Resistance of ships.....	131
Pipe-flow assumptions.....	132
Steady flow in pipe of uniform section.....	133
Influence of pipe's inclination on flow	138
Formulae of Darcy, Hagen, Thrupp, Reynolds, etc.....	139
Diagrams of pipe-flow.....	146
Values of c , x , and y in the formula $v = cm^x i^y$	153
Transmission of energy.....	156
Pressure due to shock.....	160
Flow in uniform pipe connecting two reservoirs.....	162
Losses of head due to abrupt changes of section, elbows, valves, etc..	164
Nozzles	174
Ellis' experiment on nozzles.....	177
Freeman's nozzle experiments.....	178
Motor driven by water from a pipe.....	179
Siphons	181
Inverted siphons.....	182
Air in a pipe.....	183
Flow in a pipe of varying diameter.....	184
Equivalent uniform main.....	186
Branch mains of uniform diameter.....	188
Flow in pipes leading from three reservoirs to a common junction....	191
Mains with any required number of branches.....	201
Variation of velocity in a transverse section.....	202
Gauging of pipe-flow	207
Examples	210

CHAPTER III.

FLOW OF WATER IN OPEN CHANNELS.

	PAGE
Channel-flow assumptions.....	220
Steady flow in channels of constant section.....	221
Retarding effect of air, etc.....	224
Table of slopes and mean velocities of flow.....	227
Form of channel cross-section.....	228
Best dimensions for trapezoidal channel.....	233
Aqueducts.....	240
Formulae of Prony, Eytelwein, Beardmore, and Tadini.....	247
Bazin's formulæ.....	249
Table of values of α and β	249
Table of values of γ	250
Ganguillet & Kutter's formula.....	250
Table of values of n	251
Formulae of Manning, Tutton, Humphreys & Abbott, Gankler.....	252
Variation of velocity in channel cross-section.....	257
Boileau's formulæ.....	268
Tables of erosion and viscosity.....	266
River-bends.....	269
Channels of varying cross-section.....	271
Tables of values of f for standing wave.....	281
Longitudinal profile and Rühlmann's law.....	285
Channel of rectangular section of small slope.....	287
Channel of great width as compared with the depth.....	288
Tables of values of backwater function, $\phi(z)$	290, 292
Change of section.....	293
Gauging of streams and water-courses.....	297
Determinations of mean velocity of flow.....	298
Tables of values of coefficients in formulæ of Bazin, Ganguillet & Kutter, and Manning.....	311-327
Examples.....	328

CHAPTER IV.

RAMS, PRESSES, ACCUMULATORS, WATER-PRESSURE ENGINES.

Hydraulic rams.....	334
Packing.....	336
Hydraulic press.....	337
Hydraulic jack.....	337
Punching bear.....	339
Accumulators.....	340

	PAGE
Differential accumulators.....	342
Steam-accumulator.....	344
Hydraulic engines.....	344
Losses of energy in hydraulic engines.....	351
Brakes.....	353
Examples.....	355

CHAPTER V.

IMPACT, REACTION, IMPACT AND TANGENTIAL TURBINES.

Impact upon a flat vane.....	359
" " a series of flat vanes.....	362
" " surface of revolution.....	364
" " a series of surfaces.....	366
" " a bordered vane.....	368
Impact apparatus.....	369
Coefficient of impact.....	372
Reaction.....	373
Jet propeller.....	373
Jet reaction wheel (Scotch turbine).....	375
Impact wheel.....	378
Borda turbine.....	382
Effect of friction on impact turbine.....	384
Danaïdes.....	386
Tub-wheel.....	387
Impact on a curved vane.....	388
Tangential (centrifugal) turbines.....	393
Jet turbine.....	400
Resistance to motion of a solid in a fluid.....	402
Pressure on a thin plate.....	404
Pressure on a cylindrical body.....	406
Examples.....	408

CHAPTER VI.

VERTICAL WATER-WHEELS.

Classification.....	416
Undershot wheels.....	416
Wheels in straight-race.....	418
Losses in straight-race.....	421
Mechanical effect of straight-race.....	420, 423
Poncelet wheel.....	424

	PAGE
Mechanical effect of Poncelet wheel.....	428
Efficiency of Poncelet wheel.....	428, 432
Form of buckets.....	435, 458
Sluices.....	437
Breast-wheels.....	440
Speed of wheels.....	441
Mechanical effect of wheels.....	442
Sagebien wheel.....	449
Overshot wheels.....	450
Velocity of wheels.....	450
Effect of centrifugal force in overshot wheel.....	451
Weight of water on wheel.....	452
Arc of discharge.....	452, 457
Capacity of bucket.....	458
Useful effect of overshot wheel.....	467

CHAPTER VII.

TURBINES.

Reaction and impulse turbines.....	482
Actual path of a fluid particle in a turbine.....	486
Classification of turbines.....	490
Analysis of turbine.....	497
Practical coefficients.....	519
Theory of draft-tubes.....	529
Losses and mechanical effect.....	531
Examples.....	539

CHAPTER VIII.

CENTRIFUGAL PUMPS.

General statement.....	547
Analysis of centrifugal pump.....	553
Losses in hydraulic resistance	554
Blade-angles.....	556
Volute.....	558
Whirlpool chamber.....	565
Practical coefficients.....	569
Examples.....	572

HYDROSTATIC PRINCIPLES.

FUNDAMENTAL PRINCIPLES OF HYDROSTATICS.—*Fluids* may be divided into two classes :

Liquids, which are incompressible, or nearly so, showing no sensible change of volume under changes of pressure, and

Gases, which are compressible, changing in volume with changes of pressure.

The pressure of a perfect fluid on any surface with which it is in contact is perpendicular to the surface.

The pressure of a fluid at any point of a surface is the pressure per unit of area.

The pressure at any point of a fluid is the same in every direction.

Any pressure applied to the surface of a fluid is transmitted equally to all parts of the fluid.

The density of any uniform substance is the mass of a unit of volume of the substance.

The intrinsic weight of a substance is the weight of a unit of volume of the substance, expressed in terms of some standard unit of weight. The difference in the unit due to change of locality is very slight, the ratio of polar to equatorial gravity being 32.2527 : 32.088.

The specific gravity of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of a standard substance.

If fluid volumes V, V', V'' —of densities ρ, ρ', ρ'' —are mixed together, the density of the mixture = $\Sigma(\rho V) \div \Sigma(V)$.

If fluid volumes V, V', V'' —of specific gravities s, s', s'' —are mixed together, the specific gravity of the mixture = $\Sigma(sV) \div \Sigma(V)$.

The pressure in a homogeneous fluid at rest under gravity increases uniformly with the depth, or, in other words, the difference of the pressures at any two points varies as the vertical distances between the points.

Analytically, the difference of pressure $= wz$, w being the intrinsic weight of the fluid and z the difference of level.

The free surface of a liquid at rest under gravity is a horizontal plane.

The common surface of two liquids of different densities, which do not mix, is a horizontal plane, when at rest under gravity. If a number of liquids of different densities, e.g., mercury, water, and oil, are poured into a vessel, they will come to rest with their common surfaces horizontal planes, the densities of the liquid increasing downwards.

The surfaces of equal pressure are horizontal planes.

The pressure of a liquid on any horizontal area, A , is equal to the weight of a column of the liquid of which the area is the base and of which the height, z , is equal to the depth of the area below the surface, i.e., wAz (disregarding the pressure on the free surface).

The whole pressure of a fluid on a submerged surface is the sum of all the normal pressures exerted by the fluid on every portion of the surface and (disregarding the pressure on the free surface) is equal to the weight of a column of liquid of which the base is equal to the area of the surface, and the height is equal to the depth of the centroid of the surface below the surface of the liquid. Thus:

(a) The total normal pressure on a wall of width b , sloping at θ to the vertical and retaining water which rises over a length z of the wall

$$= wbz \frac{z}{2} \cos \theta = \frac{wbz^2 \cos \theta}{2}.$$

(b) The total pressure on a circular valve of diameter d , with its centroid z below the surface $= w \frac{\pi d^2}{4} z$.

(c) The total normal pressure on a lock-gate of width b and on which the water rises to a height $z = wbz \frac{z}{2} = \frac{1}{2}wbz^2$.

The pressure *between a pair of* lock-gates = pressure on the hinge post $= \frac{1}{4}wbz^2 \sec \alpha$, 2α being the angle between the gates.

The centre of pressure of a plane area is the point of action of the resultant fluid-pressure, (R), upon the plane area.

If \bar{y} , \bar{z} are the horizontal and vertical distances of the C. of P. from the vertical and horizontal axes through the C. of G. of the area,

$$\bar{y} = \frac{wD}{R} = \frac{wD}{wAh} = \frac{D}{Ah}, \text{ and } \bar{z} = \frac{\omega I}{R} = \frac{\omega . I k^2}{\omega Ah} = \frac{k^2}{h},$$

D being the product of inertia about the axes; I the moment of inertia of the area about the axis of y ; h the depth below the surface of the centroid, and k the radius of gyration.

Ex. 1. Depth of C. of P. of a parallelogram with one edge in surface = $\frac{2}{3}$ of depth of opposite edge.

Ex. 2. Depth of C. of P. of a triangular area, the middle points of the sides being at depths d_1, d_2, d_3 below the surface, $= \frac{d_1^2 + d_2^2 + d_3^2}{a_1 + a_2 + a_3}$,

and (a) if vertex is in surface and base horizontal, depth = $\frac{2}{3}$ of depth of base;

(b) if base is in surface, depth = $\frac{1}{3}$ of depth of vertex;

(c) if vertex is in surface and y and z are depths of ends of base, the depth = $\frac{1}{2} \frac{y^3 - z^3}{y^2 - z^2}$.

The resultant pressure on the surface of a solid, wholly or partially immersed in a fluid, is equal to the weight of the displaced liquid and acts vertically upwards in a line passing through the centroid of the displaced liquid. In other words, a solid immersed in a liquid appears to lose as much of its weight as is equal to the weight of the fluid it displaces.

If a homogeneous body float in a liquid, its volume will bear to the volume immersed the inverse ratio of the specific gravities of the solid and liquid.

A body of weight W , carrying a load P , floats in a liquid, G and H being the centres of gravity of the body and of the displaced water, so that GH is vertical. If the load P is shifted, the body will heel through an angle θ and the point H , also called the centre of buoyancy, will move on a curve or surface of buoyancy to a new position H' , the line $G'H$ connecting H' with the new position of the C. of G. of the body being vertical. If θ is small, the ultimate position of M , the intersection of HG and $H'G'$, is called the *metacentre*, and M is therefore the centre of curvature of the surface of buoyancy at H . For stability of equilibrium M must be above G . Theoretically,

$$HM = \frac{wI}{W} = \frac{wAk^2}{wV} = \frac{Ak^2}{V},$$

A being the water-line area and V the volume of liquid displaced by body.

CAPILLARY PHENOMENA.—If a glass tube of fine bore is placed vertically in a liquid like water, which wets the glass, the water-surface on the outside next the glass is elevated and slightly concave, while on the inside the water-surface is concave and there is a marked elevation above the outside surface.

With a liquid which does not wet the glass, like mercury, an opposite effect is observed. There is a depression on the outside and the surface

is slightly convex, while on the inside the surface is convex and there is a marked depression below the outside surface.

SURFACE TENSION.—At the bounding surface separating air from any liquid, or between two liquids, there is a surface-tension which is the same at every point and in every direction.

At the line of junction of the bounding surface of a gas and a liquid with a solid body, or of the bounding surface of two liquids with a solid body, the surface is inclined to the surface of the solid body at a definite angle, depending upon the nature of the solid and the liquids.

The surface-tension is independent of the curvature of the surface but, if the temperature be increased, it diminishes.

USEFUL CONSTANTS.

The following abbreviations are used: Metre = m.; sq. metre = m.²; cubic metre = m.³; centimetre = cm.; sq. centimetre = cm.²; cubic centimetre = cm.³; kilometre = kilo.; grain = gr.; gramme = gm.; kilo-gramme = k.; kilogramme metre = km.

1 in.	= 2.54 cm.
1 cm.	= .3937011 in.
1 ft.	= 30.4799 cm.
1 m.	= 3.280843 ft.
1 mile	= 1.6093 kilo.
1 kilo.	= .62137 mile.
1 knot	= 1 naut. mile per hr. = 6080 ft. (av.) per hr.
1 sq. in.	= 6.4516 cm. ²
1 cm. ²	= .155 sq. in.
1 sq. ft.	= 929.03 cm. ²
1 m. ²	= 10.7639 sq. ft.
1 sq. yd.	= .836126 m. ²
1 acre	= 43,560 sq. ft. = .40468 hectare.
1 hectare	= 10,000 m. ² = 100 ares. = 2.4711 acres.
1 sq. mile	= 640 acres. = 2.59 sq. kilo. = 259 hectares.
1 sq. kilo.	= 100,000 m. ² = 24.711 acres.
1 lb.	= 16 oz. = 7000 gr. = .4535924 k. = 453.5924 gm. = 445,000 dynes.
1 k.	= 2 204622 lbs. = 981,000 dynes.

1 British ton	= 2240 lbs. = 1016 k.
1 U. S. ton	= 2000 lbs. = 907.143 k.
1 Fr. tonne	= 1000 k. = .9842 British ton. = 2204.622 lbs.
1 cu. in. of water at 4° C.	= 252.89 gr.
1 cm. ³ " "	= 1 gm.
1 cu. ft. " "	= 62.43 lbs.
1 litre " "	= 1 k.
1 imp. gal. at 62° F.	= 10 lbs.
1 cu. ft. of water at 62° F.	= 62.3 lbs.
1 cu. ft. of air at 0° C. and 1 atm.	} = .0807 lb.
1 cu. ft. of hydrogen at 0° C. and 1 atm.	} = 1.2932 gm.
1 litre of air at 0° C. and 1 atm.	} = .00557 lb.

Water compresses $\frac{1}{200000}$ th of its bulk under a change of pressure of 1 atm., or about $\frac{1}{70}$ th of its vol. under a pressure of 2 tons (of 2240 lbs.) per sq. in.

1 lb. per sq. in.	= .0703 k. per cm. ²
1 k. per cm. ²	= .0703 lb. persq.in.
1 lb. per sq. ft.	= 4.8826 k. per m. ² = 479 dynes per cm. ²

No. of lbs. per } sq. in.	= 14.223 k. per cm.	1 standard atm. } of 14.7 lbs. per sq. in.	= { 29.95 ins. of mercury.
	= { .4907 ins. of mercury.		= 760 mm.
	= { ins. of mercury ÷ 2.0378.	1 metric atm. of } 14.223 lbs. per sq. in.	= { 28.96 ins. of mercury.
No. of k per m. ² = {	4.8826 lbs. per sq. ft.	1 erg	= 1 dyne × 1 cm.
1 in. of mer- } cury at 0° C. }	= .034534 k. per cm. ²	1 gm.-cm.	= 981 ergs.
1 mm. mercury } at 0° C. }	= .0013596 per cm. ²	1 ft.-lb.	= .13825 km.
			= 1.3562 × 10 ⁷ ergs.
1 cu. in.	= 16.387 cm. ³	1 km.	= 7.233 ft.-lbs.
1 cm. ³	= .061 cu. in.		= 9.81 × 10 ⁷ ergs.
1 cu. ft.	= .028317 m. ³	No. of ft.-lb.	= 7.2178 km.
	= 28.317 litres.		= 777 B. T. U.
1 m ³ .	= 35.3148 cu. ft.		= { 1399 lbs. de- gree C.
1 litre	= 1000 cm. ³	1 B. T. U.	= 1058 joules.
	= 1.7598 pints.		= 1058 × 10 ⁷ ergs.
	= .22 imp. gal.	1 k. degree C.	= 4200 joules.
1 imp. gal.	= .1605 cu. ft.		= 4200 × 10 ⁷ ergs.
	= 277.27 cu. ins.	1 calorie	= 1 k. raised 1° C.
	= 4.545963 litres.		= 426.9 km.
1 U. S. gal.	= 231 cu. ins.		= 3080.9 ft.-lbs.
	= .83254 imp. gal.	1 watt	= 1 joule per sec.
g	= { 981 cm. per sec. per sec.		{ work done by a current of 1 amp. at 1 volt.
	= { 32.2 ft. per sec. per sec.	1 horse-power	= 550 lbs. per sec.
g at Greenwich	= 32.19078 ft.		= { 746 × 10 ⁹ ergs per sec.
	= 981.17 cm.		= 746 watts.
g at London	= 32.182 ft.		= { 1.01 forces-de- cheval.
	= 980.9 cm.		
g at Manchester	= 32.196 ft.	1 force-de-cheval = {	.9863 horse- power.
	= 981.34 cm.		= 736 watts.
g at the equator	= 32.088 ft.		= { 545 ft.-lbs. per sec.
	= 978.04 cm.		= 75 km. per sec.
g at Baltimore	= 32.152 ft.	1 radian	= 57.296 degrees.
	= 980 cm.		
g at Montreal	= 32.1765 ft.		
	= 980.73 cm.		
The inertia or } mass of a body }	= { its wt. in lbs. at London ÷ 32.2.		

To convert common into hyperbolic and hyperbolic into common logarithms; multiply the former by 2.3025 and the latter by .43429.



HYDRAULICS.

CHAPTER I.

GENERAL PRINCIPLES. FLOW THROUGH ORIFICES, OVER WEIRS, ETC.

1. Fluid Motion.—The term “hydraulics,” as its derivation ($\nu\delta\omega\rho$, water; $\alpha\upsilon\lambda\acute{o}\varsigma$, a tube or pipe) indicates, was primarily applied to the conveyance of water in a tube or pipe, but its meaning now embraces the experimental theory of the motion of fluids.

The motion of a fluid is said to be *steady* or *permanent* when the molecules successively arriving at any given point are animated with the same velocity, are subjected to the same pressure, and are the same in density. As soon as the motion of a stream becomes steady a *permanent régime* is said to be established, and hydraulic investigations are usually made on the hypothesis of a permanent régime. With such an hypothesis, any portion of the fluid mass, which leaves a given region, is replaced by a like portion under conditions which are identically the same.

The terms “steady motion” and “permanent régime” are often considered to be synonymous.

The general problem of flow is the determination of the relation which exists at any point between the density, pressure, and velocity of the molecules which successively pass that point.

The actual motion of a fluid is exceedingly complex, and, in order to simplify the investigations, various assumptions are made as to the nature of the flow.

2. (a) **Stream-line Motion.**—The molecules may be regarded as flowing along definite paths, and a succession of such molecules as forming a continuous fluid rope, which is termed an *elementary stream* or a fluid filament; or, if the motion is steady, and the paths therefore fixed in space, is termed a *stream-line*.

Experiment shows that the velocity of flow in any cross-section varies from point to point, and it is often assumed that the section is made up of an infinite number of indefinitely small areas, each area being the section of a fluid filament.

(b) **Motion in Plane Layers.**—In this motion it is assumed that the molecules, which at any given moment are found in a plane layer, will remain in a plane layer after they have moved into any new position.

(c) **Laminar Motion.**—On this hypothesis the stream is supposed to consist of an infinite number of indefinitely thin layers. The variation in velocity from point to point of a cross-section may then be allowed for, by giving the several layers different velocities based upon the law of fluid resistance between consecutive layers.

3. Density ; Compressibility ; Head ; Continuity.

The freezing-point of pure water is	32° F.	or	0° C.
“ boiling- “ “ “ “ “	212° F.	or	100° C.
“ max. density “ “ “ “	39°.1 F.	or	4° C.
“ standard mean temperature “	62° F.	or	16°.66 C.

The comparative densities and also the comparative volumes are the same at 32° F. and 46° F.

The bulk of fresh snow is 12 times the bulk of the equivalent water.

1 cu. ft. of fresh snow weighs 5.2 lbs. and its s. g. is .0833.

1 cu. ft. of ice at 32° F. weighs 57½ lbs. and its s. g. is .922.

1 cu. ft. of average sea-water at 62° F. weighs 64 lbs. and its average s. g. is 1.028.

1 cu. ft. of pure water at 32° F. weighs 62.418 lbs.

“ “ “ “ “ 39°.1 F. “ 62.425 “

“ “ “ “ “ 52°.3 F. “ 62.400 “

“ “ “ “ “ 62° F. “ 62.355 “

“ “ “ “ “ 212° F. “ 59.640 “

“ “ “ “ contains { 6.2355 gallons or
6.2328 imperial gallons.

1 cu. yd. “ “ “ 168.36 gallons.

1 cu. metre “ “ “ 220.09 “

The vol. of 1 lb. of pure water at 32° F. is .016021 cu. ft.

“ “ “ “ “ 39°.1 F. “ .016019 “

“ “ “ “ “ 52°.3 F. “ .016 “

“ “ “ “ “ 62° F. “ .016037 “

“ “ “ “ “ 212° F. “ .01677 “

The vol. of 1 ton “ “ 52°.3 F. “ 35.9 “

“ “ “ sea-water at 62° F. “ 35 “

“ “ 1 tonne of pure water at 39°.1 F. “ 35.3156 “

“ “ 1 kilog. “ “ “ .0353 “

1 gallon of pure water at 62° F. weighs 10 lbs. and its vol.
= 277.123 cu. ins. = .16037 cu. ft.

1 imperial gallon of pure water at 62° F. weighs 10.00545
lbs. and its vol. = 277.274 cu. ins.

TABLE OF EXPANSION AND DENSITY OF PURE WATER.

Temp., Fahrenheit, Degrees.	Temp., Centigrade, Degrees.	Comparative Density.	Weight in lbs. of 1 Cubic Foot.	Weight in Pounds of 1 Gallon.	Temp., Fahrenheit, Degrees.	Temp., Centigrade, Degrees.	Comparative Density.	Weight in lbs. of 1 Cubic Foot.	Weight in Pounds of 1 Gallon.
32	0	.999868	62.418	10.0101	69.8	21	.998020	62.313	9.9933
33.8	1	.999927			70.0	21.11		
35	1.66	62.422	10.0103	71.6	22	.998798		
35.6	2	.999968			73.4	23	.997566	62.275	9.9875
37.4	3	.999992			75	23.89		
39.1	4	1.000000	62.425	10.0112	75.2	24	.997324		
40	4.44	62.425	10.0112	77	25	.997072		
41	5	.999991			78.8	26	.996811	62.232	9.980
42.8	6	.999967			80	26.66		
44.6	7	.999929			80.6	27	.996540		
45	7.66	62.422	10.0103	82.4	28	.996261		
46	7.77	62.418	10.0101	84.2	29	.995972	62.182	9.972
46.4	8	.999876			85	29.44		
48.2	9	.999809			86	30	.995675		
50	10	.999728	62.409	10.0087	87.8	31	.995369		
51.8	11	.999634			89.6	32	.995055	62.133	9.964
52.3	11.28	62.400	10.0072	90	32.22		
53.6	12	.999524			91.4	33	.994732		
55	12.78	62.394	10.0063	93.2	34	.994401		
55.4	13	.999404			95	35	.994062	62.074	9.955
57.2	14	.999272			96.8	36	.993715		
59	15	.999128			98.6	37	.993359		
60	15.55	62.372	10.0053	100	37.77	62.022	9.947
60.8	16	.998972			100.4	38	.992996		
62	16.66	62.355	10.0000	102.2	39	.992625		
62.6	17	.998804			104	40	.992247		
64.4	18	.998624			150	65.55		
65	18.33	62.344	9.9982	200	93.33	60.081	9.635
66.2	19	.998434			212	100		
68	20	.998232			300	148.88	57.260	

The temperatures in this table may be taken as abscissæ, and the corresponding values in the three remaining columns as ordinates. Curves of comparative density, weight per cubic foot, and weight per gallon are thus obtained, and the values corresponding to any specified temperature can be easily and very accurately determined from these curves by direct measurement.

The weights per cubic foot in the table have been calculated by means of Rankine's approximate formula,

$$\frac{w}{62.425} = \frac{1000 T}{T^2 + 250,000},$$

w being the weight per cubic foot corresponding to the absolute temperature T , i.e., $461^\circ +$ ordinary temperature.

The specific weights obtained by this rule for the lower temperatures are very exact, but for the higher temperatures they become too large. Thus the rule gives 59.76 lbs. as the weight of a cubic foot of pure water at 212° F., while actual measurement makes the weight 59.64 lbs.

The comparative densities between 0° C. and 40° C. are the values obtained by Chappuis.

Compressibility.—Fluids are sensibly compressible under heavy pressures, and the compression is proportional to the pressure up to about 1000 lbs. (68 atmospheres) per sq. inch. Grassi's experiments indicate that the compressibility of water diminishes as the temperature increases. Water compresses about $47\frac{1}{2}$ millionths (i.e., $\frac{.95}{20,000} = \frac{1}{20,000}$, nearly) of its bulk for each atmosphere. This is equivalent, approximately, to a reduction of $\frac{1}{70}$ in the bulk under a pressure of 2 tons per sq. inch.

If a volume V of a fluid is compressed by an amount ΔV under an increase Δp of the pressure, then the amount of compression per unit of vol. is

$\frac{\Delta V}{V}$ and is called the cubical compression. The ratio of the

TABLE OF ELASTICITY OF VOLUME OF LIQUIDS.

(Reduced from Grassi's results.)

Liquid.	Elasticity of Volume.	Temperature.
Mercury ...	717,000,000	0° C.
Water.....	{ 42,000,000	0° C.
	{ 45,900,000	18° C.
Sea-water..	52,900,000	
Ether.....	{ 16,280,000	0° C.
	{ 15,000,000	14° C.
Alcohol....	{ 25,470,000	7.3° C.
	{ 23,380,000	13.1° C.
Oil.....	44,090,000	

N. B.—The value for mercury is probably erroneous.

increment of pressure to the cubical expansion, viz.,

$$\Delta p / \frac{\Delta V}{V}, \text{ or } V \frac{\Delta p}{\Delta V},$$

is termed the elasticity of volume. This is sensibly constant within wide limits, and is generally denoted by the letters *D* or *K*.

The vertical distance between the free surface of a mass of water and any datum plane is called the *head* with respect to that plane. If the water extends down to the level of the plane, a pressure *p* is produced at that level, and the value of *p*, so long as the water is at rest, is given by the equation

$$\frac{p}{w} = h + \frac{p_0}{w},$$

w being the specific weight of the water and *p*₀ the pressure at the free surface. Thus the pressure may be measured in terms of the head, and hence the expression “head due to pressure” or “pressure head.”

A column of water at 62° F. and 2.3093 ft. in height exerts a pressure of 1 lb. per sq. inch.

A column of water at 62° F. and 33.947 ft. or 10.347 metres in height exerts a pressure of 14.7 lbs. per sq. in., or one atmosphere.

A column of water at 62° F. and 1 ft. in height exerts a pressure of .433 lbs. per sq. inch.

Head.—A head of water is a source of energy. A volume of water descending from an upper to a lower level may be employed to drive a machine, which receives energy from the water and again utilizes it in overcoming the resistances of other machines doing useful work.

Let Q cu. ft. of water per second fall through a vertical distance of h ft. Then the total power of the fall = wQh ft.-lbs., = $\frac{wQh}{550}$ h.p., w being the weight of the water in pounds per cubic foot.

Let K be the proportion of the total power which is absorbed in overcoming frictional and other resistances. Then the *effective* power of the fall = $wQh(1 - K)$, and the efficiency is $1 - K$.

Continuity.—Imagine a bounding surface enclosing a space of invariable volume in the midst of a moving mass of fluid. The principle of continuity affirms that, in any interval of time, the flow into the space must be equal to the outflow during the same interval. Giving the inflow a positive and the outflow a negative sign, the principle may be expressed symbolically by

$$\Sigma Q = 0.$$

The continuity of a mass of water will be preserved so long as the pressure exceeds the tension of the air held in solution. It is on account of the pressure of this air that pumps cannot draw water to the full height of the water-barometer, or about 34 ft.

Generally speaking, the pressure at every point of a continuous fluid must be positive. A negative pressure is equivalent to a tension which will tend to break up the continuity presupposed by the formulæ. Should negative pressures result from the calculations, the inference would be that the latter are based upon insufficient hypotheses.

The pressure in water flowing through the air cannot at any point fall below the atmospheric pressure. There are cases,

however, as when water flows through a closed pipe (Art. 6, Chap. II), in which the pressure may fall below this limit and become almost nil. In this case there is a danger of the air held in solution being set free, thus tending to interrupt the continuity of the flow, which may even be wholly stopped if the air is present in sufficient volume.

Consider a length of a canal or stream bounded by two normal sections of areas A_1 , A_2 , and let v_1 , v_2 be the mean normal velocities of flow across these sections. Then, by the principle of continuity,

$$A_1 v_1 = Q = A_2 v_2,$$

and the velocities are inversely as the sectional areas.

Again, assume that a moving mass of fluid consists of an infinite number of stream-lines, and consider a portion of the mass bounded by stream-lines and by two planes of areas A_1 , A_2 at right angles to the direction of flow. If v_1 , v_2 are the mean velocities of flow across the planes,

$$v_1 A_1 = Q = v_2 A_2 \text{ if the fluid is incompressible.}$$

Assuming that the fluid is compressible, and that the mean specific weights at the two planes are w_1 and w_2 , then the *weight* of fluid flowing across A_1 is equal to the *weight* which flows across A_2 , since the weight of fluid *between* the two planes remains constant. Hence

$$w_1 A_1 v_1 = w_2 A_2 v_2.$$

4. Bernoulli's Theorem.—This theorem is based on the following assumptions:

(1) That the fluid mass under consideration is a *steadily* moving stream made up of an infinite number of stream-lines whose paths in space are necessarily fixed.

(2) That the velocities of consecutive stream-lines are not widely different, so that viscosity, or the frictional resistance between the stream-lines, is sufficiently small to be disregarded.

(3) That the fluid is incompressible, so that there can be no *internal work* due to a change of volume.

In any given stream-line let a portion AB , Fig. 1, of the fluid move into the position $A'B'$ in t seconds.

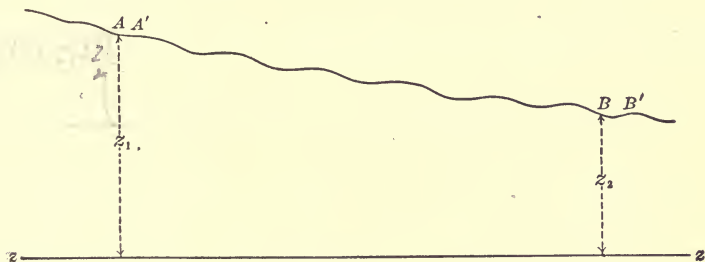


FIG. 1.

Let a_1, p_1, v_1, z_1 be the normal sectional area, the intensity of the pressure, the velocity of flow, and the elevation above a datum plane zz of the fluid at A . Let a_2, p_2, v_2, z_2 denote similar quantities at B .

Since the internal work is nil, the work done by *external* forces must be equivalent to the change of kinetic energy.

Now the external work = $\begin{cases} \text{the work done by gravity} \\ + \text{the work done by pressure.} \end{cases}$

But when the fluid AB passes into the position $A'B'$, the work done by gravity is equivalent to the work done in the transference of the portion BB' , and therefore, t being the time,

$$\begin{aligned} \text{the work done by gravity} &= wa_1 \cdot AA' \cdot z_1 - wa_2 \cdot BB' \cdot z_2 \\ &= wa_1 \cdot v_1 t \cdot z_1 - wa_2 \cdot v_2 t \cdot z_2 \\ &= wQt(z_1 - z_2), \end{aligned}$$

since $AA' = v_1 t$, $BB' = v_2 t$, and $a_1 v_1 = Q = a_2 v_2$.

$$\begin{aligned} \text{Again, the work done by the pressures on the ends } A \text{ and } B \\ &= p_1 a_1 v_1 t - p_2 a_2 v_2 t \\ &= Qt(p_1 - p_2). \end{aligned}$$

The work done by the pressure on the surface of the

stream-line between A and B is nil, since the pressure is at every point normal to the direction of motion.

The change of kinetic energy

= kinetic energy of $A'B'$ — kinetic energy of AB

= kinetic energy of BB' — kinetic energy of AA' ,

since the motion is steady, and there is therefore no change in the kinetic energy of the intermediate portion $A'B$. Thus

$$\begin{aligned} \text{the change of kinetic energy} &= \frac{w}{g} a_2 BB' \frac{v_2^2}{2} - \frac{w}{g} a_1 AA' \frac{v_1^2}{2} \\ &= \frac{w}{g} a_2 v_2 t \frac{v_2^2}{2} - \frac{w}{g} a_1 v_1 t \frac{v_1^2}{2} \\ &= \frac{w}{g} Q t \left(\frac{v_2^2}{2} - \frac{v_1^2}{2} \right). \end{aligned}$$

Hence, equating the external work and the change of kinetic energy,

$$wQ t(z_1 - z_2) + Q t(p_1 - p_2) = \frac{w}{g} Q t \left(\frac{v_2^2}{2} - \frac{v_1^2}{2} \right),$$

which may be written in the form

$$wz_1 + p_1 + \frac{w}{g} \frac{v_1^2}{2} = wz_2 + p_2 + \frac{w}{g} \frac{v_2^2}{2}, \quad \dots \quad (1)$$

or

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g}. \quad \dots \quad (2)$$

But A and B are arbitrarily chosen points, and therefore, at any point of a stream-line, the motion being steady and the viscosity nil, the gradual interchange of the energies, due to head, pressure, and velocity, is expressed by the equation

$$wz + p + \frac{w}{g} \frac{v^2}{2} = wH, \text{ a constant; } \dots \quad (3)$$

or

$$z + \frac{p}{w} + \frac{v^2}{2g} = H, \text{ a constant; } \dots \quad (4)$$

z being the elevation of the point above the datum plane, p the pressure at the point, w the specific weight, and v the velocity of flow. This is Bernouilli's theorem.

Thus the total constant energy of wH ft.-lbs. per cubic foot of fluid, or H ft.-lbs. per pound of fluid, is distributed uniformly along a stream-line, wH being made up of wz ft.-lbs. due to head, p ft.-lbs. due to pressure, $\frac{w}{g} \frac{v^2}{2}$ ft.-lbs. due to velocity, and H being made up of z ft.-lbs. due to head, $\frac{p}{w}$ ft.-lbs. due to pressure, and $\frac{v^2}{2g}$ ft.-lbs. due to velocity.

Hence the total energy is made up of three elements, and each element may be utilized by a specially designed motor. The now almost obsolete overshot-wheel is driven by the *weight* of the water filling the buckets on one side and descending from a higher to a lower level. In the breast-wheel and certain turbines, the energies, due both to the *weight* (wz) and to the *velocity* ($\frac{wv^2}{2g}$), are transformed into useful work. The rotation of impulse-wheels is due to the *kinetic energy* ($\frac{wv^2}{2g}$) of a jet of water issuing from a nozzle and impinging upon curved buckets. Finally, the piston of the hydraulic engine is actuated by water admitted into the cylinder from a closed pipe in which the water under pressure moves with a low velocity.

Assuming that

- (a) the motion is steady,
- (b) the frictional resistance may be disregarded,
- (c) the fluid is incompressible,

Bernouilli's theorem may be applied to currents of finite size at any normal section, if the stream-lines across that section are sensibly rectilinear and parallel. There is then no interior work due to a change of volume, and the distribution of the pressure in the section under consideration will be the same as if the fluid were at rest, that is, in accordance with the hydrostatic law. This is also true whether the flow takes place

under atmospheric pressure only, or whether the fluid is wholly or partially confined by solid boundaries, as in pipes and canals, or whether the flow is through another medium already occupied by a volume of the fluid at rest or moving steadily in a parallel direction. In the last case there must necessarily be a lateral connection between the two fluids, but the pressure over the section must follow the hydrostatic law throughout the separate fluids, and there can be no sudden change of pressure at the surface of separation, as this would lead to an interruption of the continuity.

The hypotheses, however, upon which these results are based, are never exactly realized in actual experience, and the results can only be regarded as tentative. Further, they can only apply to an indefinitely short length of the current, as the viscosity, which is proportional to the surface of contact, would otherwise become too great to be disregarded.

5. Applications of Bernoulli's Theorem.—If a glass tube, open at both ends, and called a piezometer (*πιέζειν*, to press;

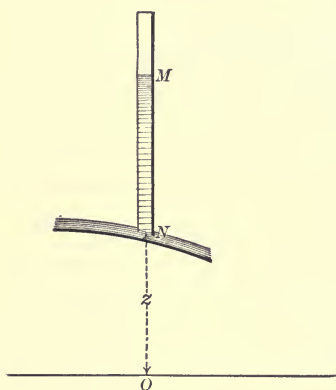


FIG. 2.

μέτρον, a measure) is inserted vertically in the current, Fig. 2, at a point *N*, *z* ft. above the point *O* in the datum line, the water will rise in the tube to a height *MN* dependent upon the pressure at *N*. The effect of the eddy motion, produced at *N* by obstructing the stream-lines, may be diminished by making this end of the tube parallel to the direction of flow. Disregarding the effect of the eddies and taking *p* and *p*₀ to

be the intensities of the pressure at *N* and of the atmospheric pressure, respectively, then,

$$\frac{p}{w} = MN + \frac{p_0}{w},$$

and therefore

$$\begin{aligned} z + \frac{p}{w} &= z + MN + \frac{p_0}{w} \\ &= ON + MN + \frac{p_0}{w} \\ &= OM + \frac{p_0}{w}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

The locus of all such points as M is often designated "the line of hydraulic gradient," or the "virtual slope," terms also used when friction is taken into account.

Let the two piezometers AB , CD , Fig. 3, be inserted in the current at any two points B and D , z_1 ft. and z_2 ft. respectively above the points E and F in the datum line.

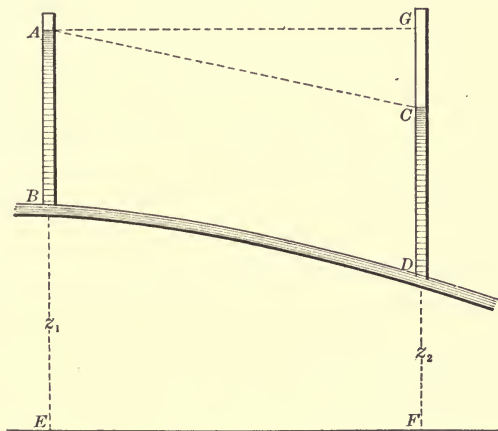


FIG. 3.

Let p_1 be the intensity of the pressure at B in pounds per square foot, p_2 that at D , and let the water rise in these tubes to the heights BA , DC . Then,

$$\frac{p_0}{w} + AE = z_1 + \frac{p_1}{w}, \quad \text{and} \quad \frac{p_0}{w} + CF = z_2 + \frac{p_2}{w},$$

and therefore

$$\left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right) = AE - CF = CG, \quad (6)$$

the line AG being parallel to the datum line.

Thus, $\left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right)$ is equal to the fall of the free surface level between the points B and D .

Let v_1, v_2 be the velocities of flow at B and D . Then, by Bernoulli's theorem,

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g}, \quad (7)$$

and therefore the fall of free surface level between B and D

$$= \left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right) = \frac{v_2^2 - v_1^2}{2g}$$

Equation (7) may also be written in the form

$$\frac{v_2^2}{2g} = \frac{v_1^2}{2g} + \left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right) = \frac{v_1^2}{2g} + CG, \quad (8)$$

so that the velocity at D is equal to that acquired by a body with an initial velocity v_1 falling freely through the vertical distance CG .

Froude illustrated Bernoulli's theorem experimentally by means of a tube of varying section, Fig. 4, conveying a current between two cisterns. The pressure at different points along the tube is measured by piezometers, and it is found that the water stands higher and the pressure is therefore greater, where the cross-section is larger and the velocity consequently less. Reynolds illustrates the principle, that the pressure in a frictionless pipe of varying section increases and diminishes with the section, by forcing water at a high velocity through a $\frac{3}{4}$ -in. pipe drawn down in the middle to a bore of .05 inch. At

this point the pressure is so much diminished, that the water hisses and boils. To the same cause is due the hissing sound heard in water-injectors and in partially opened valves. If the section of the throat at A is such, that the velocity is that acquired by a body falling freely through the vertical distance

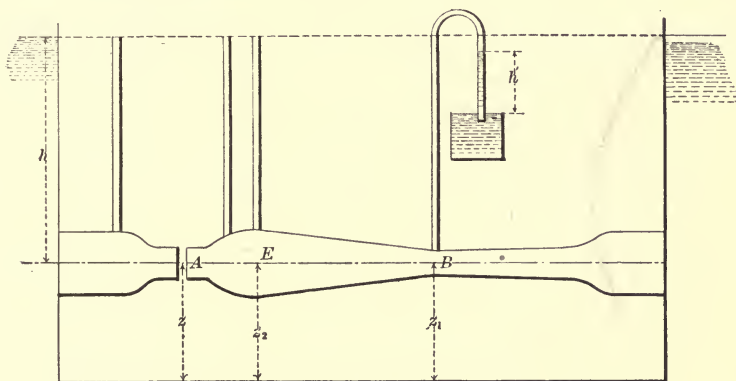


FIG. 4.

h between A and the surface level of the water in the cistern, and if p be the pressure at A , and z the elevation of A above datum, then, neglecting friction,

$$z + \frac{p}{w} + \frac{v^2}{2g} = H = z + h + \frac{p_0}{w}.$$

But $v^2 = 2gh$, and therefore $p = p_0$, so that the pressure at A is that due to atmospheric pressure only. Thus, a portion of the pipe in the neighborhood of A may be removed, as in the throat of the injector.

Again, let the cross-section in the throat at B be less than that at A . The pressure at B will be less than the atmospheric pressure, and a column of water will be lifted up in the curved piezometer to a height h' .

Let a_1, z_1, p_1, v_1 be the sectional area, elevation above datum, pressure, and velocity at B .

Let a_2, z_2, p_2, v_2 be similar symbols at E .

Then

$$z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g} = z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_1 + \frac{p_0}{w} - h' + \frac{v_1^2}{2g}. \quad (9)$$

Put $H_2 = z_2 + \frac{p_2}{w}$, the height above datum to which the water is observed to rise in the piezometer inserted at E , and also let $H_1 = z_1 + \frac{p_0}{w} - h'$. Then

$$H_2 - H_1 = \frac{1}{2g}(v_1^2 - v_2^2) = \frac{v_1^2}{2g} \frac{a_2^2 - a_1^2}{a_2^2},$$

since $a_2 v_2 = a_1 v_1$, a_2 being the sectional area at E . Therefore

$$v_1^2 = \frac{2ga_2^2}{a_2^2 - a_1^2}(H_2 - H_1),$$

an equation giving the *theoretical* velocity of flow at the throat B . Hence the *theoretical* quantity of flow across the section at B is

$$a_1 v_1 = \frac{a_1 a_2}{\sqrt{a_2^2 - a_1^2}} \sqrt{2g(H_2 - H_1)}. \quad (10)$$

This is the principle of the aspirator and also of the Venturi water-meter, which, as now used, is said to be correct to within $\frac{1}{2}$ per cent.

The actual quantity of flow is found by multiplying equation (10) by a coefficient C , whose value is to be determined by experiment and may be taken to be approximately unity.

If the pressure at E is positive, then H_2 is merely the height to which the water is observed to rise in an ordinary piezometer inserted at E .

Again, Froude also points out that when any number of combinations of enlargements and contractions occur in a pipe, the pressures on the converging and diverging portions of the

pipe will balance each other if the sectional areas and directions of the ends are the same.

Ex. 1. One cubic foot of water per second flows steadily through a frictionless pipe. At a point A , 100 ft. above datum, the sectional area of the pipe is .125 sq. ft., and the pressure is 2500 lbs. per sq. ft. Find the total energy at A per cubic foot of water. At a point B in the datum line, the pressure is 1250 lbs. per sq. ft. and the sectional area .0625 sq. ft. Find the loss of energy per cubic foot of water between A and B .

The velocity of flow at $A = \frac{1}{.125} = 8$ ft. per sec.

The total energy at A per cubic foot of water

$$= 100 + \frac{2500}{62\frac{1}{2}} + \frac{8^2}{64} = 141 \text{ ft.-lbs.}$$

The velocity of flow at $B = \frac{1}{.0625} = 16$ ft. per sec.

The total energy at B per cubic foot of water

$$= 0 + \frac{1250}{62\frac{1}{2}} + \frac{16^2}{64} = 24 \text{ ft.-lbs.}$$

Hence, the loss of energy between A and B per cubic foot of water

$$= 141 - 24 = 117 \text{ ft.-lbs.}$$

Ex. 2. A horizontal frictionless pipe, in which the pressure is 100 lbs. per square inch, gradually contracts to a throat of one tenth of the diameter and then again gradually enlarges to a pipe of uniform diameter. What will be the maximum velocity of flow at the throat?

The velocity at the throat will be greatest when the pressure there is nil. Hence, if v is the throat velocity and therefore $\frac{v}{100}$ the pipe velocity,

$$\frac{100 \times 144}{62\frac{1}{2}} + \frac{1}{64} \left(\frac{v}{100} \right)^2 = 0 + \frac{v^2}{64},$$

and $v = 121.437$ ft. per sec.

6. Rotation of a Fluid.—In any stream-line moving freely in space, let $ABCD$ be an element of mass m and normal thickness $dn(=BC)$. It is acted upon by the pressures on AD and BC , a pressure of intensity p on the area $AB(=a)$, a pressure of intensity $p + dp$ on the area CD , its weight mg

inclined at an angle α to the normal, and the centrifugal force $m \frac{v^2}{r}$, r being the radius of curvature.

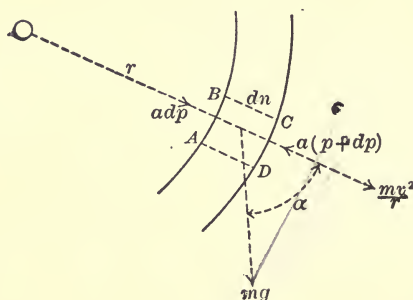


FIG. 5.

Resolving along the normal,

$$a \cdot dp - m \frac{v^2}{r} - mg \cos \alpha = 0,$$

or

$$a \cdot dp = m \left(\frac{v^2}{r} + g \cos \alpha \right) = \frac{wa \cdot dn}{g} \left(\frac{v^2}{r} + g \cos \alpha \right),$$

or

$$\frac{dp}{dn} = \frac{w}{g} \left(\frac{v^2}{r} + g \cos \alpha \right).$$

If the stream-line is in a horizontal plane, $\alpha = 90^\circ$, and then,

$$\frac{dp}{dn} = \frac{w}{g} \frac{v^2}{r}.$$

But by equation (4), Art. 4, since z is now constant,

$$\frac{dH}{dn} = \frac{1}{w} \cdot \frac{dp}{dn} + \frac{v}{g} \cdot \frac{dv}{dn} = \frac{v}{g} \left(\frac{v}{r} + \frac{dv}{dn} \right) = \frac{2v}{g} \cdot \frac{1}{2} \left(\frac{v}{r} + \frac{dv}{dn} \right).$$

The expression $\frac{1}{2} \left(\frac{v}{r} + \frac{dv}{dn} \right)$ is designated the average angular velocity, or *the rotation of the fluid*.

Again, if the stream-line is horizontal and is also circular, $dn = dr$, and

$$\frac{dp}{dr} = \frac{w v^2}{g r},$$

a differential equation connecting the pressure and the velocity. If v is a known function of r , the pressure can be at once determined.

7. Whirling Fluids.—Let a fluid mass whirl like a rigid body about a vertical axis YY , with an angular velocity ω .

Consider the relative equilibrium of an element of mass m .

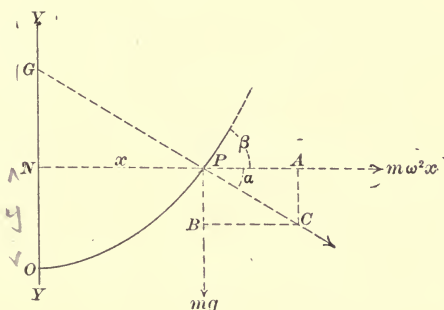


FIG. 6.

at P distant x horizontally from the axis and y vertically from the origin O in YY .

Take PA horizontally to represent the centrifugal force $m\omega^2 x$, and PB vertically to represent the weight mg . The remaining forces must be equal and opposite to the resultant of these two forces, viz., the diagonal PC of the parallelogram AB . The magnitude of this resultant is

$$PC = \sqrt{(mg)^2 + (m\omega^2 x)^2} = mg \sqrt{1 + \frac{\omega^4 x^2}{g^2}}.$$

sarily have a resultant normal to the free surface at P . Drawing the horizontal PN and the normal PG to meet the axis of rotation in N and G , PNG is evidently a triangle of forces, and therefore $\frac{NG}{PN} = \frac{mg}{m\omega^2 x} = \frac{NG}{x}$, and $NG = \frac{g}{\omega^2}$, a constant. Thus, the sub-normal is constant, and the free surface must be a paraboloid with its vertex at the point O where the free surface cuts the axis of rotation.

EX. 1. Deduce the law of pressure variation (*a*) for water in a vessel moving slowly towards a hole in the centre, the stream-lines being approximately horizontal circles and the velocity of any fluid particle inversely as its distance from the axis (*b*) for water rotating as a rigid body about an axis (as in a full centrifugal pump before delivery commences), the velocity of any fluid particle being directly proportional to its distance from the axis.

(*a*) Take $v = \frac{a}{r}$, then

$$\frac{1}{w} \frac{dp}{dr} = \frac{1}{g} \frac{v^2}{r} = \frac{1}{g} \frac{a^2}{r^3}.$$

Therefore
$$\frac{p}{w} = c - \frac{1}{2g} \frac{a^2}{r^2} = c - \frac{v^2}{2g}.$$

(*b*) Take $v = br$, then

$$\frac{1}{w} \frac{dp}{dr} = \frac{1}{g} \frac{v^2}{r} = \frac{1}{g} b^2 r.$$

Therefore
$$\frac{p}{w} = c' + \frac{1}{2g} b^2 r^2 = c' + \frac{v^2}{2g}.$$

EX. 2. A cylindrical vessel, 10 ft. in height and 1 ft. in diam., is half full of water. Find the number of revolutions per minute which the vessel must make so that the water may just reach the top, the axis of revolution being coincident with (*a*) the axis of the vessel, (*b*) a generating line.

(*a*) The free surface of the water is the paraboloid POP , Fig. 8, with its vertex at O , since the vol. of the paraboloid

$$= \frac{1}{2} \text{ vol. of circumscribing cylinder,}$$

$$= \text{vol. of water in vessel.}$$

Then
$$\frac{g}{\omega^2} = NG = \frac{\text{latus rectum}}{2} = \frac{1}{2} \frac{PN^2}{ON} = \frac{1}{80},$$

and

$$\omega = \sqrt{32 \times 80} = 16\sqrt{10}.$$

The linear speed of the rim at $P = r\omega = 8\sqrt{10}$,
 and the number of revols. per min. $= \frac{60 \times 8\sqrt{10}}{\frac{2\pi}{4} \times 1} = 482.96$.



FIG. 8.

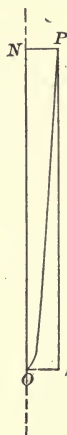


FIG. 9.

(b) The free surface is now the paraboloid OP , with its vertex at O ,
 Fig. 9.

Then
$$\frac{g}{\omega^2} = \frac{1}{2} \frac{PN^2}{ON} = \frac{1}{20},$$

and
$$\omega = \sqrt{640} = 8\sqrt{10}.$$

Thus the number of revols. per min. $= \frac{60 \times 8\sqrt{10}}{2 \cdot \frac{2\pi}{4} \cdot 1} = 241.48.$

8. Orifice in a Thin Plate.—If an opening is made in the wall or bottom of a tank containing water, the fluid particles immediately move towards the opening, and arrive there with a velocity depending upon its depth below the free surface. The opening is termed an “orifice in a thin plate,” when the water springs clear from the inner edge, and escapes without again touching the sides of the orifice. This occurs when the bounding surface is changed to a *sharp edge*, as in Fig. 10, and also when the ratio of the thickness of the bounding surface to the least transverse dimension of the orifice, does not

exceed a certain amount which is usually fixed at unity, as in Figs. 11 and 12.

Owing to the inertia acquired by the fluid filaments, there will be no sudden change in their direction at the edge of the orifice, and they will continue to converge to a point a little in front of the orifice, where the jet is observed to contract to the smallest section. This portion of the jet is called the *vena*

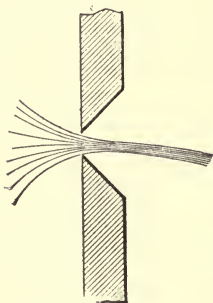


FIG. 10.

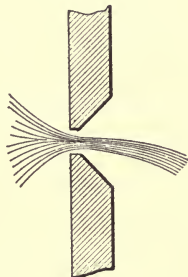


FIG. 11.

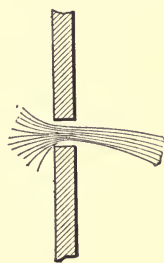


FIG. 12.

contracta, or contracted vein, and the fluid filaments flow across the minimum section in sensibly parallel lines, so that here, if the motion is steady, Bernoulli's theorem is applicable.

The dimensions of the contracted section and its distance from the orifice depend upon the form and dimensions of the orifice and upon the head of water over the orifice.

Let Fig. 13 represent the portion of the jet between a circular orifice of diameter AB and the contracted section of diameter CD , EF being the distance between AB and CD . Then, taking the *average* results of a number of observations, it is found that AB , CD and EF are in the ratios of 100 to 80 to 50.

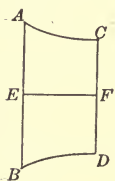


FIG. 13.

Thus the areas of the contracted section and of the orifice are in the ratio of 16 to 25, and, generally speaking, this is assumed to be the ratio whatever may be the form of the orifice.

9. Torricelli's Theorem.—Let Fig. 14 represent a jet issuing from a thin-plate orifice in the side of a vessel containing water kept at a constant level AB .

Let XX be the datum line, MN the contracted section, and consider any stream-line mn , m being in a region where the

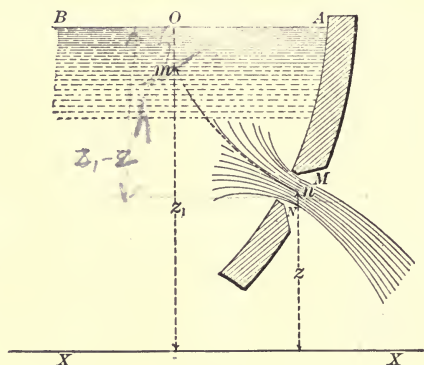


FIG. 14.

velocity is sensibly zero, and n in the contracted section. Then, by Bernoulli's theorem, the motion being steady,

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z + \frac{p}{w} + \frac{v^2}{2g}, \quad \dots \quad (1)$$

p , p_1 being the pressures at n and m , and z , z_1 their elevations above datum. Hence

$$\frac{v^2}{2g} = z_1 - z + \frac{p_1 - p}{w}. \quad \dots \quad (2)$$

If the flow is into the atmosphere,

p = the atmospheric pressure = p_0 , and

$p_1 = w \cdot Om + p_0$, $z_1 = h$

O being the point in which the vertical through m intersects the free surface. Thus

$$\frac{v^2}{2g} = z_1 - z + Om = h, \quad \dots \quad (3)$$

h being the depth of n below the free surface.

The result given by equation (3) was first deduced by Torricelli.

The depth below the free surface is very nearly the same for all points of the contracted vein, and the value of v as given by (3) is taken to be the theoretical mean velocity of flow across the contracted section.

Equation (3) is equivalent to the statement that when the orifice is opened, the hydrostatic energy of the water, viz., h ft.-lbs. per pound, is converted into the kinetic energy of $\frac{v^2}{2g}$ ft.-lbs. per pound. Thus, if the jet is directed vertically upwards, it will very nearly rise to the level of the free surface, and would reach that level were it not for air resistance, or for viscosity, or for friction against the sides of the orifice, or for a combination of these retarding causes.

If the jet issues in any other direction, it describes a parabolic arc of which the directrix lies in the free surface.

Let OTV , Fig. 15, be such a jet, its direction at the orifice at O making an angle α with the vertical.

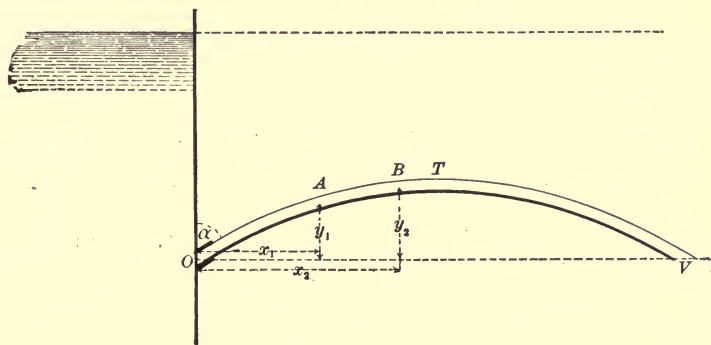


FIG. 15.

formed orifice a greater or less length of the jet will have the appearance of a glass rod, and if this portion were suddenly solidified and supported at the ends, it would stand as an arch without any shearing stress in normal sections

Again, the horizontal component of the velocity of flow at any point of the jet is constant ($= v \sin \alpha$), so that, for the unbroken portion of the jet, equidistant vertical planes will intercept equal amounts of water, and the height of the C. G. of the jet above the horizontal line OV , will be two thirds of the height of the jet.

10. Efflux through an Orifice in the Bottom or in the Side of a Vessel in Motion.—If a vessel containing water z ft. deep ascend or descend vertically with an acceleration f , the pressure p at the bottom is given by the equation

$$\pm \frac{w}{g}zf = p - p_0 - wz,$$

p_0 being the atmospheric pressure. Therefore

$$\frac{p - p_0}{w} = z \left(1 \pm \frac{f}{g} \right).$$

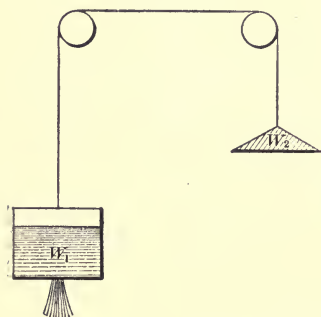


FIG. 16.

Before an orifice is opened, if the heavier vessel is reduced to rest by applying an upward acceleration f , the pressure at the depth z is changed from wz to $wz \left(1 + \frac{f}{g} \right)$, while in the other vessel it would be changed from wz to $wz \left(1 - \frac{f}{g} \right)$.

If now an orifice is opened at the bottom, the velocity of efflux v is still taken as being due to the head of the pressure p , and therefore by Torricelli's Theorem

$$\frac{v^2}{2g} = z \left(1 \pm \frac{f}{g} \right).$$

Let W_1 be the weight of the vessel and water, and let the vessel be connected with a counterpoise of weight W_2 by

means of a rope, passing over a pulley. Then by Newton's second law of motion, and neglecting pulley friction,

$$\frac{f}{g} = \frac{\pm T \mp W_1}{W_1} = \frac{\pm W_2 \mp T}{W_2} = \frac{\pm W_2 \mp W_1}{W_2 \mp W_1},$$

T being the tension of the rope. Therefore, also, $T = \frac{2W_1W_2}{W_1 + W_2}$.

Next let a cylindrical vessel, Fig. 17, of radius r and containing water, rotate with an angular velocity ω about its axis, Art. 7. The surface of the water assumes the form of a paraboloid with its vertex at O and its latus rectum equal to $\frac{2g}{\omega^2}$. If an orifice is made at Q

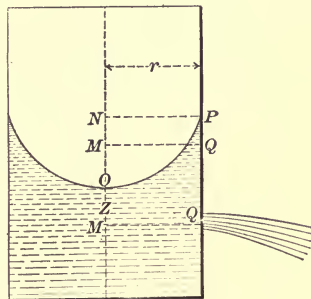


FIG. 17.

in the side of the vessel, at a vertical distance z from O , the water will flow out with a velocity v due to the head of pressure at the orifice. This head is PQ , and

$$PQ = ON \pm \varepsilon = \frac{\omega^2 r^2}{2g} \pm \varepsilon,$$

the sign being *plus* or *minus*, according as the orifice is below or above O . Hence, by Torricelli's theorem,

$$\frac{v^2}{2g} = \frac{\omega^2 r^2}{2g} \pm z,$$

or

$$v^2 = \omega^2 r^2 \pm 2gz.$$

11. Application to the Flow through a Frictionless Pipe of Gradually Changing Section (Fig. 18).—Let the pipe be supplied from a mass of water of which the free surface is H ft. above datum.

Let a_1 , p_1 , v_1 be the sectional area, pressure, and velocity of flow at any point A , z_1 ft. above datum and h_1 ft. below the free surface.

Let a_2 , p_2 , v_2 be similar symbols for a second point B , z_2 ft. above datum and h_2 ft. below the free surface.

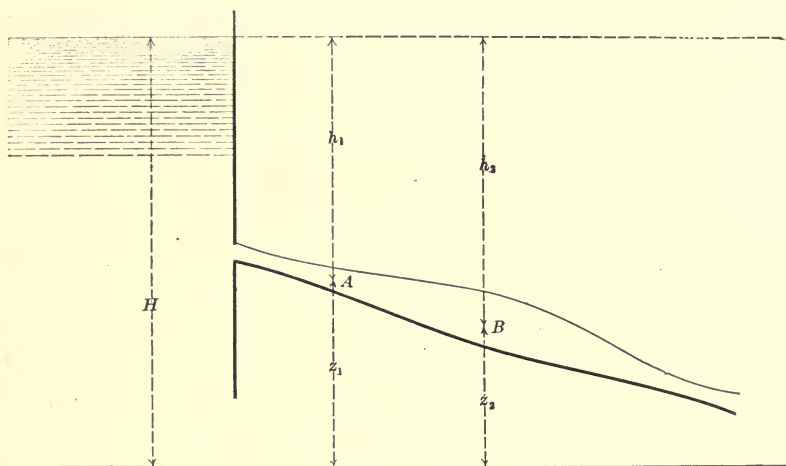


FIG. 18.

Then by the condition of continuity,

$$a_1 v_1 = a_2 v_2,$$

and by Torricelli's theorem,

$$\frac{v_1^2}{2g} = h_1 + \frac{p_0 - p_1}{w},$$

and

$$\frac{v_2^2}{2g} = h_2 + \frac{p_0 - p_2}{w}.$$

Hence

$$\begin{aligned} \frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 &= z_1 + h_1 + \frac{p_0}{w} = H + \frac{p_0}{w} \\ &= z_2 + h_2 + \frac{p_0}{w} = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2, \end{aligned}$$

so that Bernoulli's theorem, viz.,

$$\frac{v^2}{2g} + \frac{p}{w} + z = H + \frac{p_0}{w} = \text{a constant},$$

holds true for the assumed conditions.

12. Hydraulic Coefficients.—These are coefficients introduced to correct the discrepancies between the results deduced by theoretical considerations and the actual results of practice.

Numerous experiments have been made to determine the values of these coefficients, and with the same object in view, special apparatus has been designed and installed in the hydraulic laboratory of McGill University. A main feature of this apparatus is a cast-iron tank, square in section, 28 ft. in height, and having a sectional area of 25 sq. ft. Care has been taken to make the inside surfaces of the tank perfectly flush, and to this end the flanges, by which the several sections are bolted together, are placed on the outside.

The valve, Fig. 19, in the side of the tank is a gun-metal disc $\frac{1}{4}$ in. in thickness and 24 ins. in diameter, fitted into a recess of the same di-

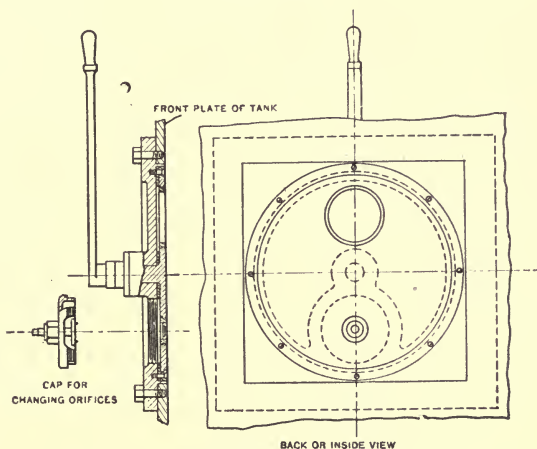


FIG. 19.

mensions in a cast-iron cover-plate, with gun-metal bearing faces, forming a water-tight joint for the face of the disc. This cover-plate is bolted to an opening in the front of the tank, and the inner faces of the cover-plate and disc are flush with the inner surface of the tank.

In the disc, and on opposite sides of the centre, there are two screwed openings, the one 3 ins. and the other 7 ins. in diameter. By rotating the disc each opening can be made concentric with a screwed $7\frac{1}{2}$ -in. opening in the body of the valve. The disc is rotated by means of a spindle through its centre, passing through a gland in the front of the valve body, and operated by a lever on the outside. Gun-metal bushes, with the required orifices, are screwed into the disc openings, and when screwed home have their inner surfaces flush with the valve surface, and therefore with the inside surface of the tank. By means of a simple device, these bushes can be readily removed and replaced by others without the loss of more than a pint of water. A cap with a central gland is screwed into the $7\frac{1}{2}$ -in. opening of the valve body and forms a practically water-tight cover. The valve is rotated so as to bring the bush opposite the opening, and it is then unscrewed by means of a special key projecting through the cap-gland. The valve is now turned back until the opening is closed, when the cap is unscrewed, the bush taken out, and another put in its place. The cap is again screwed into position, the valve rotated until the openings in the disc and tank-side are concentric, when the bush is screwed home by the key.

A gun-metal bush screwed into each of the two openings in the main disc, carries a series of orifice plates. The larger bush takes plates with openings up to 4 ins. in diameter, and the smaller bush takes plates with openings up to $1\frac{1}{4}$ ins. in diameter. The plates are provided with a shouldered edge, which fits against the corresponding rim of the bush, and are screwed with the orifice in any required position by means of an annular screwed ring fitting the interior surface of the bushing. The orifice plates are gun-metal discs, $4\frac{1}{2}$ ins. in diameter by $\frac{1}{4}$ in. thick for the large bush, and 2 ins. in diameter by $\frac{1}{8}$ in. thick for the small bush.

The utmost care has been taken to form the orifices with the greatest possible accuracy. The orifices are worked in the discs approximately to the sizes required, and are then stamped out with hardened-steel punches, the sizes of which have been determined with great exactness by means of Brown & Sharpe micrometers. The diameters of the orifices are also checked by a Rogers' comparator and a standard scale. In some cases a discrepancy has been found between the sizes of the die and its orifice, but the size obtained for the orifice is the one which has been invariably used in the calculations.

(a) *Coefficients of Velocity*.—The actual velocity v at the vena contracta is a little less than $\sqrt{2gh}$, the theoretical velocity (Art. 9), and the ratio of v to $\sqrt{2gh}$ is called the coefficient of velocity. Denoting this coefficient by c_v , then,

$$v^2 = c_v^2 \cdot 2gh,$$

and the equations for the velocities of discharge in the case of moving vessels (Art. 10) become

$$v^2 = c_v^2 \cdot 2(g \pm f)h$$

and

$$v^2 = c_v^2(w^2r^2 \pm 2gz).$$

A *mean* value of c_v for well formed simple orifices is .974. Assuming that the face of the orifice is vertical and that the jet issues horizontally with a velocity of v ft. per second, under a head of h ft. of water, and assuming also that in t secs., a

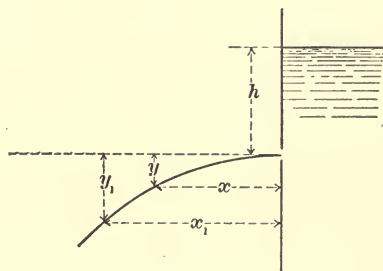


FIG. 20.

fluid particle reaches a point y ft. measured vertically and x ft. measured horizontally from the point of discharge, then, disregarding the effect of air resistance and other disturbing causes,

$$x = vt,$$

$$y = \frac{1}{2}gt^2,$$

and therefore

$$\frac{x^2}{y} = \frac{2v^2}{g} = \frac{2}{g}c_v^2 2gh = 4c_v^2 h,$$

or

$$c_v^2 = \frac{x^2}{4hy}.$$

If x_1 , y_1 are the coordinates of the fluid particle in any other position, then, also,

$$c_v^2 = \frac{x_1^2}{4hy}.$$

Hence

$$c_v^2 = \frac{x_1^2 - x^2}{4h(y_1 - y)},$$

which is the formula used in the McGill laboratory in the experimental determination of coefficients of velocity. The position of the jet is defined by vertical measurements from a straight-edge, supported horizontally above the jet, by a bracket on the tank face at one end, and at the other on a bearing, which admits of a vertical adjustment, Fig. 21.

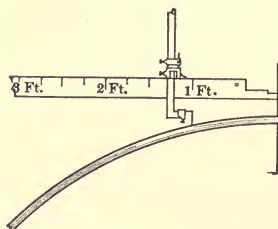


FIG. 21.

The straight-edge is of machinery-steel, is $5\frac{1}{2}$ ft. in length, $2\frac{1}{2}$ ins. in depth, $\frac{3}{8}$ in. in width, and is graduated so as to give the horizontal distances from the inner face of the orifice plate. The vertical ordinates are measured by a Vernier caliper specially adapted for the purpose. The flat face of the movable limb is made to rest upon the upper surface of the straight-edge, and the caliper-arm hangs vertically. A bent piece of wire, with a needle-point, is clamped to the other limb, and,

by means of the screw adjustment, can be readily moved until it just touches the upper or lower surface of the jet.

By means of the above method, an extended series of experiments with $\frac{1}{4}$ -in., $\frac{1}{2}$ -in., and 1-in. sharp-edge orifices, and under heads varying from 6 to 20 ft., gave .99 as the average value of the coefficient of velocity (c_v).

Let the direction of the jet, Fig. 22, at the point of discharge make an angle α with the horizontal, and let $x_1, y_1,$

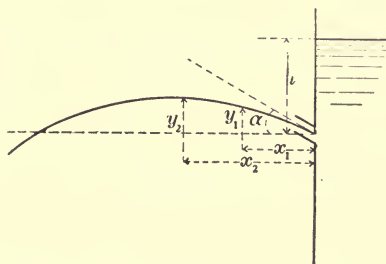


FIG. 22.

$x_2, y_2,$ be the coordinates defining the position of a fluid particle after intervals of t_1 secs. and t_2 secs. Then

$$x_1 = v \cos \alpha \cdot t_1 \quad \text{and} \quad y_1 = v \sin \alpha \cdot t_1 - \frac{1}{2}gt_1^2,$$

$$x_2 = v \cos \alpha \cdot t_2 \quad \text{and} \quad y_2 = v \sin \alpha \cdot t_2 - \frac{1}{2}gt_2^2.$$

These equations give

$$\tan \alpha = \frac{x_2^2 y_1 - x_1^2 y_2}{x_1 x_2 (x_2 - x_1)}$$

and

$$\frac{x_1^2 \sec^2 \alpha}{x_1 \tan \alpha - y_1} = \frac{2v^2}{g} = 4h \cdot c_v^2 = \frac{x_2^2 \sec^2 \alpha}{x_2 \tan \alpha - y_2},$$

from which α and then c_v can be calculated.

(b) *Coefficient of Resistance*.—Let h_v be the head required to produce the velocity v . Let h_r be the head required to overcome the frictional resistance. Then

$$h, \text{ the total head, } = h_v + h_r = h_v(1 + c_r),$$

where $h_r = c_r h_v$.

c_r is termed the coefficient of resistance, and is approximately constant for varying heads with simple sharp-edged orifices. Again,

$$c_v^2 h = \frac{v^2}{2g} = h_v.$$

Hence

$$h = c_v^2 h(1 + c_r),$$

and therefore

$$\frac{1}{c_v^2} = 1 + c_r;$$

so that c_r can be found when c_v is known, and *vice versa*.

(c) *Coefficient of Contraction*.—The ratio of the area a of the vena contracta to the area A of the orifice is called the coefficient of contraction, and may be denoted by c_c .

The value of c_c must be determined in each case, but in sharp-edged orifices an average value of c_c , as already pointed out, is $\frac{16}{25} = .64$. *Cæteris paribus*, c_c increases as the orifice area and the head diminish.

The following are some of the conditions which tend to modify the value of c_c :

(1) The contraction is *imperfect* and will be suppressed over the lower edge of a square orifice at the bottom of a vessel, and over a side as well if the orifice is in a corner. In fact, the contraction is more or less imperfect for any orifice

within three diameters from the side or bottom of the vessel. Thus, the cross-section of the vena contracta is increased, and experiment shows that the discharge is also increased.

(2) c_c is increased or diminished according as the surface surrounding the orifice is convex or concave to the interior of the vessel.

FIG. 23.

(3) The contraction is imperfect and c_c is increased, if the orifice is placed in a confined part of the vessel, or if the water approaches the orifice through a channel, as in Fig. 23, the velocity of the fluid filaments being thereby considerably increased.

(4) If the inner edge of an orifice is rounded, as shown by Figs. 24 and 25, the contraction is more or less imperfect.



FIG. 24.



FIG. 25.

The value of c_c varies from .64 for a sharp-edged orifice to very nearly *unity* for a perfectly rounded orifice.

(5) The contraction is *incomplete* when a border or rim is placed round a part of the edge of the orifice, projecting inwards or outwards. According to Bidone,

$$c_c = .62 \left(1 + .152 \frac{n}{p} \right) \text{ for rectangular orifices,}$$

and

$$c_c = .62 \left(1 + .128 \frac{n}{p} \right) \text{ for circular orifices,}$$

n being the length of the edge of the orifice over which the border extends, and p the perimeter of the orifice.

(6) If the sides of the orifice are curved so as to form a bell-mouth of the proportions shown by Fig. 26, and corre-

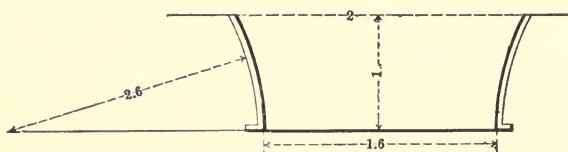


FIG. 26.

sponding approximately to the shape of the vena contracta, the whole of the contraction will take place within the bell-mouth, and c_c is unity if the area of the orifice is taken to be the area of the smaller end.

For such an orifice Weisbach gives the following table of values of c_v :

Head over Orifice in Feet.	c_v .
.66.....	.959
1.64.....	.967
11.48.....	.975
55.77.....	.994
337.93.....	.994

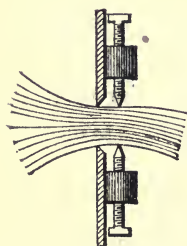


FIG. 27.

The dimensions of the jet at the contracted section or at any other point, may be directly measured by means of set-screws of fine pitch, arranged as in Fig. 27. The screws are adjusted so as to touch the surface of the jet, and the distance between the screw-points is then measured.

Measurements of very great accuracy can be made with the jet-measurer, Fig. 28, designed and constructed in the McGill laboratories which may be described as follows—One end of a horizontal 2-in. bar

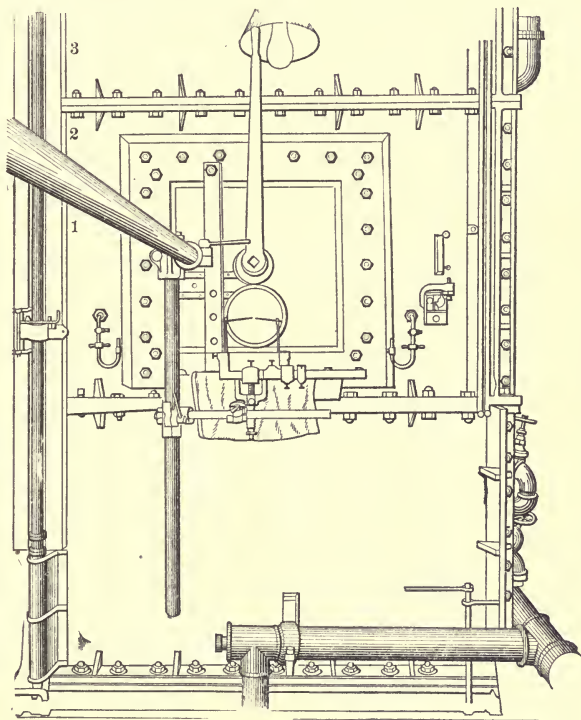


FIG. 28.

is attached to the front of the tank (Fig. 28) and the other is supported on a frame bolted to the sides of the flume. A split sleeve slides along this bar and may be clamped by a tightening screw in any desired position. Upon the cross-head there is another split boss, or sleeve, through which a second bar passes at right angles to the first, and carries a similar cross-head to that on the 2-in. bar, so that provision is made for a rough adjustment in a vertical plane. Through the latter cross-head passes a smaller bar, and along this bar slides a third adjustable cross-head, or caliper-holder, by which the caliper can be swung round and receive its final adjustment. For the measurements a 12-in. Brown & Sharpe vernier caliper is used. A capstan head rod is clamped to each leg and can be swivelled through any angle. Steel needle-pointers are inserted in the heads, and are clamped in such position as may be required. In making a measurement the steel points are first made to

touch and the corresponding readings taken. The points are then separated by sliding the caliper-heads apart, and the whole apparatus is moved into position. The points are finally adjusted so as to touch the surfaces of the jet at opposite points, and readings are again taken. From the two sets of readings the transverse dimension of the jet can be at once determined, to the one-thousandth of an inch, and at any point between 72 ins. and $\frac{1}{8}$ in. from the inner surface of the orifice-plate. Rigidity in the apparatus is, however, most essential.

(d) *Coefficient of Discharge.*—If Q is the discharge in cubic feet per second across the contracted section, then

$$Q = av = c_c A c_v \sqrt{2gh} = cA \sqrt{2gh},$$

where $c = c_c c_v$, is the coefficient of discharge and is to be determined by experiment.

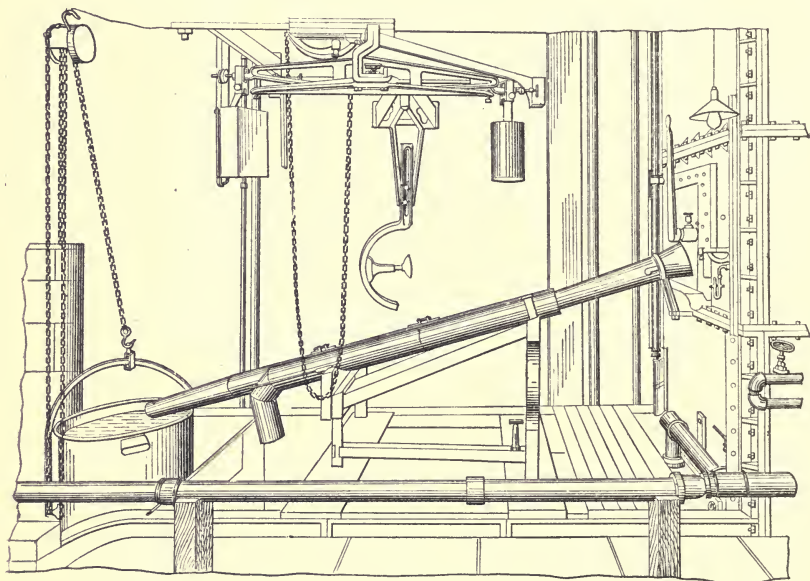


FIG. 29.

In the experiments made in the McGill laboratory, the water, on leaving the orifice, passes either to waste, or to the measuring tank through a bifurcated galvanized-iron tubing, supported in a pivoted frame, Fig. 29. The water is first run to waste through one of the branches until a steady head has been obtained, and the frame is then rapidly swung through a small angle by means of a lever, when the water passes through the other branch to the tank. As soon as the tank is suffi-

ciently full, the frame is swung back and the water again runs to waste. At first the water discharged from the tank was replaced by water admitted into the top of the tank through a hose terminating in a rose submerged just below the surface. Although the utmost care had been taken in the design of this rose to reduce the eddy motion at efflux to a minimum, there was always an appreciable disturbance. The hose was therefore extended until the rose rested on the bottom of the tank, 8 feet below the orifice; with this arrangement a series of orifice-flow experiments were made, the time in each case being the mean of that given by two stop-watches and the values of the coefficients of discharge are given in Tables A and B.

TABLE A.

TRIANGULAR ORIFICE OF .05 SQ. IN. AREA AND REMAINING ORIFICES OF .0625 SQ. IN. AREA.

Head in Feet.	Circular.		Equilateral Tri- angle with Horizontal Base Uppermost.		Square with Vertical Sides.		Rectangle with Vertical Sides Equal to Four Times the Width		Rectangle with Vertical Sides Equal to Sixteen Times the Width	
	T	S	T	S	T	S	T	S	T	S
1	.678	.620	.657	.631	.643	.627	.662	.640	.688	.671
2	.613	.613	.646	.623	.631	.621	.643	.629	.655	.657
4	.610	.605	.628	.616	.620	.615	.631	.620	.642	.643
6	.607	.601	.628	.613	.615	.612	.627	.616	.634	.636
8	.606	.601	.621	.610	.613	.609	.624	.613	.631	.632
10	.604	.600	.618	.608	.612	.608	.621	.613	.629	.629
12	.603	.598	.617	.607	.611	.607	.621	.611	.626	.627
14	.602	.598	.617	.607	.610	.606	.620	.610	.623	.625
16	.602	.598	.616	.606	.609	.606	.619	.609	.622	.625
18	.601	.597	.615	.605	.607	.605	.618	.608	.622	.623
20	.601	.597	.615	.605	.607	.604	.618	.608	.621	.622

The presence of the hose in the tank was not satisfactory, as it necessarily interfered with the stream-line motion, and therefore affected to a greater or less extent the values of the coefficient of discharge. The hose was discarded, and the water is now admitted into a 3-inch chamber extending right across the bottom of the tank and containing perforations on the lower surface through which the water flows to the bottom and is there deflected upwards. Twelve inches above the bottom the water is made to pass through a baffle-plate perforated with $\frac{3}{8}$ -in. holes, and 6 inches higher there is a second baffle-plate also perforated with $\frac{3}{8}$ -in. holes. In order to equalize as much as possible the flow from all points, the pitch of the holes in the upper plate was determined by the projections on a horizontal plane of equal distances on a sphere of 10 ft. diam. with its centre at the centre of the orifice of discharge.

There are two outlet pipes for fast and slow discharge, and there are two inflow pipes, the one 3 ins. and the other $1\frac{1}{2}$ ins. in diameter. Each of these pipes is controlled by a stop-valve.

TABLE B.
ORIFICES OF .197 SQ. IN. AREA.

Head in Feet.	Circular.		Equilateral Triangle with Horizontal Side Uppermost.		Square with Vertical Sides.		Square with Diagonal Vertical.		Rectangle with Vertical Sides Equal to Four Times the Width.		Rectang. with Vertical Sides Equal to One- quarter Width.	Rectang. with Vertical Sides Equal to Sixteen Times Width.	Rectang. with Vertical Sides Equal to One-six- teenth Width.
	T	S	T	S	T	S	S	T	S	S	S	S	S
1	.624	.618	.627	.627	.623	.628	.623	.635	.640	.641	.658	.659	
2	.616	.611	.620	.621	.613	.621	.619	.626	.633	.632	.646	.646	
4	.610	.607	.615	.615	.606	.617	.614	.619	.629	.629	.637	.637	
6	.607	.605	.613	.613	.604	.614	.612	.616	.625	.627	.634	.633	
8	.606	.604	.612	.612	.603	.612	.612	.614	.625	.625	.631	.631	
10	.606	.604	.611	.611	.602	.610	.611	.612	.624	.623	.630	.629	
12	.605	.603	.611	.611	.601	.610	.611	.611	.622	.622	.627	.626	
14	.604	.603	.610	.610	.600	.610	.609	.611	.622	.621	.624	.625	
16	.606	.602	.610	.610	.600	.610	.609	.610	.620	.621	.624	.624	
18	.605	.602	.610	.610	.600	.610	.609	.609	.620	.620	.623	.623	
20	.604	.601	.609	.609	.600	.610	.609	.602	.620	.620	.622	.622	

N.B.—In Tables A and B, T indicates a thickness of plate of .16-in., }
 and S indicates that the orifice is sharp-edged. }

The time is also measured electrically. In the forward and return movements, the lever, controlling the angular movement of the galvanized-iron tubing, makes and breaks an electric contact, so that the interval of time occupied by an experiment is registered on a chronograph.

With this new arrangement, the following values for the coefficient of discharge have been deduced for sharp-edged orifices, the area in each case being practically the same, viz., .19635 sq. ins., and equivalent to the area of a circle of $\frac{1}{2}$ in. diameter:—

Head in Feet.	Circular.	Square.		Rectangular Ratio of Sides 4 : 1.		Rectangular Ratio of Sides 16 : 1.		Triangular
		Sides Vertical.	Diagonals Vertical.	Long Side Vertical.	Long Side Horizontal.	Long Side Vertical.	Long Side Horizontal.	
1	.6199	.6267	.6276	.6419	.6430	.6633	.6644	.6359
2	.6131	.6204	.6277	.6335	.6355	.6503	.6510	.6280
4	.6081	.6162	.6177	.6281	.6293	.6409	.6415	.6228
6	.6073	.6137	.6156	.6255	.6266	.6368	.6372	.6202
8	.6056	.6127	.6138	.6234	.6252	.6342	.6346	.6189
10	.6050	.6116	.6132	.6224	.6240	.6323	.6327	.6183
12	.6040	.6109	.6123	.6217	.6230	.6311	.6314	.6177
14	.6038	.6104	.6118	.6207	.6222	.6304	.6304	.6176
16	.6032	.6099	.6113	.6203	.6215	.6301	.6298	.6171
18	.6031	.6096	.6110	.6200	.6212	.6299	.6293	.6163
20	.6029	.6094	.6108	.6198	.6210	.6291	.6285	.6160

At least two sets of measurements were made for each head, and the mean was adopted as correct, if the results did not differ by more than 3 in 10,000.

Numerous experiments with a 1-in. sharp-edged orifice give .6 as an average value of the coefficient of discharge for heads varying from 1 to 20 ft.

The jet springs clear from the orifice in all cases represented in the above tables, and the following inferences may be drawn from an inspection of the same:—

(1) The coefficient of discharge diminishes as the head increases, but at a diminishing rate.

(2) The coefficients for the thick-plate orifices are in all cases greater than the corresponding coefficients for sharp-edged orifices, excepting in the case of the longest rectangular orifice. Under a head of 1 ft. the coefficient of discharge for this orifice still exceeds that of the same orifice with a sharp edge, while for heads exceeding 1 ft. the coefficient seems to be a little less, but is practically the same. It may be noted that the thickness of the plate is 2.56 times the width of the orifice, and the contraction for the thick-plate orifice is consequently increased.

(3) The coefficient for rectangular orifices seems to be practically the same whether the longest side is vertical or horizontal.

(4) The coefficient increases with the area of the orifice, excepting when the head is very small. The coefficient for orifices of small area then rapidly increases.

(5) With rectangular orifices, the coefficient increases as the width of the orifice diminishes, i.e., as the orifice becomes more elongated.

The two last results are in accordance with similar results deduced by Weisbach, Buff, and others.

NOTE.—The manner in which the head of water in the tank is defined is both simple and effective. A glass gauge, of $1\frac{1}{2}$ in. diam., is fixed to the tank by iron brackets and extends from the top to the bottom. On one

side of the gauge there is a brass scale graduated from a zero point in the same horizontal plane as the centre of the orifice of discharge. A carrier, with a horizontal wire passing in front of the gauge, slides up and down, and any required head is obtained by bringing the necessary scale graduation, the surface of the water in the gauge, the wire and its reflection in a mirror at the back of the gauge, into the same horizontal plane. There is a second indicator on the opposite side of the tank, consisting of a float attached to an ordinary water-proof silk fishing-cord passing over a large light frictionless pulley and then vertically downwards in front of the tank. The cord is kept taut by a weight at the bottom, and carries a friction-tight pointer which can be easily and rapidly adjusted to indicate any required mark on a brass plate fixed in a convenient position on the tank face, so that the operator working the valves has it constantly under observation. As soon as the head of water in the tank has been determined by means of the glass gauge, the pointer is moved into position opposite the mark, and is kept there throughout the experiment. This obviates the necessity of constantly watching the level of the water in the gauge, which, on account of the height of the tank, is often very inconvenient and troublesome. Occasionally, however, it is advisable to check the position of the pointer by observing the water-level in the gauge, as the cord indicator is extremely sensitive, and the cord itself necessarily varies slightly in length, so that small errors might otherwise be introduced.

The head of water is brought to any required level by means of a three-way valve through which the water can either be admitted or allowed to escape. The valve is provided with a long vertical spindle, upon which handles are arranged at different points in such manner that one can be easily reached and operated from any position in the height of the tank. Close to the cord indicator and within the reach of the operator there is a small $\frac{1}{4}$ -in. pipe with valve, which is useful for a fine adjustment when the inflow is only slightly in excess of the discharge.

EX. 1. A vessel, 6 ft. in diar., is full of water and makes 100 revols. per min. Find the velocity of efflux through an orifice 2 ft. below the surface of the water at the centre, assuming the coefficient of velocity to be *unity*.

The linear velocity of the vessel's periphery

$$= 3\omega = 3 \cdot \frac{2\pi \cdot 100}{60} = \frac{220}{7} \text{ ft. per sec.}$$

rev = linear

Hence the velocity of efflux

$$= \sqrt{\left(\frac{220}{7}\right)^2 + 2 \cdot 32 \cdot 2}$$

$$= \sqrt{\frac{48,400}{49} + 128} = 33.4 \text{ ft. per sec.}$$

EX. 2. The area of an orifice in a thin plate was 36.3 cm.², the discharge under a head of 3.396 m. was found to be .01825 m.³ per sec., and the velocity of flow at the contracted section, as determined by measurements of the position of the axis of the jet, was 7.98 m. per second. Find the coefficients of velocity, discharge, contraction, and resistance, taking $g = 9.81$ m.

$$v = c_v \sqrt{2gh}.$$

Therefore, $7.98 = c_v \sqrt{2 \times 9.81 \times 3.396},$

and $c_v = .97729,$

$$Q = cA \sqrt{2gh}.$$

Therefore $.01825 = c \times \frac{36.3}{(100)^2} \sqrt{2 \times 9.81 \times 3.396},$

and $c = .6159,$

$$c = \frac{c}{c_v} = \frac{.6159}{.97729} = .632,$$

$$c_v = \frac{1}{c_v^2} - 1 = \left(\frac{1}{.97729} \right)^2 - 1 = .046.$$

EX. 3. The jet from an orifice of .008 sq. ft. area, under a head of 16 ft., issues horizontally and falls 1 ft. vertically in a horizontal range of 7.68 ft. Find the coefficient of velocity.

$$c_v^2 = \frac{(7.68)^2}{4 \times 1 \times 16} = .9216,$$

and $c_v = .96.$

EX. 4. If .625 is the coefficient of discharge in the preceding example, find the discharge in gallons per minute. The orifice is rectangular and is .2 ft. wide by .04 feet deep. Find the discharge when the contraction is suppressed over the lower edge by means of a projecting rim.

$$Q, \text{ in cub. ft. per sec.}, = .625 \times .008 \sqrt{2 \cdot 32 \cdot 16} = .16,$$

and therefore

$$\begin{aligned} \text{the discharge in gallons per minute} &= 60 \times .16 \times 6\frac{1}{2} \\ &= 60. \end{aligned}$$

When the contraction is suppressed over the lower edge,

$$\text{the coeff. of contraction} = .62 \left(1 + .152 \frac{.2}{2(.2 + .04)} \right) = \frac{1.9778}{3}.$$

Therefore

$$\text{the coeff. of discharge} = .96 \times \frac{1.9778}{3} = .632896.$$

Hence the discharge in cubic feet per second

$$\begin{aligned} &= .632896 \times .008 \times \sqrt{2.32.16} = .162 \\ &= 60,758 \text{ gallons per minute.} \end{aligned}$$

13. Miner's Inch. (*Tr. Can. Soc. C. E.*, 1900).—The miner's inch of water is an arbitrary module adopted in mining districts for selling water. It is variously defined as being the amount of water discharged per minute by an orifice 1 in. square, or an equivalent fraction of a larger orifice, with a head of from 6 to 9 ins., the thickness of the orifice being usually 2 inches.

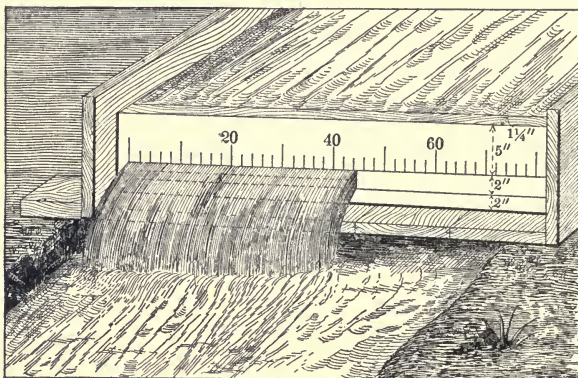


FIG. 30.

One great difficulty is that this is a variable quantity depending upon the specified head, and therefore all such modules should also define the flow in cubic feet per minute.

There are many practical difficulties in the way of delivering absolutely exact quantities of water, but the definition of the module or unit should be correct within a reasonable limit of error. If it is a definition of a single miner's inch from an orifice of 1 sq. in., it should go no further; but if the inch is defined as being some fractional part of the discharge from a

larger orifice, it should be limited to the capacity of that orifice. Further, as it is a term of local signification only, the discharge should be given in cubic feet per minute, convenient discharges being $1\frac{1}{2}$ and 2 cu. ft. The flow under low heads is irregular. Heads of 1 ft. or more are not suitable, because the water is delivered from ditches or flumes in which the depth is never great.

The question thus resolves itself into a choice of a standard module or unit from a flow under one of two conditions, viz.:

(1) With a low head of $6\frac{1}{4}$ ins. above the centre of the orifice giving a discharge of $1\frac{1}{2}$ cu. ft. per minute, with the advantage that it is already practically recognized as the miner's inch, and with the disadvantage that the flow is irregular.

(2) With a head of $11\frac{1}{2}$ ins. above the centre of the orifice, and a discharge of 2 cu. ft. per minute, the flow being much more regular, but the quantity discharged is not recognized in practice.

The flow under the first condition is chosen as being the one now in use in British Columbia, and the following specification is given of the miner's inch, including discharges of from 1 to 100 miner's inches of $1\frac{1}{2}$ cu. ft. per minute:—

The water taken into a ditch or sluice shall be measured at the ditch or sluice head. It shall be taken from the main ditch, flume, or canal, through a box or reservoir arranged at the side, and the water shall have no appreciable velocity of approach. The orifice shall be fixed vertically at right angles to the delivering waterway, and the edges and corners shall be square and sharp, the top, bottom, and sides of the orifice being at right angles to the pressure-board. The issuing vein shall be fully contracted, and the discharge shall pass freely into the air. The distance between the sides and bottom of the orifice and the sides and bottom of the waterway shall be at least three (3) times the least dimension of the orifice. The miner's inch of water shall mean $\frac{1}{12}$ of the quantity which shall

be discharged through an orifice six (6) ins. wide and two (2) ins. high, made of 2-in. planks, planed, made smooth and painted. The water shall have a constant head of $6\frac{1}{4}$ ins. above the centre of the orifice, and the amount discharged shall be estimated at $1\frac{1}{2}$ cu. ft. per minute.

Discharges up to and including 101.55 miner's inches of $1\frac{1}{2}$ cu. ft. of water per minute shall be as in the following table:

Dimensions of Orifice in Inches.		Head in Inches over Centre of Orifice.	Number of Miner's Inches of $1\frac{1}{2}$ Cubic Feet per Minute.
Width.	Depth.		
6	2	6.25	11.9858
12	2	6.25	24.2485
18	2	6.25	36.3851
24	2	6.25	48.6865
4	4	6.25	15.6998
6	4	6.25	23.5560
12	4	6.25	47.2853
18	4	6.25	71.6296
$25\frac{1}{2}$	4	6.25	101.5495

T. Drummond, B.A.Sc., has made an interesting series of experiments (Trans. Can. Soc. C. E., vol. XIV, 1900) on the Miner's Inch, in the Hydraulic Laboratory, McGill University.

The discharges recorded were made under low heads of from 6 to 12 ins., and with two kinds of orifices, viz.:

(1) Standard sharp-edged rectangular orifices in brass from 1 to 4 sq. ins. in area.

(2) Square-edged rectangular orifices in wood, 2 ins. thick, 2 to 4 ins. in height, and $\frac{1}{2}$ to 24 ins. in width.

The formula adopted for the discharge was

$$Q = \frac{2}{3}CB\sqrt{2g}(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}) \text{ (see Article 22),}$$

in which C is the coefficient of discharge;

g is 32.176;

Q is the discharge in cubic feet per second;

B is the width of the orifice;

H_1 and H_2 the heads over the top and bottom of the orifice.

No corrections were made for changes in temperature.

The shape of the orifice has a very sensible effect upon the discharge. Circular orifices give the least discharge, the greatest discharges occur with rectangular orifices, while the discharges with square orifices are intermediate. The coefficient of discharge (C) diminishes as the size of the orifice increases, the same form of orifice being maintained. For the same orifice C diminishes as the head increases. In rectangular orifices of constant depth the coefficient of discharge increases with the width. If the width remains constant, the coefficient increases as the depth diminishes.

These experiments illustrate a curious point, namely, that various small orifices, 2 ins. thick (made in a 2-in. plank), run full like a short tube, and these orifices therefore discharge more water than they theoretically should if the vein were contracted. The $\frac{1}{2}$ -in. \times 2-in., 1-in. \times 2-in., and 2-in. \times 2-in. orifices run full under these conditions, as also does the 1-in. \times 1-in. orifice.

The 1-in. \times 2-in. orifice, 2 ins. thick, is just on the margin between flow with contraction and full-bore flow. If it is fixed in the vertical position, with the longest diameter vertical, the vein contracts. If it is fixed in the horizontal position, with the longest diameter horizontal, it will also contract, but if it is rubbed with the fingers on the edge, it will run full for a time and then contract again. If kept running full in this way, it will discharge about 1 cu. ft. of water per minute more than when full contraction takes place.

The 2-in. \times 2-in. orifice runs partly full, that is to say, the lower half of the orifice, where the issuing vein curves down, runs full, while the upper half contracts. This largely increases both the discharge and the coefficient of discharge, but the flow becomes irregular and it is therefore practically impossible to measure a simple miner's inch. For this reason $\frac{1}{12}$ of the flow from the 6-in. \times 2-in. orifice was chosen as the standard for the unit miner's inch, and this miner's inch actually discharges 1.4982 cu. ft. per minute.

14. Inversion of the Jet.—The phenomenon of the inversion of the jet was first noticed by Bidone, and has been subsequently investigated by Poncelet, Lesbros, Magnus, Lord Rayleigh, the author, and others.

Sectional Elevation.

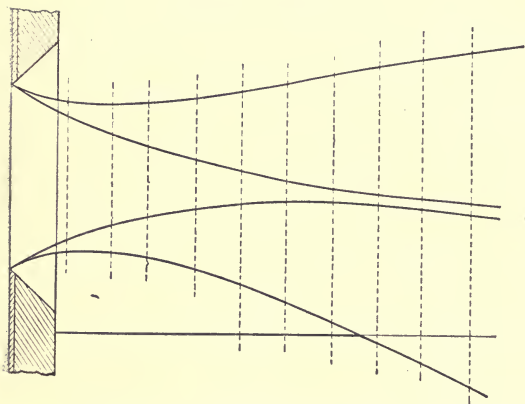


FIG. 31.

Cross-section.

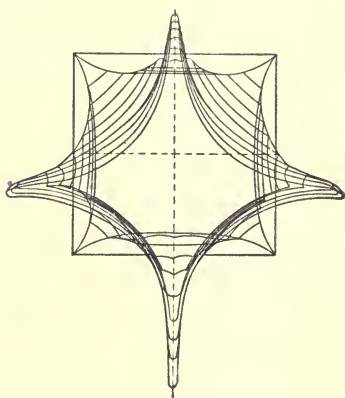


FIG. 32.

When a jet issues from an orifice in a vertical surface, the sections of the jet at points along its path assume singular forms dependent upon the nature of the orifice.

With a square or rectangular orifice the section of the jet is a star of four sheets at right angles to the sides, Figs. 31, 32, 33.

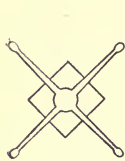


FIG. 33.

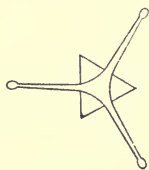


FIG. 34.

With a triangular orifice the section is a star of three sheets at right angles to the sides, Fig. 34.

In general, with a polygonal orifice of n sides, the section is a star of n sheets at right angles to the sides.

These jets from non-circular orifices have central cores, and the sheets at the edges are thickened out into beads, Figs. 33 and 34, which are approximately elliptical in section with major diameters double the minor diameters. Many exact measurements of these jets have been made and are partially described in a paper by Farmer and Strickland in the *Trans. Can. Roy. Soc.*, vol. IV. sec. 3.

With a semicircular orifice the section has a more or less semicircular boundary and a single sheet at right angles to the diameter.

The common explanation of this phenomenon is that the fluid particles issuing along different parabolic stream-lines impinge upon each other, and by their mutual reactions cause the jet to spread out and assume sectional forms depending upon the shape of the orifice.

Thus the fluid particles issuing horizontally and freely at *B*, with a velocity $\sqrt{2gAB}$, describe a parabola *BD*. The particle issuing at *C* with a velocity $\sqrt{2gAC}$ describe a parabola *CD* of less curvature than *BD*. The particles cannot pass simultaneously through the point *D* and must necessarily press upon each other. They are therefore compelled to move out of their natural paths, and the jet spreads into sheets.

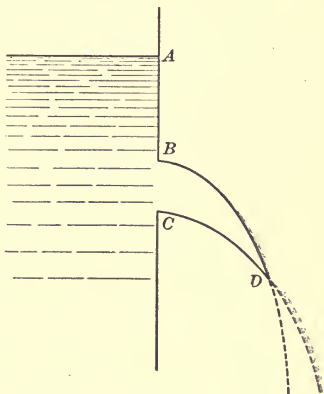


FIG. 35.

A theory which seems more fully to account for the whole of the facts is that the peculiar changes in form are really due to surface tension and to the differences between the atmospheric pressure and the internal pressure of the jet.

In the case, for example, of a jet flowing through an elliptical orifice with the major axis vertical, the stream-lines

in the vein are convergent and mutually react upon each other, causing the jet to contract vertically and elongate horizontally at a rate gradually increasing to a maximum, when the section is a circle in form.

At this stage the rates of elongation and contraction are the same. The elongation and contraction still continue, but at a diminishing rate, until the movement is stopped by the effect of surface tension, when the section is again elliptical, with the major axis horizontal and the minor axis vertical. The new major and minor axes then again begin respectively to contract and to elongate, the section of the jet passing through the circular form to its initial elliptical form.

This process is repeated over the whole length of the unbroken jet, and, in fact, in this portion of the jet the surface tension produces an effect similar to that which would be produced if the jet were surrounded by an elastic envelope.

If the orifice is small and the head not large, the jet, on leaving the contracted section at the orifice, spreads out into sheets and then diminishes to a contracted section similar to the first, after which it again spreads out into sheets, bisecting the angles between the first set of sheets, and again diminishes to a contracted section. This action is repeated so long as the jet remains unbroken. A comparatively few experiments made in the laboratory indicate that if the head h is not large,

$$\text{the wave-length} \propto \sqrt{h} \propto v.$$

15. Emptying and Filling a Canal Lock.—When the head varies, as in filling or emptying a reservoir or a lock, in filling a vessel by means of an orifice under water, or in emptying water out of a vessel through a spout, Torricelli's theorem is still employed.

If the lock or vessel is to be filled, Fig. 36, let X sq. ft. be the area of the water-surface when it is x ft. below the surface of the outside water.

If the lock or vessel is to be emptied, Fig. 37, then X sq. ft. is the area of the water-surface when it is x ft. above the orifice.

In each case x ft. is the effective head over the orifice, and is the head under which the flow takes place.

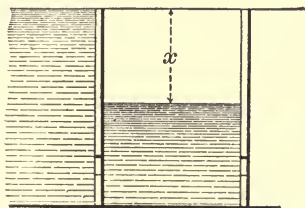


FIG. 36.

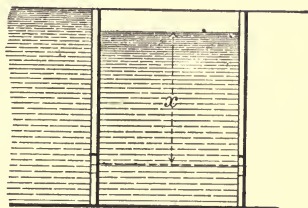


FIG. 37.

In the time dt the water-surface in the lock or vessel will rise or fall by an amount dx . Then

$$\begin{aligned} -X \cdot dx &= \text{quantity which has entered the lock} \\ &= cA \sqrt{2gx} \cdot dt, \end{aligned}$$

A being the area of the orifice.

Hence

$$t = \int_x^h \frac{Xdx}{cA \sqrt{2gx}},$$

an equation giving the time of filling or emptying the lock between the level x and h . The value of c for submerged orifices seems to be somewhat less than when the flow occurs freely, but it is usual to take .6 or .625 as a mean value.

Ex. 1. A paraboloidal vessel with a latus-rectum of 1 ft. and 5 ft. in height, is immersed in water to a depth of 4 feet. How long will it take to fill the vessel to the level of the outside surface through an orifice 1 inch in diar. at the vertex? (Take $c = \frac{5}{8}$.)

Let $2y$ ft. be the diar. of the free surface when it is x ft. above the orifice. Then

$$y^2 = x.$$

Also, if the water rises dx ft. in dt secs.,

$\pi y^3 dx$ = amount entering vessel in dt secs.

= quantity flowing through orifice under a head of $4 - x$ ft. in dt secs.

$$= \frac{5}{8} \pi \cdot \frac{1}{4} \cdot \frac{1}{144} \sqrt{2 \cdot 32(4 - x)} \cdot dt,$$

$$= \frac{5\pi}{576} (4 - x)^{\frac{1}{2}} dt,$$

and therefore

$$y^2 \cdot dx = x dx = \frac{5}{576} (4 - x)^{\frac{1}{2}} dt,$$

or

$$\begin{aligned} dt &= \frac{576}{5} \frac{x}{(4 - x)^{\frac{1}{2}}} dx = \frac{576}{5} \left\{ \frac{4 - (4 - x)}{(4 - x)^{\frac{1}{2}}} \right\} dx \\ &= \frac{576}{5} \left\{ 4(4 - x)^{-\frac{1}{2}} - (4 - x)^{\frac{1}{2}} \right\} dx. \end{aligned}$$

Integrating between the limits $x = 0$ and $x = 4$ ft., the required time in secs.

$$\begin{aligned} &= \frac{576}{5} \left\{ 4 \cdot 2 \cdot 4^{\frac{1}{2}} - \frac{2}{3} \cdot 4^{\frac{3}{2}} \right\} \\ &= 1228.8. \end{aligned}$$

EX. 2. The horizontal section of a lock-chamber is approximately a rectangle and its length is 360 feet. The side walls have a batter of 1 in 12, and the width of the free surface when the lock is full of water is 45 feet. How long will it take to empty the lock through two sluices in the gates, each 8 ft. by 2 ft., the sluice horizontal centre-line being 13 ft. below the free surface in the lock and 4 ft. below that of the canal on the down-stream side?

Let the level of the water in the lock fall x ft. in t seconds.

The area of the water-surface is then

$$= 360 \left(45 - \frac{x}{6} \right).$$

If the level now sinks dx ft. in dt secs.,

$360 \left(45 - \frac{x}{6} \right) dx$ = amount of water which has flowed out through the sluices

16000

$$= 2 \cdot \frac{5}{8} \cdot 2 \cdot 8 \cdot \sqrt{2 \cdot 32 \cdot x} \cdot dt$$

$$= 160x^{\frac{1}{2}} \cdot dt.$$

Therefore

$$dt = \frac{2}{4} \left(45x^{-\frac{1}{2}} - \frac{1}{6}x^{\frac{1}{2}} \right) dx.$$

Integrating between the limits $x = 0$ and $x = 9$ ft., the required time

$$\begin{aligned} \text{in secs.} &= \frac{2}{4} \left(90.9^{\frac{1}{2}} - \frac{1}{9} 9^{\frac{3}{2}} \right) \\ &= 600\frac{3}{4}. \end{aligned}$$

16. General Equations.—Bernouilli's theorem may be easily deduced from the general equations of fluid motion, as follows:—

Let p be the pressure and ρ the density at any point whose co-ordinates parallel to the axes are x, y, z .

Let u, v, w be the velocities of flow at the same point parallel to the axes, and let X, Y, Z be the accelerating forces. Then three equations result from the principle of the equality of pressure in all directions, viz.:

$$\frac{1}{\rho} \frac{dp}{dx} = X - \frac{d(u)}{dt} = X - \frac{du}{dt} - u \frac{du}{dx} - v \frac{du}{dy} - w \frac{du}{dz}; \quad (1)$$

$$\frac{1}{\rho} \frac{dp}{dy} = Y - \frac{d(v)}{dt} = Y - \frac{dv}{dt} - u \frac{dv}{dx} - v \frac{dv}{dy} - w \frac{dv}{dz}; \quad (2)$$

$$\frac{1}{\rho} \frac{dp}{dz} = Z - \frac{d(w)}{dt} = Z - \frac{dw}{dt} - u \frac{dw}{dx} - v \frac{dw}{dy} - w \frac{dw}{dz}. \quad (3)$$

If the motion is steady, so that the velocity at any point is a function of the position only, then $\frac{du}{dt} = 0 = \frac{dv}{dt} = \frac{dw}{dt}$, and u, v, w may be expressed as the differential coefficients of a function F . Thus,

$$u = \frac{dF}{dx}; \quad v = \frac{dF}{dy}; \quad w = \frac{dF}{dz};$$

and therefore

$$\frac{du}{dy} = \frac{d^2F}{dydx} = \frac{dv}{dx};$$

$$\frac{du}{dz} = \frac{d^2F}{dzdx} = \frac{dw}{dx};$$

$$\frac{dv}{dz} = \frac{d^2F}{dzdy} = \frac{dw}{dy}.$$

Hence equations 1, 2, and 3 may be written

$$\frac{1}{\rho} \frac{dp}{dx} = X - u \frac{du}{dx} - v \frac{dv}{dx} - w \frac{dw}{dx}; \quad . \quad . \quad . \quad (4)$$

$$\frac{1}{\rho} \frac{dp}{dy} = Y - u \frac{du}{dy} - v \frac{dv}{dy} - w \frac{dw}{dy}; \quad . \quad . \quad . \quad (5)$$

$$\frac{1}{\rho} \frac{dp}{dz} = Z - u \frac{du}{dz} - v \frac{dv}{dz} - w \frac{dw}{dz}. \quad . \quad . \quad . \quad (6)$$

Multiplying eq. (4) by dx , eq. (5) by dy , and eq. (6) by dz , and adding, then

$$\begin{aligned} \frac{dp}{\rho} = & Xdx + Ydy + Zdz - u \left(\frac{du}{dx} dx + \frac{du}{dy} dy + \frac{du}{dz} dz \right) \\ & - v \left(\frac{dv}{dx} dx + \frac{dv}{dy} dy + \frac{dv}{dz} dz \right) \\ & - w \left(\frac{dw}{dx} dx + \frac{dw}{dy} dy + \frac{dw}{dz} dz \right), \end{aligned}$$

which may be written

$$\frac{dp}{\rho} = Xdx + Ydy + Zdz - (u du + v dv + w dw).$$

Integrating, and assuming the fluid to be homogeneous,

$$\frac{p}{\rho} = \int (Xdx + Ydy + Zdz) - \frac{u^2 + v^2 + w^2}{2} + \text{a constant.}$$

Hence, if gravity is the only force, and if V is the resultant velocity at the point,

$$X=0=Y; \quad Z=-g; \quad u^2+v^2+w^2=V^2;$$

and the last equation becomes

$$\begin{aligned} \frac{p}{\rho} &= - \int g dz - \frac{V^2}{2} + \text{a constant} \\ &= -gz - \frac{V^2}{2} + \text{a constant}; \end{aligned}$$

and therefore

$$z + \frac{p}{\rho g} + \frac{V^2}{2g} = \text{a constant.}$$

17. Loss of Energy in Shock.—An abrupt change of section at any point in a length of piping destroys the parallelism of the fluid filaments, breaks up the fluid, and energy is dissipated in the production of eddy and other motions. The energy thus wasted is termed "*energy lost in shock.*"

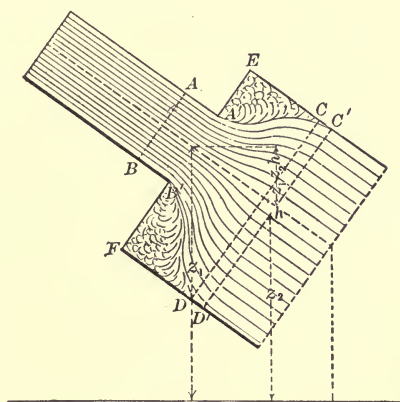


FIG. 38.

In a short length of piping, Fig. 38, where the section suddenly changes from $A'B'$ to EF , consider the fluid mass between the two transverse sections AB , where the motion of

the fluid filaments has been undisturbed and is in parallel lines, and CD , where the parallelism has been again re-established.

In an indefinitely short interval of time t let the mass move forward into the position bounded by the plane sections $A'B'$ and $C'D'$.

Let a_1, v_1, p_1 be the sectional area, velocity of flow, and mean intensity of pressure at $A'B'$.

Let a_2, v_2, p_2 be similar symbols for $C'D'$.

Let z_1, z_2 be the elevation above datum of the C. G.s of the sectional areas at $A'B'$ and $C'D'$.

Let h be the vertical distance between the C. G.s of the areas EF and $A'B'$.

Let P be the mean intensity of pressure over the annular surface between EF and $A'B'$.

The resultant force acting in the direction of motion upon the mass of fluid under consideration

= component of weight of mass in this direction

+ pressure on $A'B'$

+ pressure on annular surface between EF and $A'B'$

− pressure on $C'D'$

$$= wa_2 \cdot EC' \frac{z_1 - z_2 - h}{EC'} + p_1 a_1 + P(a_2 - a_1) - p_2 a_2$$

$$= wa_2(z_1 - z_2 - h) + a_2(p_1 - p_2),$$

assuming that $P = p_1$, or that the mean intensity of pressure is unchanged throughout the whole of the section EF .

The normal reaction of the pipe-surface between EF and $C'D'$ has no component in the direction of motion, and frictional resistances are disregarded.

Hence the impulse of the resultant force

$$= wa_2(z_1 - z_2 - h)t + a_2(p_1 - p_2)t$$

= change of momentum in the same direction of the fluid masses $CDD'C'$ and $ABB'A'$, since the momentum of the mass between $A'B'$ and CD remains unchanged

$$\begin{aligned}
 &= \frac{w}{g} a_2 v_2 \cdot v_2 t - \frac{w}{g} a_1 v_1 \cdot v_1 t \\
 &= \frac{w}{g} a_2 (v_2^2 - v_1 v_2) t,
 \end{aligned}$$

since, by the condition of continuity,

$$a_1 v_1 = a_2 v_2.$$

Dividing throughout by the factor $wa_2 t$, the equation becomes

$$z_1 - z_2 - h + \frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{g} - \frac{v_1 v_2}{g},$$

which may be written in the form

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + h + \frac{p_2}{w} + \frac{v_2^2}{2g} + \frac{(v_1 - v_2)^2}{2g}.$$

Now the pipes are nearly always axial, and in such case $h = 0$, so that the last equation becomes

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g} + \frac{(v_1 - v_2)^2}{2g}.$$

If there had been no abrupt change of section, or if the change between $A'B'$ and CD had been gradual, then no internal work would have been done in destroying the parallelism of the fluid filaments, and no energy wasted. Therefore, by Bernouilli's theorem, the relation

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g}$$

would have then held good.

Thus $\frac{(v_1 - v_2)^2}{2g}$ ft.-lbs. of energy per pound of fluid is the *loss in shock* between $A'B'$ and CD .

Experiment justifies the assumption $P = p_1$.

Ex. At a point A , 150 ft. above datum, a line of piping suddenly doubles in sectional area. If the velocity of flow in the larger pipe is 8 ft. per sec., and if the pressure at A is 125 lbs. per sq. in., find the pressure per sq. in. at B , 8 ft. above datum, the motion being steady.

The velocity of flow in the smaller pipe is evidently 16 ft. per second. Therefore the loss of head in shock at the sudden change of section

$$= \frac{(16 - 8)^2}{2 \cdot 32} = 1 \text{ ft.}$$

Hence, if p is the pressure per sq. in. at B ,

$$8 + \frac{p \times 144}{62\frac{1}{2}} + \frac{8^2}{164} + 1 = 150 + \frac{125 \times 144}{62\frac{1}{2}} + \frac{16^2}{64},$$

or
$$p \cdot \frac{144}{62\frac{1}{2}} = 432,$$

and
$$p = 187\frac{1}{2} \text{ lbs. per sq. in.}$$

18. Mouthpieces. — (a) *Borda's Mouthpiece.* — This is merely a short pipe projecting inwards, as in Fig. 39, which

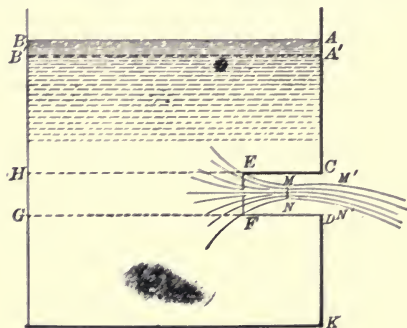


FIG. 39.

represents a jet flowing through a re-entrant mouthpiece of sectional area A , fixed in the vertical side of a vessel of constant horizontal section and containing water kept at a constant level. The mouthpiece is sufficiently long to allow of the jet springing clear from the end EF without adhering to the inside surface.

The velocity of the fluid molecules along AC and DK , is sufficiently small to be disregarded, so that the pressure over this portion of the vessel is distributed in accordance with the hydrostatic law. The same may also be said of the pressure on the remainder of the vessel's surface.

Again, the only unbalanced pressure is that on the surface HG immediately opposite the mouthpiece, and the resultant horizontal force in the direction of the axis of the mouthpiece

$$= (p_0 + wh)A - p_0A = whA,$$

h being the depth of the axis below the water-surface and p_0 the intensity of the atmospheric pressure.

The jet converges to a minimum, or contracted section MN , of area a .

In a unit of time let the fluid mass between AB and MN take up the position bounded by $A'B$ and $M'N'$. Then

$$\begin{aligned} whA &= \text{impulse of force in direction of motion} \\ &= \text{change of momentum in same direction in a unit} \\ &\quad \text{of time} \\ &= \text{difference between the momenta of } MNV'M' \text{ and} \\ &\quad ABB'A', \text{ since the momentum of the mass} \\ &\quad \text{between } A'B' \text{ and } MN \text{ remains unchanged} \\ &= \text{momentum of } MNV'M', \text{ since the momentum of} \\ &\quad ABB'A' \text{ is vertical} \\ &= \frac{w}{g}av \cdot v = \frac{w}{g}av^2, \end{aligned}$$

v being the mean velocity of flow across the contracted section.
Hence

$$whA = \frac{w}{g}av^2 = \frac{w}{g}a \cdot 2gh,$$

and therefore

$$A = 2a,$$

or

$$\frac{1}{2} = \frac{a}{A} = \text{coefficient of contraction.}$$

This result has been very closely verified by experiment, the coefficient having been found to be .5149 by Borda, .5547 by Bidone, and .5324 by Weisbach.

Borda's mouthpiece gives a smaller discharge than a sharp-edged orifice, but a discharge which is much more uniform, and hence it is generally used in vessels from which water is to be distributed by measure.

NOTE.—Let Fig. 40 represent a jet flowing through a re-entrant mouthpiece of sectional area A , fixed in the sloping side of a reservoir containing water kept at a constant level, and suppose that the reservoir is of such size that GHL may represent a cylindrical fluid mass, coaxial with the mouthpiece and so large that the velocity at its surface is sensibly nil. Let h' , h be the depths below the water-surface of the C. G.s of the areas GH and KL , respectively.

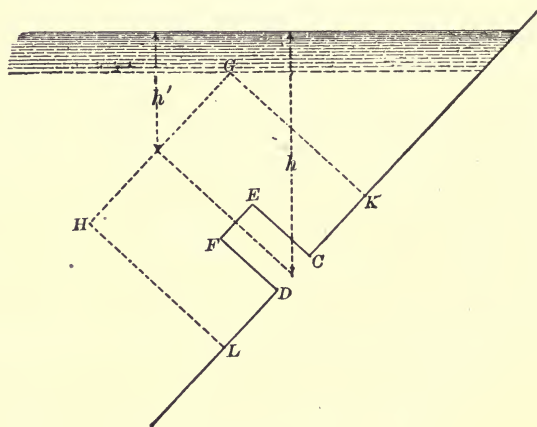


FIG. 40.

Then the resultant force along the axis of the mouthpiece
 = pressure on GH — pressure on CK and on DL

— pressure on EF

+ component of the weight of the fluid mass GHL

$$\begin{aligned}
 &= (p_0 + wh') \text{ area } GH - (p_0 + wh) (\text{area } CK + \text{area } DL) \\
 &\quad - p_0 \cdot \text{area } EF + w \cdot \text{area } GH \cdot GK \cdot \frac{h - h'}{GK}, \text{ very nearly} \\
 &= whA.
 \end{aligned}$$

Hence, in a unit of time,

whA = impulse of this force

= change of momentum in direction of axis

$$= \frac{w}{g} av \cdot v = \frac{w}{g} av^2 = \frac{w}{g} a \cdot 2gh,$$

a being the area of the contracted section, while h is also very approximately the depth of its C. G. below the water-surface.

Thus, as before,

$$\text{the coefficient of contraction} = \frac{a}{A} = \frac{1}{2}.$$

(b) *Ring-nozzle*.—The ring-nozzle (see Fig. 41) is often used with a fire-engine jet, and consists of a re-entrant pipe of sectional area a_1 fixed in a pipe of sectional area a_2 . The length of the re-entrant portion is such that the water springs clear from the inner end and, without again touching the surface of the mouthpiece, converges to a minimum or

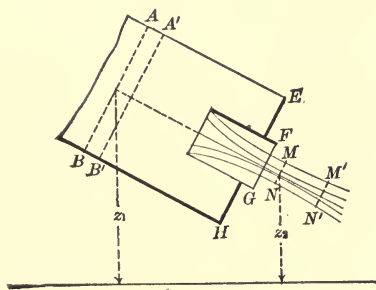


FIG. 41.

contracted section of area a at MN .

Consider the fluid mass between MN and a transverse section AB , and in a unit of time let it move into the position bounded by the planes $M'N'$ and $A'B'$.

It is assumed that the motion is steady and that there is no internal work due to the production of eddies or other motions.

Let p_0 , v be the intensity of the atmospheric pressure and the velocity at MN .

Let p_1 , v_1 be the mean intensity of pressure and the velocity at AB .

Let P be the mean intensity of the pressure over the annular surface EF , GH .

Let z_0 , z_1 be the elevations above datum of the C. G.s of the sections MN and AB .

Then

$$wa_2(z_1 - z_0) + p_1a_2 - P(a_2 - a_1) - p_0a_1$$

= impulse in direction of motion

= change of momentum in same direction in a unit of time

= difference of the momenta of the fluid masses $MNN'M'$ and $ABB'A'$

$$= \frac{w}{g}(av^2 - a_2v_1^2).$$

Assuming that $P = p_1$, the last equation becomes

$$wa_2(z_1 - z_0) + a_1(p_1 - p_0) = \frac{w}{g}(av^2 - a_2v_1^2). \quad (1)$$

By Bernoulli's theorem,

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_0 + \frac{p_0}{w} + \frac{v^2}{2g},$$

and therefore

$$z_1 - z_0 + \frac{p_1 - p_0}{w} = \frac{v^2 - v_1^2}{2g}. \quad (2)$$

Now $z_1 - z_0$ is very small and may be disregarded without sensible error, and then, by eqs. (1) and (2),

$$\frac{v^2 - v_1^2}{2g} = \frac{p_1 - p_0}{w} = \frac{1}{g} \frac{av^2 - a_2v_1^2}{a_1}.$$

Hence

$$\frac{2}{a_1} = \frac{v^2 - v_1^2}{av^2 - a_2v_1^2} = \frac{(a_2^2 - a^3)v_1^2}{(aa_2^2 - a^3a_2)v_1^2} = \frac{1}{a} + \frac{1}{a_2},$$

since $a_2v_1 = av$.

If the sectional area a_2 of the pipe is very large as compared with a , so that $\frac{1}{a_2}$ may be disregarded without sensible error, then $\frac{2}{a_1} = \frac{1}{a}$, and therefore the coefficient of contraction $= \frac{a}{a_1} = \frac{1}{2}$, as in Borda's mouthpiece.

(c) *Cylindrical Mouthpiece*.—When water issues from a cylindrical mouthpiece (see Fig. 42) at least two to two and one-half diameters in length, the jet issues full bore, or without contraction at the point of discharge.

If A be the sectional area of the mouthpiece, h the depth of its axis below the water-surface, and Q the amount of the discharge, then experiment shows that

$$Q = .82A \sqrt{2gh}. \quad (1)$$

The coefficient .82 is the product of the coefficients of velocity and contraction, but the

coefficient of contraction is unity, and therefore the coefficient of velocity is .82. Now the mean coefficient of velocity in the case of a simple sharp-edged orifice is .947, and the difference between .947 and .82 cannot be wholly accounted for by frictional resistances, but is in part due to a loss of head. In fact, the water, as it clears the inner edge of the mouthpiece, con-

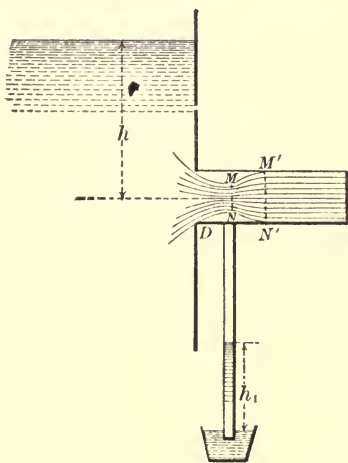


FIG. 42.

verges to a minimum section MN , of area a , and then swells out until at $M'N'$ it again fills the mouthpiece.

Energy is wasted in eddy motions between MN and $M'N'$, where the action is similar to that which occurs at an abrupt change of section.

Let p, v be the intensity of the pressure and the mean velocity of flow at the point of discharge.

Let p_1, v_1 be similar symbols for the contracted section MN .

Let p_0 be the intensity of the atmospheric pressure.

Remembering that $\frac{(v_1 - v)^2}{2g}$ is the loss of head "due to shock" between MN and $M'N'$, then, by Bernoulli's theorem,

$$h + \frac{p_0}{w} = \frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p}{w} + \frac{v^2}{2g} + \frac{(v_1 - v)^2}{2g}. \quad (2)$$

Hence

$$\frac{p_0 - p}{w} = \frac{v_1^2}{2g} - h, \quad . \quad . \quad . \quad (3)$$

and

$$\begin{aligned} h + \frac{p_0 - p}{w} &= \frac{v^2}{2g} \left\{ 1 + \left(\frac{v_1}{v} - 1 \right)^2 \right\} \\ &= \frac{v^2}{2g} \left\{ 1 + \left(\frac{A}{a} - 1 \right)^2 \right\} \\ &= \frac{v^2}{2g} \left\{ 1 + \left(\frac{1}{c_c} - 1 \right)^2 \right\}, \end{aligned}$$

where c = coefficient of contraction = $\frac{a}{A} = \frac{v}{v_1}$. Therefore

$$v^2 = \frac{2g \left(h + \frac{p_0 - p}{w} \right)}{1 + \left(\frac{1}{c_c} - 1 \right)^2}, \quad . \quad . \quad . \quad (4)$$

an equation giving the velocity of flow at the point of discharge.

If the discharge is into the atmosphere, $p_0 = p$ and equation (4) becomes

$$v^2 = \frac{2gh}{1 + \left(\frac{1}{c_c} - 1\right)^2} = c_v^2 \cdot 2gh, \quad . \quad . \quad . \quad (5)$$

where

$$\frac{1}{c_v^2} = 1 + \left(\frac{1}{c_c} - 1\right)^2. \quad . \quad . \quad . \quad (6)$$

If $c_c = .62$, then $c_v = .85$, while experiment gives .82 as the value of c_v . The small difference between .85 and .82 is probably due to frictional resistance. The value .82 for c_v makes c_c approximately .617.

Again, the discharge from a simple sharp-edged orifice of same sectional area as the mouthpiece is $.62A \sqrt{2gh}$, or more than 24 per cent less than the discharge from the cylindrical mouthpiece.

The loss of head between MN and $M'N'$

$$\begin{aligned} &= \frac{(v_1 - v)^2}{2g} = \frac{v^2}{2g} \left(\frac{1}{c_c} - 1\right)^2 \\ &= hc_v^2 \left(\frac{1}{c_c^2} - 1\right) \quad (\text{by eqs. (5) and (6)}) \\ &= h(1 - c_v^2) = h \times .3276 = .487 \times \frac{v^2}{2g} \\ &= \frac{h}{3}, \text{ approximately.} \end{aligned}$$

Thus the effective head is only $\frac{2}{3}h$, instead of h .

By eq. (3), the difference between the pressure-heads at MN and at the point of discharge

$$\begin{aligned} &= \frac{p_0 - p}{w} = \frac{v_1^2}{2g} - h \\ &= \frac{1}{c_c^2} \frac{v^2}{2g} - h = h \left(\frac{c_v^2}{c_c^2} - 1\right) \\ &= \frac{3}{4}h, \text{ very nearly.} \end{aligned}$$

Now if one end of a tube is inserted in the mouthpiece at the contracted section (Fig. 42) and the other end immersed in a vessel of water, the water will at once rise to a height h_1 in the tube, showing that the pressure at the contracted section is less than that due to the atmosphere. By careful measurement it is found that h_1 is very nearly equal to $\frac{3}{4}h$, which verifies the theory.

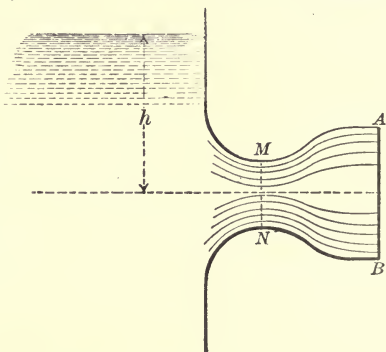


FIG. 43.

(d) *Divergent Mouthpiece.*

—Suppose that for the cylindrical mouthpiece in (c) there is substituted a divergent mouthpiece of the exact form of the issuing jet, Fig. 43. Then

(1) The mouthpiece will run full bore.

(2) There will be no loss of head between the minimum section MN and the plane of discharge AB , as there is now no abrupt change of section.

Hence by Bernoulli's theorem, and retaining the same symbols as in (c),

$$\frac{p_0}{w} + h = \frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p}{w} + \frac{v^2}{2g}. \quad \dots \quad (1)$$

If the discharge is into the atmosphere, $p = p_0$, and therefore

$$v^2 = 2gh; \quad \dots \quad (2)$$

or, introducing a coefficient c_v ($= .98$, nearly, for a smooth well-formed mouthpiece),

$$v^2 = c_v^2 2gh, \quad \dots \quad (3)$$

and the discharge is

$$Q = c_v A \sqrt{2gh}. \quad \dots \quad (4)$$

From the last equation it would appear as if the discharge would increase indefinitely with A , but this is manifestly impossible.

In fact, by eq. (1), the flow being into the air, and taking $c_v = 1$,

$$\frac{p_1}{w} = \frac{p_0}{w} - \frac{v^2}{2g} \left(\frac{v_1^2}{v^2} - 1 \right) (5)$$

$$= \frac{p_0}{w} - h \left(\frac{A^2}{a^2} - 1 \right), (6)$$

since $av_1 = Av$. But p_1 cannot be negative, and therefore

$$\frac{p_0}{w} \geq h \left(\frac{A^2}{a^2} - 1 \right),$$

so that

$$\frac{A}{a} = \sqrt{\frac{p_0}{wh} + 1} (7)$$

gives a maximum limit for the ratio of A to a .

Now $\frac{p_0}{w} = 34$ ft. very nearly, and the last equation may be written

$$\frac{A}{a} = \sqrt{\frac{34 + h}{h}} (8)$$

By eqs. (4) and (7),

$$Q = c_v a \sqrt{2g \left(h + \frac{p_0}{w} \right)}, (9)$$

which is also the expression for the discharge through the minimum section a into a vacuum.

If, however, the sectional areas of the mouthpiece at the point of discharge and at the throat are in the ratio of A to a , as given by eq. (7), it is found that the full-bore flow will be interrupted either by the disengagement of air, or by any slight disturbance, as, for example, a slight blow on the

mouthpiece, and hence, in practice, it is usual to make the ratio of A to a sensibly less than that given by eq. (7).

(e) *Convergent Mouthpiece*.—With a convergent mouthpiece (Fig. 44) two points are to be noted:

(1) There is a contraction within the mouthpiece, followed by a swelling out of the jet until it again fills the mouthpiece.

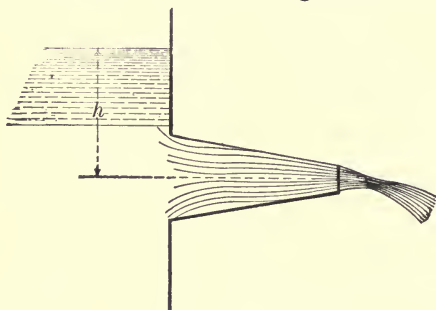


FIG. 44.

Thus, as in the case of cylindrical mouthpieces, there is a "loss of head" between the contracted section and the point of discharge, and also a consequent diminution in the velocity of discharge.

(2) There is a second contraction outside the mouthpiece due to the convergence of the fluid filaments. The mean velocity of flow (v') across the section is

$$v' = c_v' \sqrt{2gh},$$

c_v' being the coefficient of velocity and h the effective head above the centre of the section.

Also, the area of this section

$$= c_c' \times \text{area of mouthpiece at point of discharge} \\ c_c' \cdot A,$$

c_c' being the coefficient of contraction. Hence the discharge Q is given by

$$Q = c_v' c_c' A \sqrt{2gh} = c' A \sqrt{2gh},$$

$c' (= c_v' c_c')$ being the coefficient of discharge.

The coefficients c_v' and c depend upon the angle of convergence, and Castel found that a convergence of $13^\circ 24'$ gave a maximum discharge through a mouthpiece 2-6 diameters in length, the smallest diameter being .05085 foot.

TABLE GIVING CASTEL'S RESULTS.

Angles of Convergence.	C_c'	C_v'	C'	Angles of Convergence.	C_c'	C_v'	C'
0° 0'	.999	.830	.829	13° 24'	.983	.962	.946
1 36	1.000	.866	.866	14 28	.979	.966	.941
3 10	1.001	.894	.895	16 36	.969	.971	.938
4 10	1.002	.910	.912	19 28	.953	.970	.924
5 26	1.004	.920	.924	21 0	.945	.971	.918
7 52	.998	.931	.929	23 0	.937	.974	.913
8 58	.992	.942	.934	29 58	.919	.975	.896
10 20	.987	.950	.938	40 20	.887	.980	.869
12 4	.986	.955	.942	48 50	.861	.984	.847

19. Energy and Momentum of a Jet.—(a) *Jet from a sharp-edge orifice.*

The energy of the jet = $wav \frac{v^2}{2g}$ ft.-lbs. per second

$$= \frac{wav^3}{2g} \text{ ft.-lbs. per second}$$

$$= wavhc_v^2 \text{ ft.-lbs. per second}$$

$$= \frac{wavhc_v^2}{550} \text{ h. p. (horse-power)}$$

$$= \frac{pavc_v^2}{550} \text{ h. p.,}$$

$p(=wh)$ being the hydrostatic pressure due to the head h , and the average value of c_v being .62.

$$\begin{aligned} \text{The momentum of the jet} &= \frac{w}{g} av \cdot v = wa \frac{v^2}{g} = 2wahc_v^2 \\ &= 2pac_v^2, \end{aligned}$$

and this is equal to the pressure in pounds produced by the jet against a fixed plane perpendicular to its direction. Neglect-

ing c_v^2 , the thrust is double the hydrostatic pressure due to the head h .

(b) *Jet from a Cylindrical Mouthpiece.*

The energy of the jet $= wAv \frac{v^2}{2g}$ ft.-lbs. per second

$$= \frac{wAv^3}{2g} \text{ ft. lbs. per second}$$

$$= c_v^3 wA \sqrt{2gh^3} \text{ ft.-lbs. per second}$$

$$= \frac{pAv c_v^2}{550} \text{ h. p.,}$$

the average value of c_v being .82.

The momentum of the jet $= \frac{w}{g} Av \cdot v = wA \frac{v^2}{g} = 2wA h c_v^2$.

EX. 1. Water flows through a Borda mouthpiece of A sq. ft. sectional area under a head of h feet. If the jet springs clear from the inner edge, the discharge is 29.2% less and the jet's energy 41.4 % greater than when the mouthpiece runs full.

Let v be the mean velocity of flow across the contracted section MN ;

u be the mean velocity of flow at the mouth CD when the mouthpiece runs full. Then $v = 2u$.

Let Q_1, E_1 be the discharge and energy of the jet when it springs clear;

Q_2, E_2 be the discharge and energy of the jet when the mouthpiece runs full. Then

$$Q_1 = \frac{A}{2} v,$$

and
$$E_1 = \frac{wQ_1 v^2}{g \cdot 2} = \frac{wA}{g} \frac{u^3}{4}.$$

When the mouthpiece runs full, the loss of head between MN and CD

$$= \frac{(v - u)^2}{2g} = \frac{u^2}{2g}.$$

Hence

$$h + \frac{p_0}{w} = 0 + \frac{p_0}{w} + \frac{u^2}{2g} + \frac{u^2}{2g},$$

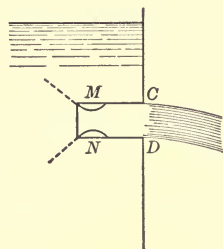


FIG. 45.

or $u^2 = gh = \frac{v^2}{2}.$

Therefore $Q_2 = Au = \frac{A}{\sqrt{2}}v,$

and $E_2 = \frac{wQ_2}{g} \frac{u^2}{2} = \frac{wA}{g} \cdot \frac{v^3}{4\sqrt{2}}.$

Hence $\frac{Q_1}{Q_2} = \frac{\sqrt{2}}{2} = .707$ and $\frac{Q_2 - Q_1}{Q_2} = .292.$

Also, $\frac{E_1}{E_2} = \frac{4\sqrt{2}}{4} = 1.414$ and $\frac{E_1 - E_2}{E_2} = .414.$

EX. 2. Determine the discharges and energies of a jet under a head of 100 ft., issuing from a 6-in. mouthpiece which is (a) cylindrical, (b) divergent (bell-mouth), (c) convergent, the angle of convergence being $29^\circ 58'.$

(a) $v = .82 \sqrt{64 \cdot 100} = 65.6$ ft. per sec.

$Q = \frac{22}{7} \frac{1}{4} \left(\frac{6}{12} \right)^2 \times 65.6 = 12\frac{3}{8}\frac{1}{2}$ cu. ft. per sec. $= 80\frac{1}{2}\frac{5}{8}$ gals. per sec.

Energy $= \frac{62\frac{1}{2}}{32} \times 12\frac{3}{8}\frac{1}{2} \frac{(65.6)^2}{2} = 54,152\frac{3}{4}$ ft.-lbs. $= 98\frac{3}{7}\frac{21}{10}$ H. P.

(b) $v = .98 \sqrt{64 \cdot 100} = 78.4$ ft. per sec.

$Q = \frac{22}{7} \frac{1}{4} \left(\frac{6}{12} \right)^2 \times 78.4 = 15.4$ cu. ft. per sec. $= 96\frac{1}{2}$ gals. per sec.

Energy $= \frac{62\frac{1}{2}}{32} \times 15\frac{2}{5} \frac{(78.4)^2}{2} = 92,438\frac{1}{2}$ ft.-lbs. $= 168\frac{7}{10}$ H. P.

(c) $v = .896 \sqrt{64 \cdot 100} = 71.68$ ft. per sec. (See Castel's Table.)

$Q = \frac{22}{7} \frac{1}{4} \left(\frac{6}{12} \right)^2 \times 71.68 = 14.08$ cu. ft. per sec. $= 88$ gals. per sec.

Energy $= \frac{62\frac{1}{2}}{32} \times 14\frac{2}{5} \frac{(71.68)^2}{2} = 70647.807$ ft.-lbs. $= 128.45$ H. P.

EX. 3. There is a 36-ft. head of water over the 2-in. throat of a bell-mouth. Find the greatest diameter of the mouth when open to the atmosphere and running full, the height of the water-barometer being 34 feet.

Let p, v be the pres. and vel. at the throat;

v_0 be the vel. at the mouth.

Then $\frac{p}{w} + \frac{v^2}{2g} = 34 + \frac{v_0^2}{2g} = 36.$

Therefore $\frac{v_0^2}{64} = 2,$ or $v_0 = 8\sqrt{2}$ ft. per sec.

The velocity in the throat is greatest when the pressure, p_1 , is least, i.e., when $p_1 = 0$, and then

$$0 + \frac{v^2}{2g} = 36, \quad \text{or} \quad v = 48 \text{ ft. per sec.}$$

If D ins. is the diameter of the mouth, the discharge

$$\begin{aligned} &= \frac{1}{144} \frac{\pi D^2}{4} 8 \sqrt{2} = \frac{1}{144} \cdot \frac{\pi \cdot 2^2}{4} \cdot 36 \\ &= \frac{11}{14} \text{ cu. ft. per sec.,} \end{aligned}$$

and $D^2 = 12.726,$

or $D = 3.56 \text{ ins.}$

20. Radiating Current.—As an application of Bernoulli's theorem, consider the steady plane motion of a body of water flowing radially between two horizontal planes a ft. apart, and symmetrical with respect to a central axis (Fig. 46).

Let v ft. per second be the velocity at the surface of a cylinder of radius r ft. described about the same axis. Then the volume Q crossing the surface per second is

$$Q = 2\pi r \cdot av,$$

and therefore

$$rv = \frac{Q}{2\pi a} = \text{a constant,}$$

since Q is constant.

Thus v increases as r diminishes, and becomes infinitely great at the axis; but it is evident that the current must take a new course at some finite distance from the axis.

If p is the pressure at any point of the cylindrical surface z ft. above datum, then, by Bernoulli's theorem,

$$z + \frac{p}{w} + \frac{v^2}{2g} = \text{a constant} = h = y + \frac{v^2}{2g},$$

denoting the *dynamic head* $z + \frac{p}{w}$ by y . Hence

$$h - y = \frac{v^2}{2g} = \frac{Q^2}{8\pi^2 a^2 r^2 g} = \frac{\text{a constant}}{r^2},$$

and therefore

$$r^2(h - y) = \text{a constant},$$

FIG. 46.

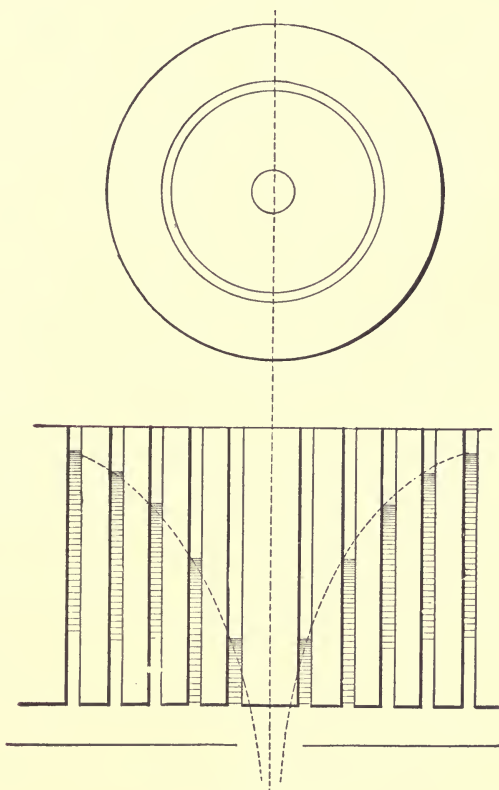


FIG. 47.

is an equation giving the free surfaces of the pressure columns (Fig. 47). These surfaces are thus generated by the revolution of what is called Barlow's curve.

The surfaces of equal pressure are also given by an equation of the same form.

21. Vortex Motion.—A vortex is a mass of rotating fluid, and the vortex is termed *free* when the motion is produced naturally and under the action of the forces of weight and pressure only.

In the radiating current already discussed, assume that the direction of motion at each point is turned through a right angle so that the mass of water will now revolve in circular layers about the central axis. Also, if there is a slow radial movement, so that fluid particles travel from one circular stream-line to another, it is assumed that these particles freely take the velocities proper to the stream-lines which they join. Such a motion is termed a free *circular* vortex.

The motion being steady and horizontal, the equation

$$z + \frac{p}{w} + \frac{v^2}{2g} = \text{a constant} = H \quad . \quad . \quad . \quad (1)$$

holds good at every point of a circular stream of radius r .

Again,

$$\begin{aligned} w \cdot d\left(z + \frac{p}{w}\right) &= \text{increment of dynamic pressure between two} \\ &\quad \text{consecutive elementary stream-lines} \\ &= \text{deviating force} \\ &= \text{centrifugal force of an element between the} \\ &\quad \text{two stream-lines} \\ &= \frac{w v^2}{g r} \cdot dr. \end{aligned}$$

But, by eq. (1),

$$d\left(z + \frac{p}{w}\right) = - \frac{v \cdot dv}{g}.$$

Hence

$$w \cdot d\left(z + \frac{p}{w}\right) = - \frac{w}{g} v dv = \frac{w v^2}{g r} \cdot dr,$$

and therefore

$$\frac{dv}{v} + \frac{dr}{r} = 0,$$

so that $vr = \text{a constant}$, and v varies inversely as r , as in the case of the radiating current. Therefore the curves of equal pressure will also be the same as in a radiating current.

Free Spiral Vortex.—Suppose that the motion of a mass of water with respect to an axis O is of such a character that at any point M , the components of the velocity in the direction of OM , and perpendicular to OM , are each inversely proportional to the distance OM from O . The motion is thus equivalent to the superposition of the motions in a radiating current and in a free circular vortex; and if θ is the angle between OM and the direction of the stream-line at M , $v \cos \theta$ and $v \sin \theta$ are each inversely proportional to OM , and therefore θ must be constant. Hence the stream-lines must be equi-angular spirals, and the motion is termed a free spiral vortex.

This result is of value in the discussion of certain turbines and centrifugal pumps. A *steady* free surface in the case of a free spiral vortex is impossible, as the stream-lines cross the surfaces of equal pressure, which are the same as before.

Also, if p_0 , r_0 , v_0 are the pressure, radius, and velocity at any other point at the same elevation z above datum, then

$$z + \frac{p}{w} + \frac{v^2}{2g} = z + \frac{p_0}{w} + \frac{v_0^2}{2g},$$

and the increase of pressure-head

$$= \frac{p - p_0}{w} = \frac{v_0^2 - v^2}{2g} = \frac{v^2}{2g} \left(\frac{r^2}{r_0^2} - 1 \right) = \frac{v_0^2}{2g} \left(1 - \frac{r_0^2}{r^2} \right).$$

Forced Vortex.—A forced vortex is one in which the law of motion is different from that in a free vortex. The simplest and most useful case is that in which all the particles have an equal angular velocity, so that the water will revolve bodily,

the velocity at any point being directly proportional to the distance from the axis.

As before,

$$d\left(z + \frac{p}{w}\right) = \frac{v^2}{g} \frac{dr}{r}.$$

But

$$v \propto r = \omega r,$$

ω being the constant angular velocity of the rotating mass. Therefore

$$d\left(z + \frac{p}{w}\right) = \frac{\omega^2}{g} r \cdot dr.$$

Integrating,

$$z + \frac{p}{w} = \frac{\omega^2 r^2}{2g} + \text{a constant} = \frac{v^2}{2g} + \text{a constant}.$$

Hence, if p_0 , r_0 , v_0 are the pressure, radius, and velocity for any second point at the same elevation z above datum, then

$$\frac{p - p_0}{w} = \frac{\omega^2}{2g}(r^2 - r_0^2) = \frac{1}{2g}(v^2 - v_0^2).$$

If the second point is on the axis of revolution, then $r_0 = 0$, and the last equation becomes

$$\frac{p - p_0}{w} = \frac{\omega^2}{2g} r^2.$$

Thus the free surface of the pressure columns is evidently a paraboloid of revolution with its vertex at O , as in Fig. 48.

A *compound* vortex is produced by the combination of a central forced vortex with a free circular vortex, the free surface being formed by the revolution of a Barlow curve and a parabola.

For example, the fan of a centrifugal pump draws the water into a forced vortex and delivers it as a free spiral vortex into a whirlpool-chamber (Chap. VIII).

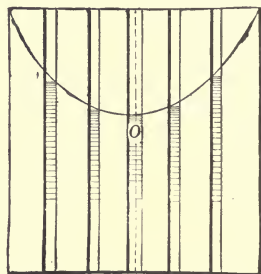


FIG. 48.

In this chamber there is thus a gain of pressure-head, and the water is therefore enabled to rise to a corresponding additional height. James Thomson adopted the theory of the compound vortex as the principle of the action of his vortex turbine.

Ex. A centrifugal pump of 2 ft. interior and 4 ft. exterior diar., makes 336 revols. per minute. The water gradually fills up and flows very slowly through the wheel into a chamber of comparatively much larger diar., from which it passes away into the discharge-pipe. The pressure at the inlet may be taken to be one atmosphere, or 2116 lbs. per sq. foot.

Basing the flow through the wheel upon the hypothesis that the velocity v of any fluid particle is *directly* proportional to its distance r

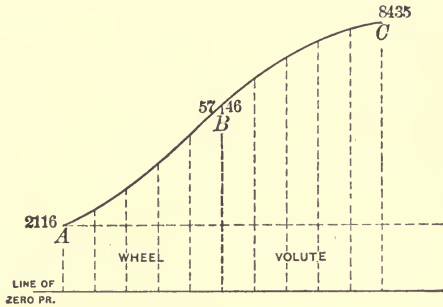


FIG. 49.

from the axis of rotation, the law connecting the pressure p and the velocity v may be expressed in the form (Ex. 1, p. 21)

$$\frac{p}{w} = c' + \frac{v^2}{2g}.$$

At the inlet $p = 2116$, and let $v = v_1$. Then

$$\frac{2116}{w} = c' + \frac{v_1^2}{2g},$$

so that
$$p = 2116 + \frac{wv_1^2}{2g} \left(\frac{v^2}{v_1^2} - 1 \right).$$

But

$$\frac{wv^2}{2g} = \frac{125}{128} \left(\frac{336}{60} \frac{22}{7} \right)^2 = 1210, \text{ and } \frac{v^2}{v_1^2} = \frac{r^2}{1^2} = r^2.$$

Therefore

$$p = 2116 + 1210 (r^2 - 1) = 906 + 1210r^2,$$

Giving r , successively,

the values 1, 1.2, 1.4, 1.6, 1.8, 2 ft.,

the corresponding values

of p are 2116, 2648.4, 3277.6, 4003.6, 4826.4, 5746 lbs.

Thus the curve AB , obtained by plotting these values, shows the variation of the pressure inside the wheel.

The hypothesis of the flow in the surrounding chamber is that the velocity of any fluid particle is *inversely* proportional to its distance from the axis of rotation; and in this case the pressure and velocity are connected by the relation (Ex. 1, p. 21)

$$\frac{p}{w} = c - \frac{v^2}{2g}.$$

At the wheel outlet, i.e., where $r = 2$ ft., $p = 5746$ lbs. per sq. ft., and let $v = v_2$.

Then

$$\frac{5746}{w} = c - \frac{v_2^2}{2g};$$

therefore

$$p = 5746 + \frac{wv_2^2}{2g} \left(1 - \frac{v^2}{v_2^2} \right).$$

But

$$\frac{wv_2^2}{2g} = \frac{125}{128} \left(\frac{336}{60} \frac{22}{7} 4 \right)^2 = 4840, \quad \text{and} \quad \frac{v}{v_2} = \frac{2}{r};$$

therefore

$$p = 5746 + 4840 \left(1 - \frac{4}{r^2} \right) = 10586 - \frac{19360}{r^2}.$$

Giving r , successively, the values 2, 2.2, 2.4, 2.6, 2.8, and 3 ft., the corresponding values

of p are 5746, 6586, 7225, 7723, 8117, and 8435 lbs.

Thus the curve BC , obtained by plotting these values, shows the variation of the pressure in the chamber surrounding the wheel.

22. Large Orifices in Vertical Plane Surfaces.—The issuing jet is approximately of the same sectional form as the orifice, and the fluid filaments converge to a minimum section as in the case of simple sharp-edged orifices.

(a) *Rectangular Orifice* (Fig. 50).—Let E , F be the upper and lower edges of a large rectangular orifice of breadth

B , and let H_1, H_2 be the depths of E and F , respectively, below the free surface at A . If u be the velocity with which the water reaches the orifice, then $H = \frac{u^2}{2g}$, is the fall of free surface which must have been expended in producing the velocity u .

Hence $H_1 + H$ and $H_2 + H$ are the true depths of the edges E and F below the surface of still water.

Let MN be the minimum or contracted section, and assume that it is a rectangle of breadth b .

Let h_1, h_2 be the depths of M and N , respectively, below the free surface at A .

Then $h_1 + H, h_2 + H$ are the true depths of M and N below the surface of still water.

First. Let the flow be into the air, the orifice being clear above the tail-water level, Fig. 50.

Consider a lamina of the fluid at the section MN , of the

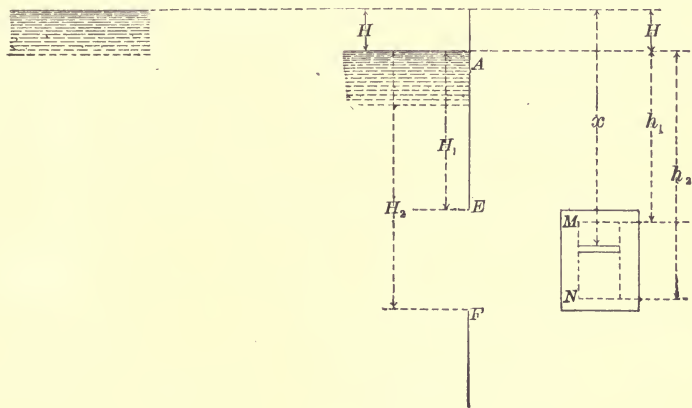


FIG. 50.

width of the section, and between the depths x and $x + dx$ below the surface of still water.

The elementary discharge dq , in this lamina, is

$$dq = bdx \sqrt{2gx},$$

and therefore the total discharge Q across the section MN is

$$Q = \int dq = \int_{h_1+H}^{h_2+H} b \cdot dx \sqrt{2gx}$$

$$= \frac{2}{3} b \sqrt{2g} \{ (h_2 + H)^{\frac{3}{2}} - (h_1 + H)^{\frac{3}{2}} \}.$$

$$\text{Put } c = \frac{b \{ (h_2 + H)^{\frac{3}{2}} - (h_1 + H)^{\frac{3}{2}} \}}{B \{ (H_2 + H)^{\frac{3}{2}} - (H_1 + H)^{\frac{3}{2}} \}}.$$

Then

$$Q = \frac{2}{3} c B \sqrt{2g} \{ (H_2 + H)^{\frac{3}{2}} - (H_1 + H)^{\frac{3}{2}} \}. \quad (1)$$

The coefficient c is by no means constant, but is found to vary both with the head of water and also with the dimensions of the orifice, and can only be determined by experiment.

Second. Let the orifice be partially (Fig. 51) submerged, and let H_3 be the depth between the surface of the tail-race water and the free surface at A .

By what precedes, the discharge Q_1 through EG , the portion of the orifice clear above the tail-race, is

$$Q_1 = \frac{2}{3} c_1 B \sqrt{2g} \{ (H_3 + H)^{\frac{3}{2}} - (H_1 + H)^{\frac{3}{2}} \}. \quad (2)$$

Every fluid filament flows through the portion GF of the orifice under an effective head $H_3 + H$, and therefore with a velocity equal to

$$\sqrt{2g(H_3 + H)}.$$

Hence the discharge Q_2 through GF is

$$Q_2 = c_2 B (H_2 - H_3) \sqrt{2g(H_3 + H)}, \quad (3)$$

and the total discharge Q is equal to $Q_1 + Q_2$.

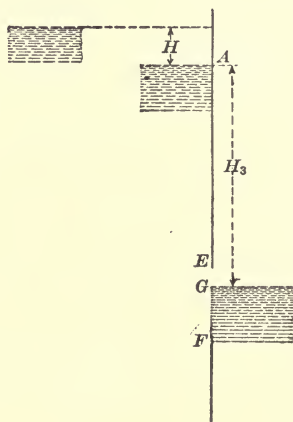


FIG. 51.

The coefficients c_1 , c_2 are to be determined by experiment, and if $c_1 = c_2 = c$,

$$Q = Q_1 + Q_2 = cB \sqrt{2g} \left[\frac{2}{3} \{ (H_3 + H)^{\frac{3}{2}} - (H_1 + H)^{\frac{3}{2}} \} + (H_2 - H_3) \sqrt{H_3 + H} \right]. \quad (4)$$

Third. Let the orifice be wholly submerged (Fig. 52).

Then the total discharge Q is evidently

$$Q = cB \sqrt{2g} (H_2 - H_1) \sqrt{H_3 + H}, \quad (5)$$

c being a coefficient to be determined by experiment.

If the velocity of approach, u , is sufficiently small to be disregarded without sensible error, then $H = 0$, and equations (1), (4), and (5), respectively, become

$$Q = \frac{2}{3} cB \sqrt{2g} (H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}); \quad (6)$$

$$Q = cB \sqrt{2g} \left\{ H_3^{\frac{1}{2}} \left(H_2 - \frac{H_3}{3} \right) - \frac{2}{3} H_1^{\frac{3}{2}} \right\}; \quad (7)$$

$$Q = cB \sqrt{2g} (H_2 - H_1) H_3^{\frac{1}{2}}. \quad (8)$$

FIG. 52.

(b) *Circular Orifices.*—Let Fig. 53 represent the minimum section of the circular jet issuing from a circular orifice.

Let 2θ be the angle subtended at the centre by the fluid lamina between the depths x and $x + dx$ below the surface of still water.

Let r be the radius of the section so that $2r = h_2 - h_1$, h_1 and h_2 being, as in (a), the depths of the highest and lowest points of the orifice below the free surface at A .

H , as before, is the head corresponding to the velocity of approach.

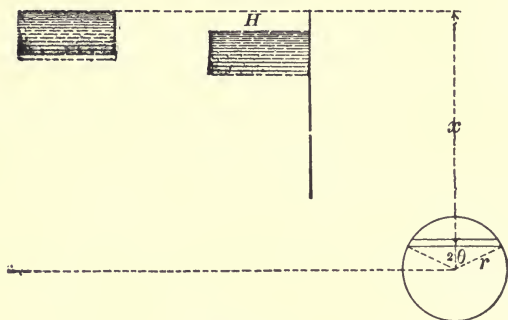


FIG. 53.

Then the area of the lamina under consideration

$$= 2r \sin \theta \cdot dx,$$

and the elementary discharge, dq , in this lamina, is

$$dq = 2r \sin \theta \cdot dx \sqrt{2gx}.$$

$$\text{But } x = \frac{h_1 + H + h_2 + H}{2} - r \cos \theta = \frac{h_1 + h_2 + 2H}{2} - r \cos \theta,$$

and therefore

$$dx = r \sin \theta \cdot d\theta.$$

Hence

$$dq = 2r^2 \sin^2 \theta \sqrt{2g \left(\frac{h_1 + h_2 + 2H}{2} - r \cos \theta \right)} d\theta,$$

and the total discharge Q is

$$Q = 2r^2 \sqrt{2g} \int_0^\pi \sin^2 \theta \left(\frac{h_1 + h_2 + 2H}{2} - r \cos \theta \right)^{\frac{1}{2}} d\theta. \quad (9)$$

Ex. The free surface on the up-stream side is 5 ft. and on the down-stream side 1 ft. above the sill of a rectangular sluice 12 ft. wide. How much must the sluice be raised to give 105,000 gals. per minute?

$$105,000 \text{ gals. per. min.} = \frac{105,000}{6\frac{1}{4} \times 60} = 280 \text{ cu. ft. per sec.}$$

Let x ft. be the opening above the sill. For a depth of 1 ft. above the sill the discharge is under a constant head of $5 - 1 = 4$ ft. For the remainder of the opening the discharge takes place freely through a rectangular orifice, with its upper and lower boundaries respectively $(5 - x)$ ft. and 4 ft. below the up-stream surface. Then

$$280 = \frac{5}{8} \cdot 12 \cdot 1 \cdot \sqrt{64} \cdot 4^{\frac{3}{2}} + \frac{2}{3} \cdot \frac{5}{8} \cdot 12 \cdot \sqrt{64} \{-(5 - x)^{\frac{3}{2}} + 4^{\frac{3}{2}}\}$$

$$= 440 - 40(5 - x)^{\frac{3}{2}}.$$

Therefore $(5 - x)^{\frac{3}{2}} = 4$, and $x = 2.48$ ft.

23. Notches and Weirs.—When an orifice extends up to the free-surface level it becomes what is called a *notch*.

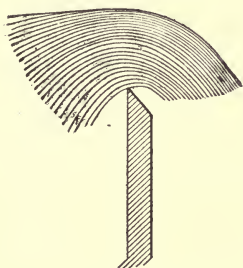


FIG. 54.

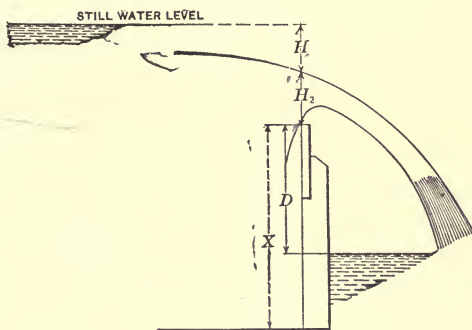


FIG. 55.

A *weir* is a structure over which the water flows, the discharge being in the same conditions as for a notch, and is very useful for gauging the flow of small streams, the amount of water supplied to hydraulic motors, etc.

Rectangular Notch or Weir.—The discharge may be found by putting $H_1 = 0$.

Thus equation (I) becomes

$$Q = \frac{2}{3} cB \sqrt{2g} \{ (H_2 + H)^{\frac{3}{2}} - H^{\frac{3}{2}} \}. \quad (10)$$

If the velocity of approach be disregarded, then $H = 0$, and the last equation becomes

$$Q = \frac{2}{3} cB \sqrt{2g} H_2^{\frac{3}{2}}, \quad (11)$$

and H_2 is the depth to the bottom of the notch or to the crest of the weir.

Great care should be taken in obtaining the accurate value of H_2 . A hook or a stiff vertical rod, with a sharp point, may be fixed, at a suitable distance (5 to 8 ft.) from the back of the weir, with the point on a level with the crest of the weir. The flume is then filled with water rising slightly above the crest and producing a capillary elevation of the surface at the point. The water is now allowed to subside until the elevation is barely perceptible, when a hook-gauge (Chap. III) is adjusted and a reading taken. A second reading is taken for any required discharge over the weir, and the difference between the two readings is the depth, H_2 , of the water on the crest.

It has been found that the discharge (Q) is appreciably affected by vibration, and it is therefore of importance that the weir should be made as rigid as possible. The up-stream face of the weir is nearly always vertical and at right angles to the direction of flow.

To diminish the effect of the velocity of approach, the water-section in the flume should be large as compared with the section of the waterway on the crest, and the depth of the weir should therefore be at least *twice* the depth H_2 of the water on the crest.

The crest should be horizontal and, generally speaking, it consists of a plate with a bevelled edge, Fig. 54, on the up-stream side, or of a thin plate, Fig. 55, so that the water springs clear from the inner edge.

A *rounded* edge, Fig. 56, diminishes the discharge and should be avoided, as its effect is uncertain.

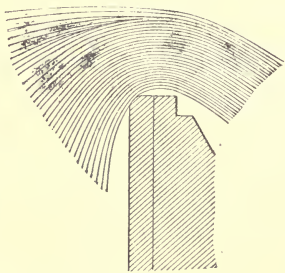


FIG. 56.

The length B of the crest should be at least *three* times the depth H_2 .

The effective sectional area of the water flowing through a rectangular notch, or over a weir, is less than BH_2 , because of (a) crest contraction, (b) end contraction, (c) the fall of the free surface towards the point of discharge.

It is reasonable to assume that the diminution of the actual sectional area, BH_2 , due to crest contraction and to the fall of the free-surface level is proportional to the width B of the opening.

Suppressed Weir, or Weir without End Contractions.—If a weir occupies the whole width of the stream, or flume, Figs. 57 and 59, the contraction at each end is wholly suppressed,

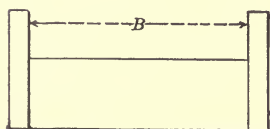


FIG. 57.

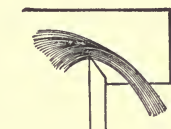


FIG. 58.

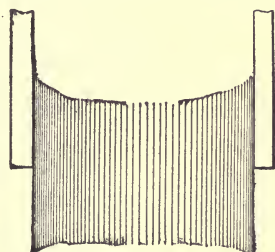


FIG. 59.

and crest contraction only takes place, i.e., the falling sheet of water is reduced in *thickness* near the crest. Air must be freely admitted below the falling sheet, as otherwise a partial or complete vacuum will be produced and the sheet will be depressed or will adhere to the face of the weir, while the discharge Q will be very sensibly modified. Francis effected the free admission of air and also prevented the lateral spreading of the sheet, after leaving the crest, by prolonging the upper portions of the flume sides a short distance beyond the weir, Fig. 58. The discharge was thereby diminished by about .4 per cent.

Weir with End Contractions.—These contractions occur when the sides of the weir, or notch, Figs. 60 and 61, are at a distance from the sides of the channel, and they have the

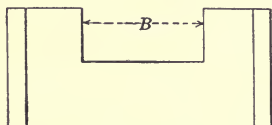


FIG. 60.

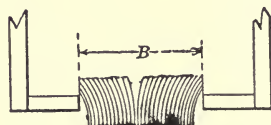


FIG. 61.

effect of diminishing the discharge. The contraction is *complete*, i.e., as great as it can be made, when the distance of the weir side from the channel side is not less than about the depth H_2 .

Other things being equal, the contraction and its effect upon the discharge increase with H_2 . The effect of end contractions is almost inappreciable and may be disregarded when the length B of the crest is not less than about H_2 ; but as the ratio $\frac{B}{H_2}$ diminishes, the effect rapidly increases. Francis found that the discharge for a weir with perfect end contractions and in which $B = 4H_2$, was diminished 6 per cent.

In his Lowell weir experiments he also found that, for depths $H_2 + H$ over the crest varying from 3 ins. to 24 ins., and for widths B not less than *three* times the depth, a perfect end contraction had the effect of diminishing the width of the fluid section by an amount approximately equal to one-tenth of the depth, or $\frac{H_2 + H}{10}$, so that the effective width =

$$B - \frac{H_2 + H}{10}.$$

Thus, if there are n end contractions, the effective width = $B - \frac{n}{10}(H_2 + H)$, and the equation giving the discharge

becomes

$$Q = \frac{2}{3}c \left\{ B - \frac{n}{10}(H_2 + H) \right\} \sqrt{2g} \left\{ (H_2 + H)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\}. \quad (12)$$

According to Francis the average value of c in this equation is .622.

Then $\frac{2}{3}c \sqrt{2g} = 3.33$, very nearly, and therefore

$$Q = 3.33 \left\{ B - \frac{n}{10}(H_2 + H) \right\} \left\{ (H_2 + H)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\}. \quad (13)$$

In experiments carried out by Fteley and Stearns with suppressed weirs, as described above, the total variation in the value of the coefficient was found to be about $2\frac{1}{2}$ per cent. The depths H_2 were measured 6 ft. from the weir, and for values of H_2 exceeding .07 ft. they deduced the formula

$$Q = B(3.31H_2^{\frac{3}{2}} + .007),$$

in which the velocity of approach is disregarded.

Allowance may be made for the velocity of approach by substituting for H_2 the expression $H_2 + 1\frac{1}{2}H$ according to Fteley and Stearns, but $H_2 + 1\frac{1}{3}H$ according to Hamilton Smith, Jr., who bases his conclusions upon a comparison of the experiments of Fteley and Stearns with those of Francis and others.

If the weir has n end contractions, $B - n \frac{H_2 + H}{10}$ must be substituted for B , and allowance is made for the velocity of approach by substituting for H_2 the expression $H_2 + 2.05H$, according to Fteley and Stearns, or $H_2 + 1.4H$, according to Hamilton Smith, Jr.

Hunking and Hart give the formula

$$Q = 3.33\mu \left(B - n \frac{H_2}{10} \right) H_2^{\frac{3}{2}},$$

in which μ is very nearly $= 1 + \frac{1}{4} \left(\frac{H_2}{S} \right)^2$, where

$$S = \frac{\text{sectional area of waterway}}{B - n \frac{H_2^2}{10}}.$$

Bazin gives the formula

$$Q = c \sqrt{2g} B H_2^{\frac{3}{2}},$$

in which, if the velocity of approach is disregarded,

$$c = .405 + \frac{.00984}{H_2},$$

but if allowance is to be made for the velocity of approach,

$$c = \left(.405 + \frac{.00984}{H_2} \right) \left\{ 1 + .55 \left(\frac{H_2}{H_2 + x} \right)^2 \right\},$$

x being the height of the weir.

Bazin considers that, with suppressed weirs, as already described and which are not very low, the results obtained with this coefficient are accurate within 1 per cent.

Submerged, or Drowned, Dams (or Weirs).—In these the surface of the tail-race water rises above the top of the dam,

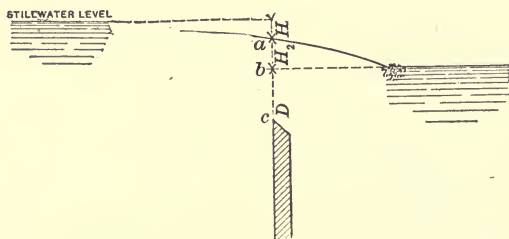


FIG. 62.

Fig. 62. It may be assumed that between a and b the flow is as over the crest of a weir, the depth of water on the crest

being $H_2 + H$, and that between b and c the flow is equivalent to that through a submerged orifice under a constant head $H_2 + H$. Hence, if H' is the depth of the top of the dam below the surface of the tailwater, and if c is the coefficient of discharge both for the flow between a and b and also that between b and c ,

$$Q = \frac{2}{3}c\sqrt{2g}B\{(H_2 + H)^{\frac{3}{2}} - H'^{\frac{3}{2}}\} + c\sqrt{2g}B(H_2 - H')(H_2 + H)^{\frac{1}{2}}.$$

The following table gives approximate values of c corresponding to different values of the ratio $\frac{H'}{H_2 + H + H'}$, as deduced from experiments carried out by Francis, the head over the crest varying from 1 to 2.32 ft., and by Fteley and Stearns, the head varying from .325 to .815 ft.:

Values of $\frac{H'}{H_2 + H + H'}$	Corresponding Values of c as deduced from the experiments of Francis, Fteley and Stearns.	
.05.....	.623 to .632
.10.....	.620 " .630	.625 to .635
.20.....	.610 " .625	.618 " .628
.30.....	.598 " .615	.600 " .610
.40.....	.586 " .610	.590 " .600
.50.....	.585 " .607	.585 " .595
.60.....	.585 " .607	.583 " .593
.70.....	.585 " .607	.580 " .590
.80.....	.585 " .607	.581 " .591
.90.....590 " .600
.95.....610 " .615

(*Trautwine.*)

Inclined Weirs.—If the up-stream face of a weir, instead of being vertical, is inclined up-stream, Fig. 63, the discharge is diminished, the depression of the upper surface of the falling sheet of water commences near the crest, while the lower surface rises higher, above the crest, and moves backwards.

If the face is inclined down-stream, Fig. 64, the discharge is increased, the depression of the upper surface commences at a point farther from the crest than when the face is vertical,

while the lower surface becomes more flattened and moves away from the weir.

Values of the coefficient of discharge for inclined weirs have been deduced by Bazin and are given in a subsequent article.

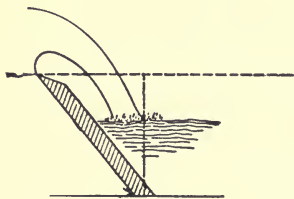


FIG. 63.

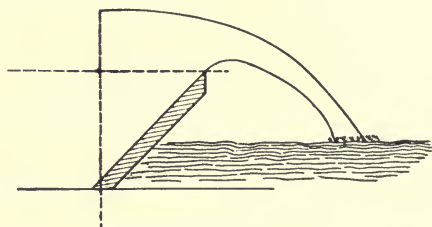


FIG. 64.

The discharge is increased by rounding the up-stream edge of the weir.

Circular Notch.—In equation (9), Art. 22, put $h_1 = 0$ and $h_2 = 2r$. Then

$$Q = 2r^2 \sqrt{2g} \int_0^\pi \sin^2 \theta \left(H + 2r \sin^2 \frac{\theta}{2} \right)^{\frac{1}{2}} d\theta,$$

and if the velocity of approach be disregarded, so that $H = 0$,

$$\begin{aligned} Q &= 2r^{\frac{5}{2}} \sqrt{4g} \int_0^\pi \sin^2 \theta \cdot \sin \frac{\theta}{2} d\theta \\ &= \sqrt{r^5 g} \int_0^\pi \left(2 \sin \frac{\theta}{2} - \sin \frac{5\theta}{2} + \sin \frac{3\theta}{2} \right) d\theta \\ &= \frac{64}{15} \sqrt{r^5 g}. \end{aligned}$$

EX. 1. A dam with a rectangular notch 6 ft. wide is formed across a channel; and the depth of the water over the sill is 12 ins. Find the quantity of flow when the notch has (a) no side contraction; (b) one side contraction; (c) two side contractions.

Disregarding the velocity of approach, and assuming the coefficient of discharge to be the same in each case, viz., $\frac{2}{3}$,

$$(a) \quad Q_1 = \frac{2}{3} \cdot \frac{5}{8} \cdot \sqrt{64} \cdot 1\frac{2}{3} \cdot 6 = 20 \text{ cu. ft. per sec.}$$

$$(b) \quad Q_2 = \frac{2}{3} \cdot \frac{5}{8} \cdot \sqrt{64} \cdot 1\frac{2}{3}(6 - \frac{1}{10}) = 19\frac{3}{8} \text{ cu. ft. per sec.}$$

$$(c) \quad Q_3 = \frac{2}{3} \cdot \frac{5}{8} \cdot \sqrt{64} \cdot 1\frac{2}{3}(6 - \frac{2}{10}) = 19\frac{1}{8} \text{ cu. ft. per sec.}$$

EX. 2. 400 cu. ft. of water per second are conveyed by a channel of rectangular section 25 ft. wide, when the water runs 4 ft. deep. Find the height of a dam built across the channel which will increase the depth 50 per cent, taking into account the velocity of approach.

$$\text{The velocity of approach} = \frac{400}{25 \times 6} = \frac{8}{3} \text{ ft. per sec.}$$

$$\text{The corresponding head} = \frac{(\frac{8}{3})^2}{64} = \frac{1}{9} \text{ ft.}$$

Let x ft. be the height of the dam.

First. Assume that the dam is not *drowned*, i.e., that its crest rises above the water-surface on the down-stream side. Then

$$400 = \frac{2}{3} \cdot \frac{5}{8} \cdot 25 \cdot \sqrt{64} \left\{ (6 - x + \frac{1}{9})^{\frac{3}{2}} - (\frac{1}{9})^{\frac{3}{2}} \right\},$$

$$\text{or} \quad (6 - x + \frac{1}{9})^{\frac{3}{2}} = 4.837037 = (6.111 - x)^{\frac{3}{2}},$$

and $x = 3.25$ ft., which is *less than* 4 ft., and therefore the assumption that the dam is not drowned is incorrect.

Second. Assuming that the dam is drowned, the discharge now takes place under a constant head of $(2 + \frac{1}{9})$ ft. for a depth of $(4 - x)$ ft., and as over a weir for a depth of 2 ft. Then

$$400 = \frac{5}{8} \cdot 25(4 - x) \sqrt{64} (2 + \frac{1}{9})^{\frac{1}{2}} + \frac{2}{3} \cdot \frac{5}{8} \cdot 25 \cdot \sqrt{64} \cdot \left\{ (2 + \frac{1}{9})^{\frac{3}{2}} - (\frac{1}{9})^{\frac{3}{2}} \right\},$$

$$\text{or} \quad 1.1798 = (4 - x)(2\frac{1}{9})^{\frac{1}{2}},$$

and $x = 3.188$ ft., which is *less than* 4 ft., and therefore the assumption of a drowned dam is correct.

EX. 3. If x is the depth of water over the crest of a rectangular notch, then, disregarding the velocity of approach,

$$Q = \frac{2}{3} c B \sqrt{2g} x^{\frac{3}{2}}.$$

Let dQ be the change in the discharge corresponding to a change dx in the depth on the sill. Then

$$dQ = \frac{2}{3} cB \sqrt{2g} \cdot \frac{3}{2} x^{\frac{1}{2}} \cdot dx.$$

Hence

$$\frac{dQ}{Q} = \frac{3}{2} \frac{dx}{x}.$$

Thus a change of 6 per cent in the discharge corresponds to a change of 4 per cent in the sill depth, and a change of 10 per cent in this depth corresponds to a change of 15 per cent in the discharge.

24. Triangular Notch.—Disregard the velocity of approach and let B be the width of the free surface.

As before, consider a lamina of fluid between the depths x and $x + dx$.

The area of the lamina

$$= \frac{B}{H_2} (H_2 - x) dx,$$

and the discharge in this lamina is

$$dq = c \frac{B}{H_2} (H_2 - x) dx \sqrt{2gx}.$$

Hence the total discharge Q is

$$\begin{aligned} Q &= c \int_0^{H_2} \frac{B}{H_2} (H_2 - x) dx \sqrt{2gx} \\ &= c \frac{B}{H_2} \sqrt{2g} \int_0^{H_2} (H_2 x^{\frac{1}{2}} - x^{\frac{3}{2}}) dx \\ &= \frac{4}{15} cB \sqrt{2g} H_2^{\frac{5}{2}} \text{ cu. ft. per sec.} \quad (14) \end{aligned}$$

c is a coefficient introduced to allow for contraction, etc., and Professor James Thomson gives .617 as its mean value for a sharp-edged triangular notch.

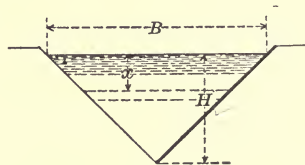


FIG. 65.

Now the ratio $\frac{B}{H_2}$ is constant in a triangular notch and varies in a rectangular notch. Hence Thomson inferred and showed by experiment that the value of c is more uniform for triangular than for rectangular notches, and therefore also the former must give more accurate results.

If the flow is through a 90° notch, $B = 2H_2$, and

$$Q = \frac{8}{15} c \sqrt{2g} H_2^{\frac{5}{2}} = 2.64 H_2^{\frac{5}{2}} \text{ cu. ft. per sec., approximately,}$$

or

$$= 158.385 H_2^{\frac{5}{2}} \text{ cu. ft. per min.,}$$

c being .617 and $g = 32.176$.

Ex. 1. A reservoir discharges through a sharp-edge triangular notch, and in t secs. the depth of the water in the notch falls from H ft. to x ft.

Let S be the sectional area of the reservoir corresponding to the x ft. depth; let mx be the width of the free surface on the notch corresponding to the x ft. depth, m being a numerical coefficient depending upon the notch angle.

Then, since the water sinks dx ft. in dt secs.,

— $S \cdot dx$ = discharge from reservoir in dt secs.

= amount flowing through notch in dt secs.

$$= \frac{4}{15} \sqrt{2g} c \cdot mx^{\frac{5}{2}} \cdot dt,$$

or

$$dt = - \frac{15S}{4 \sqrt{2gcm}} x^{-\frac{5}{2}} \cdot dx.$$

Hence the time in secs. in which the depth falls from H ft. to x ft.

$$= + \frac{15}{4 \sqrt{2gcm}} \int_x^H S x^{-\frac{5}{2}} \cdot dx.$$

If the horizontal sectional area S is constant,

$$\text{the time in secs.} = \frac{5S}{2 \sqrt{2gcm}} \left(\frac{1}{x^{\frac{3}{2}}} - \frac{1}{H^{\frac{3}{2}}} \right).$$

For a 90° notch $m = 2$, and taking $g = 32$ and $c = \frac{5}{8}$,

$$\text{the time in secs.} = \frac{S}{4} \left(\frac{1}{x^{\frac{3}{2}}} - \frac{1}{H^{\frac{3}{2}}} \right).$$

The time becomes infinite when $x = 0$, which indicates that the flow diminishes indefinitely with the depth in the notch.

EX. 2. Find the discharge in gallons per minute through a 90° sharp-edge notch when the water runs 4 ft. deep. If the reservoir supplying the water has a constant horizontal sectional area of 80,000 sq. ft., in what time will the level sink 3 ft.?

$$Q = \frac{4}{15} \sqrt{64} \cdot \frac{5}{8} \cdot 2 \cdot 4^{\frac{5}{2}} = 85\frac{1}{2} \text{ cu. ft. per sec.} = 85\frac{1}{2} \times 6\frac{1}{2} \times 60 \text{ gals. per min.} \\ = 32,000 \text{ gals. per min.}$$

$$\text{the time} = \frac{80000}{4} \left(1 - \frac{1}{4^{\frac{3}{2}}} \right) = 17,500 \text{ secs.} = 4\frac{5}{8} \text{ hours.}$$

25. Broad-crested Weir.—Let Fig. 66 represent a stream flowing over a broad-crested weir. On the up-stream side the

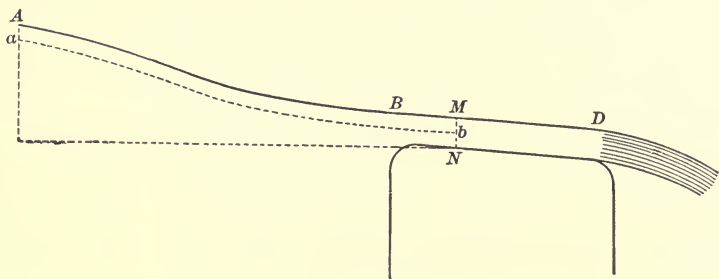


FIG. 66.

free surface falls from A to B . For a distance BD on the crest the fluid filaments are sensibly rectilinear and parallel; the inner edge of the crest is rounded so as to prevent crest contraction.

Consider a filament ab , the point a being taken in a part of the stream where the velocity of flow is so small that it may be disregarded without sensible error.

Let λ be the thickness MN of the stream at b .

Let the horizontal plane through N be the datum plane.

Let z_1, z be the depths below the free surface of a and b .

Let h_1 be the elevation of a above datum.

Let p_0 , p_1 , p be the atmospheric pressure and the pressures at a and b .

Let v be the velocity of flow at b .

Then, by Bernouilli's theorem,

$$h_1 + \frac{p_1}{w} = \lambda - z + \frac{p}{w} + \frac{v^2}{2g}.$$

But

$$\frac{p_1}{w} = z_1 + \frac{p_0}{w} \quad \text{and} \quad \frac{p}{w} = z + \frac{p_0}{w};$$

therefore

$$h_1 + z_1 + \frac{p_0}{w} = \lambda - z + z + \frac{p_0}{w} + \frac{v^2}{2g},$$

and hence

$$\frac{v^2}{2g} = h_1 + z_1 - \lambda = H_2 - \lambda,$$

H_2 being the depth of the crest of the weir below the surface of still water.

Thus, if B be the width of the weir, the discharge Q is

$$Q = B\lambda \sqrt{2g(H_2 - \lambda)}. \quad \dots \quad (16)$$

From this equation it appears that Q is nil both when $\lambda = 0$ and when $\lambda = H_2$. Hence there must be some value of λ between 0 and H_2 for which Q is a maximum. This value may be found by putting

$$dQ = 0 = B \sqrt{2g} \left(\sqrt{H_2 - \lambda} - \frac{1}{2} \frac{\lambda}{\sqrt{H_2 - \lambda}} \right) d\lambda,$$

and therefore

$$\lambda = \frac{2}{3} H_2,$$

and the expression for the discharge becomes

$$Q = \frac{2}{3\sqrt{3}}BH_2\sqrt{2gH_2} = .385B\sqrt{2g}H_2^{\frac{3}{2}}, \quad (17)$$

which is the maximum discharge for the given conditions.

Experiment shows that the more correct value for the discharge is

$$Q = .35B\sqrt{2g}H_2^{\frac{3}{2}}. \quad (18)$$

If the water approaches the weir with an appreciable velocity u , corresponding to the head H , so that $\frac{u^2}{2g} = H$, then

$$\frac{v^2}{2g} = H_2 + H - \lambda,$$

and

$$Q = .35B\sqrt{2g}(H_2 + H)^{\frac{3}{2}}.$$

This formula agrees with the ordinary expression for the discharge over a weir as given by equation (11), if $c = .525$.

It might be inferred that for broad-crested weirs and large masonry sluice-openings the discharge should be determined by means of equation (18) rather than by the ordinary weir formula, viz., equation (11).

It must be remembered, however, that in deducing equation (17), frictional resistances have been disregarded and the gratuitous assumption has been made that the stream adjusts itself to a thickness t which will give a maximum discharge. The theory is therefore incomplete.

The discharge over a sharp-crested weir is sensibly the same as that over a weir with an apron, as in Fig. 66, so long as the depth of the water on the crest is not less than about 15 ins., but below this limit, the discharge over the apron

rapidly diminishes with the depth. For example, the discharge over a sharp-crested weir is approximately double that over a weir with an apron when the depth is about 1 in., is 20 per cent greater when the depth is 6 ins., and 10 per cent greater when the depth is 12 ins.

26. Reservoir Sluices.—The water flows into the receiving channel either freely, as in Fig. 67, or under water, as in Fig. 68.

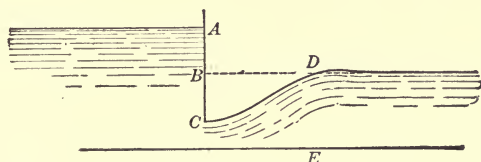


FIG. 67.

In the first case, the stream-lines converge to a contracted

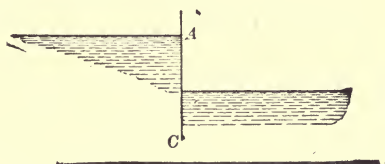


FIG. 68.

section, and between the sluice and a certain section *DE* there is a sudden swell, the height of swell being given by

$$BC = \frac{v_1^2}{2g} - \frac{v_2^2}{2g},$$

and v_1 , v_2 being the velocities of flow across the contracted section and the section at *DE*.

Let A_1 , A_2 be the areas of the sluice and section at *DE*; and let $nA_2 = A_1$. Then

$$BC = \frac{v_1^2}{2g} \left(1 - \frac{v_2^2}{v_1^2} \right) = \frac{v_1^2}{2g} \left(1 - \frac{c_c^2}{n^2} \right),$$

c_c being the coefficient of contraction.

Thus the swell will be found to be further from or nearer the sluice, according as the difference between the depths of the stream and the sluice is $>$ or $<$ BC .

If f_2 is the coefficient of hydraulic resistance, then

$$(1 + f_2) \frac{v_1^2}{2g} = AB,$$

and f_2 may be .1 or even greater; but if the sluice edges are smoothed and rounded so that f_2 can be disregarded, then

$$\frac{v_1^2}{2g} = AB = BC + \frac{v_2^2}{2g},$$

and therefore $AB - BC = AC = \frac{v_2^2}{2g}$.

It is assumed that the water in the reservoir retains the same level, but where the flow commences there is a depression in the surface due to the velocity of flow, and the amount of this depression should be deducted from the total head.

When the backwater rises above the sluice, as in Fig. 68,

AC = head required to produce v_2 + head "lost in shock"

$$= \frac{v_2^2}{2g} + \frac{(v_1 - v_2)^2}{2g} = \frac{v_2^2}{2g} \left\{ 1 + \left(\frac{n}{c_c} - 1 \right) \right\},$$

and AC increases with n , i.e., as A_1 diminishes as compared with A_2 .

27. Bazin's Flow Over Weirs.—This article is the résumé of Bazin's valuable papers on this subject published in the *Annales des Ponts et Chaussées*. The symbols are changed to correspond with the preceding articles of the present chapter.

Let c_s , B_s , H_s be the coefficient, length of crest, and head over crest for a standard weir.

Let c , B , H_2 be the corresponding symbols for an experimental weir. Then, disregarding the velocity of approach,

$$c_s B_s H_s^{\frac{3}{2}} \sqrt{2g} = Q = c B H_2^{\frac{3}{2}} \sqrt{2g},$$

and

$$c = c_s \frac{B_s}{B} \left(\frac{H_s}{H_2} \right)^{\frac{3}{2}}.$$

Experiments with the standard weir give the value of c_s , the ratio $\frac{B_s}{B}$ is usually *unity*, and the ratio $\frac{H_s}{H_2}$ is found by observation. Hence the value of c can be at once calculated.

In practice it seems impossible, with the data at present available, to make a rational selection of the proper value of c , which varies between wide limits and is affected not only by the form of the weir but by other conditions, amongst which may be enumerated the following:—

- (a) The *velocity of approach*, which cannot be disregarded when the weir is of small height.
- (b) The *height* of the weir.
- (c) The *crest contraction*, which depends both upon the height of the weir and the form of the crest.
- (d) The *end contractions*, which have a considerable influence when the weirs are of comparatively small width, but are not of so much importance when the weirs are long.
- (e) The *form of the nappe*, which may vary considerably, and which in every case should be the subject of a careful investigation.

Sharp-crested Weir (Figs. 54, 55).—The simplest and best defined case, and one which admits of an exact determination of the coefficient of discharge c , is that of a free nappe (or sheet), the sheet of water flowing over the weir without end contraction, and with its lower as well as upper surface fully exposed to atmospheric pressure. Allowance may be made for the influence upon the discharge of the velocity of approach, u , by substituting for the head, H_2 , over the crest in the discharge formula, the expression $H_2 + \alpha \frac{u^2}{2g}$, α being a coefficient which

has not been accurately determined. Thus

$$Q = \mu B \sqrt{2g} \left(H_2 + \alpha \frac{u^2}{2g} \right)^{\frac{3}{2}},$$

μ being the modified value of c .

But $\frac{u^2}{2gh}$ is very small, rarely exceeding a few centimeters, and therefore, approximately,

$$Q = \mu B H_2^{\frac{3}{2}} \sqrt{2g} \left(1 + \frac{3}{2} \alpha \frac{u^2}{2g H_2} \right).$$

Let x be the height of the weir. Then

$$u B (H_2 + x) = Q = c B \sqrt{2g} H_2^{\frac{3}{2}},$$

and therefore

$$\frac{u^2}{2g H_2} = c^2 \left(\frac{H_2}{H_2 + x} \right)^2.$$

Hence, putting $K = \frac{3}{2} \alpha c^2$,

$$Q = \mu \left\{ 1 + K \left(\frac{H_2}{H_2 + x} \right)^2 \right\} B \sqrt{2g} H_2^{\frac{3}{2}},$$

so that

$$c = \mu \left\{ 1 + K \left(\frac{H_2}{H_2 + x} \right)^2 \right\}.$$

Bazin has deduced the values of α , K , and μ by comparative experiments on five weirs of different heights.

α and K are not constant, but their mean values are $\frac{5}{8}$ and .55 respectively. The coefficient μ slowly diminishes as the head h increases.

Thus

	for heads = 0 ^m .05	0 ^m .10	0 ^m .20	0 ^m .30	0 ^m .40	0 ^m .50
the corresponding values of μ =	.448	.432	.421	.417	.414	.412

For values of $H_2 > 0^m.10$, it is sufficiently accurate to take

$$\mu = .405 + \frac{.003}{H_2},$$

and therefore

$$Q = \left(.405 + \frac{.003}{H_2} \right) \left\{ 1 + .55 \left(\frac{H_2}{H_2 + x} \right)^2 \right\}^B \sqrt{2g} H_2^{\frac{3}{2}}$$

so that

$$c = \left(.405 + \frac{.003}{H_2} \right) \left\{ 1 + .55 \left(\frac{H_2}{H_2 + x} \right)^2 \right\}.$$

Generally, for values of H_2 between 0^m.10 and 0^m.30, μ may be made equal to .425, and, taking $K = .5$,

$$c = .425 \left\{ 1 + \frac{1}{2} \left(\frac{H_2}{H_2 + x} \right)^2 \right\} = .425 + .212 \left(\frac{H_2}{H_2 + x} \right)^2,$$

a suitable form for practical use when errors of 2 to 3 per cent are not too large to be of importance.

The absolute values of c having been found for a sharp-crested weir with a free nappe and a vertical face on the up-stream side, it does not follow that the same method should be adopted to determine the corresponding coefficients for other forms of weir. In fact, if c' is the coefficient for any other given weir, when the head over the crest is the same, the influence of the velocity of approach may be largely eliminated by finding the ratio $\frac{c'}{c}$. The ratio corresponding to two different inclinations is sensibly constant for all heads, and the following table gives the values of $\frac{c'}{c}$ for varying face-slopes:—

For an up-stream face-slope of 1 hor. to 1 vert.....	$\frac{c'}{c} =$.93
“ “ “ 2 “ 3 “	“ =	.94
“ “ “ 1 “ 3 “	“ =	.96
“ “ vertical face.....	“ =	1.00
For a down-stream face-slope of 1 hor. to 3 vert.....	“ =	1.04
“ “ “ 2 “ 3 “	“ =	1.07
“ “ “ 1 “ 1 “	“ =	1.10
“ “ “ 2 “ 1 “	“ =	1.12
“ “ “ 4 “ 1 “	“ =	1.09

It may be noted that the coefficient (or ratio) gradually increases from .93, corresponding to a slope of 45° on the up-stream side, to 1.12, corresponding to a slope of about 30° on the down-stream side.

When the air cannot pass underneath the sheet of water flowing over the crest, the nappe either encloses a volume of air at less than the atmospheric pressure and is *depressed*, Fig. 69, or the air is entirely ex-

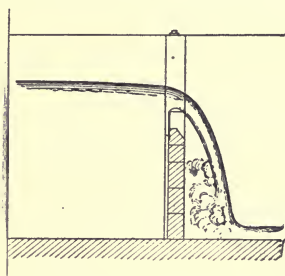


FIG. 69.—Depressed Nappe.

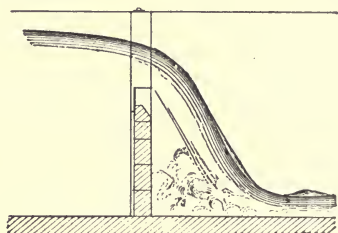


FIG. 70.—Drowned Nappe.

cluded and the nappe is wetted *underneath* or *drowned*, Fig. 70. The latter condition, when the nappe encloses an eddying mass of fluid, gives a more uniform motion, as the pressure of an enclosed volume of air may vary from the accidental admission of new air. The discharge is slightly greater than with the free nappe, and may be increased almost 10 per cent when the nappe is on the point of being drowned. So long as the head exceeds a certain limit, the nappe will not be in contact with the weir face. The drowned nappe may be either independent of or influenced by the down-stream level according as the *rise* produced beyond the nappe is at a distance from the foot of the nappe or partially encloses the foot.

Rise at a Distance from the Foot of the Nappe.—In this case

$$\frac{c'}{c} = .878 + .128 \frac{x}{H_2},$$

but the max. value of $\frac{x}{H_2}$ cannot exceed $2\frac{1}{2}$, as the *drowned* condition no

longer holds when $H_2 < \frac{2}{5}x$. The value of $\frac{c'}{c}$, corresponding to this maximum, is 1.2, and if $H_2 = x$, the coefficients c' and c are sensibly the same. Applying this formula to weirs of different heights, it is found that the absolute values of the coefficients of discharge are sensibly given by the formula

$$c' = .47 + .0075 \frac{x^2}{H_2^2}.$$

Rise Enclosing the Foot of the Nappe.—if D is the difference of level between the weir-crest and the down-stream surface,

$$\frac{c'}{c} = 1.06 + .16 \left(\frac{D}{x} - .05 \right) \frac{x}{H_2},$$

for which it is usually sufficiently accurate to substitute the simpler expression

$$\frac{c'}{c} = 1.05 + .15 \frac{D}{H_2}.$$

These formulæ are only true for values of D between certain limits. If $H_2 + D$ is greater than about $\frac{3}{4}x$, the rise is moved beyond the foot of the nappe, and the formulæ in the preceding case become applicable. Again, if the head H_2 is not sufficient to enable the nappe to push back the rise, the down-stream surface level must be sufficiently high to prevent the admission of air below the nappe.

The drowned nappe preserves its characteristic profile even when the down-stream surface is on a level with the weir crest, Fig. 71, but if the difference of level between the up- and down-stream surfaces still continues to diminish, a point is reached at which the nappe suddenly and with an *undulating* movement again forms part of the surface. This change, which is very apparent, does not seem to have much influence on the coefficient of discharge.

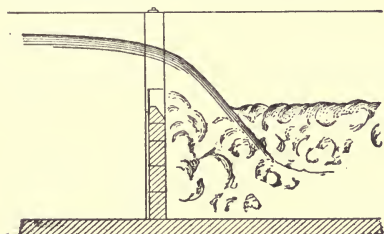


FIG. 71.—Drowned Nappe.

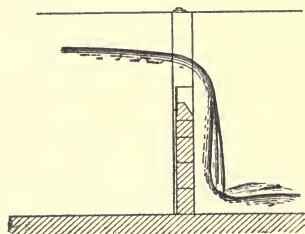


FIG. 72.—Adhering Nappe (H_2 not very small).

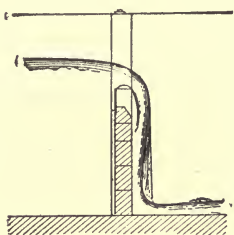


FIG. 73.—Adhering Nappe, springing clear above crest.

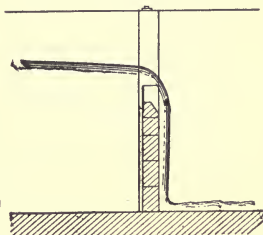


FIG. 74.—Adhering Nappe (H_2 small).

On certain rare occasions, and under conditions governed by the thickness of the weir and by the construction of the upper portion carrying the crest, the nappe becomes adherent, Figs. 72 to 76, the sheet of water remaining in contact with the weir face. The coefficient c' is then

increased and may become as large as 1.36, corresponding to an absolute value of .55 or .56.

From what has been said it may be at once inferred that the discharge over a weir is largely influenced by the form of the nappe.

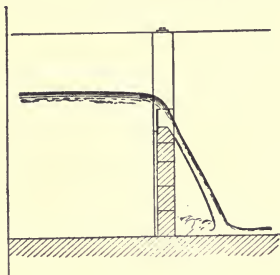


FIG. 75.—Nappe adhering on crest only.

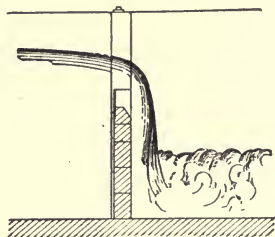


FIG. 76.

Taking, for example, a sharp-crested weir 0.75 m. high, it was found that for a head over the crest of 0.2 m. the coefficient of discharge c was

- .433 for a free nappe.
- .46 " a depressed nappe.
- .497 " a drowned nappe.
- .554 " an adhering nappe.

Beam Weirs.—These weirs are formed of squared timbers laid one above the other to any required height, the weir faces being vertical and the crest, or sill, having a width e equal to that of the timbers.

Free Nappe.—The nappe may either spring clear from the up-stream edge, when the case becomes that of a sharp-crested weir, or it may remain in contact with the sill and spring clear from the down-stream edge. The first case is at once realized if H_2 exceeds $2e$, and may occur for any value of h between $2e$ and $1\frac{1}{2}e$, the change being produced by any such extraneous disturbing cause as the admission of air or the passage of a floating body, etc.

When the nappe remains in contact with the sill,

$$c' = c \left(.7 + .185 \frac{H_2}{e} \right),$$

an expression depending essentially on the value of $\frac{H}{e}$.

$$\begin{array}{lcl} \text{For } \frac{H}{e} = .5 \dots \frac{c'}{c} = .79 \\ \quad \text{"} = 1.0 \dots \text{"} = .88 \\ \quad \text{"} = 1.5 \dots \text{"} = .98 \\ \quad \text{"} = 2.0 \dots \text{"} = 1.07 \end{array} \left. \vphantom{\begin{array}{l} .79 \\ .88 \\ .98 \\ 1.07 \end{array}} \right\} \begin{array}{l} \text{If the nappe remains in} \\ \text{contact with the sill.} \end{array}$$

The ratio $\frac{c'}{c}$ is *unity* for all values of $\frac{H_2}{e}$ above 2, and if the nappe springs clear from the up-stream edge, for all values of $\frac{H_2}{e}$ between $1\frac{1}{2}$ and 2.

With sills of considerable width, e.g., 1 or 2 m., the above formula still gives results which are approximately correct. The ratio $\frac{H_2}{e}$ may diminish to a few tenths or even less than .35. With a 2-m. flat-crested weir experiment gave for a head of .45 m., $\frac{c'}{c} = .755$, the corresponding absolute value of c' being .337. The formula gives $\frac{c'}{c} = .742$, the corresponding value of c' being .331.

The *rounding* of the up-stream edge of the sill has a very sensible influence upon the flow, and the effect of a radius of only 1 or 2 cm., as usually results from ordinary wear, must by no means be disregarded in gauging the discharge. Fteley and Stearns observed that the effect of a small radius R , not exceeding $\frac{1}{2}$ in., or 0.012 m., was to increase the head by $.7R$, and therefore the coefficient c' in the ratio of

$H_2^{\frac{3}{2}}$ to $(H_2 + .7R)^{\frac{3}{2}}$, or approximately 1 to $1 + \frac{R}{H_2}$. This approxima-

tion is not sufficiently accurately for sensibly greater radii. With two weirs, the one .8 m. and the other 2 m. wide, the up-stream edges being rounded to a radius of .10 m., the discharge was increased 14 per cent in the first and 12 per cent in the second case. With the 2-m. weir the coefficient c' for the greatest head used in the experiments was found to be .373, which is very nearly the same as the value theoretically deduced on the assumption that the flow over the weir is in fluid filaments parallel to the sill. This condition is only imperfectly realized in practice as the surface of the nappe invariably has an undulatory movement.

Depressed and Drowned Nappes.—With a sharp-crested weir the coefficient for a depressed nappe is always greater than that for a free nappe. With a beam weir, such as that now under consideration, the coefficients differ only slightly, that for the depressed weir being at first a little less, then about the same, and finally a little greater than the coefficient for the free nappe. When the nappe is *drowned*, the influence of contact with the sill is complicated by the fact that it is impossible to define exactly the point at which the nappe is freed from the sill, and this separation no longer corresponds to a certain constant value of $\frac{H_2}{e}$.

It may again occur either before or after the establishment of the *drowned* condition. Two cases may be distinguished. If x (the height of weir) $> 5e$, the separation takes place in advance of the drowned state, and in this intermediate condition the nappe does not differ from that which

flows over a sharp-crested weir. If $x < 5e$, the nappe is not freed from the sill before it assumes the drowned form, and at the moment of the change is very unstable.

So long as the contact with the sill continues, its influence predominates, and the formula

$$c' = c \left(.7 + .185 \frac{H_2}{e} \right)$$

is fairly applicable to the drowned nappe.

But when the nappe has left the sill, the phenomenon becomes more and more nearly the same as for a sharp-crested weir, and the formula now applicable is

$$c' = c \left(.878 + .128 \frac{x}{H_2} \right).$$

These two formulæ give the same value for c' for a certain limiting value of H_2 , given by

$$\frac{H'_2}{e} = \frac{1}{2} \left(1 + \sqrt{\frac{3x}{e}} \right).$$

The first formula holds when the heads are less than H'_2 , but the coefficients are a little too small although the errors are never more than 3 or 4 per cent. If the heads are greater than H'_2 , the second formula is to be used, but the results are again too small and the error in this case may be as much as 8 per cent at the moment when the nappe is separated from the sill. The error then rapidly diminishes as the head increases.

Wide-crested Weirs with Sloping Faces.—In these the coefficient c' , depending upon the head (H_2), the width (e) of crest, and the degree of face-slope, is now extremely variable and each case must be subjected to a special investigation. The face-slope on the up-stream side has the effect of diminishing the contraction and therefore increasing the discharge. The down-stream face-slope, on the other hand, produces an effect similar to the widening of the crest and diminishes the discharge. The rounding of the up-stream edge of the crest considerably diminishes the contraction and may increase c' by 10 or 15 per cent. Finally, c' is very largely increased for weirs with completely curved profiles.

Bazin has prepared Tables comprising a sufficient number of particular cases which may serve as a guide in practice. It is impracticable to establish a general formula which will take into account all the variable elements referred to.

Drowned Weirs with Sharp Crests.—When the water on the down-stream side does not stand much above the crest of the weir, Bazin gives the somewhat complicated formula

$$\frac{c'}{c} = 1.06 + \frac{1}{4} \frac{D}{x} - \left\{ .008 + \frac{1}{3} \frac{D}{x} + \frac{1}{3} \left(\frac{D}{x} \right)^2 \right\} \frac{x}{H_2}.$$

In the majority of cases, however, the following simpler formula is applicable:

$$\frac{c'}{c} = \left(1.08 + .18 \frac{H_2'}{x} \right) \sqrt[3]{\frac{H_2 - D}{H_2}}.$$

These two formulæ, established so as to represent as accurately as possible the particular experiments by which they have been deduced, may be replaced by

$$\frac{c'}{c} = \left(1.05 + .21 \frac{H_2'}{x} \right) \sqrt[3]{\frac{H_2 - D}{H_2}},$$

which will give results differing from those obtained with the other formulæ by not more than about 1 or 2 per cent, unless $\frac{H_2}{x}$ and $\frac{H_2'}{x}$ are very small, when the difference may be as much as 4 or 5 per cent, but in the latter case the determination of c' is always somewhat uncertain. The effect of drowning is not the same for wide-crested weirs. The flow on the up-stream side is not affected by the depth of the water on the down-stream side until the down-stream surface rises considerably above the weir crest, and the effect diminishes as the width of the crest increases. In the case of a sharp-crested weir the influence upon the up-stream flow is felt *before* the down-stream surface has reached the level of the crest. As the width of a wide-crested weir increases it loses its weir characteristics and approximates more and more closely to an open channel with horizontal bed.

Thickness of Nappe on Weir Crest.—Let t = thickness of nappe.

For a *sharp-crested* weir and *free* nappe $\frac{t}{H_2}$ varies from .85 to .86.

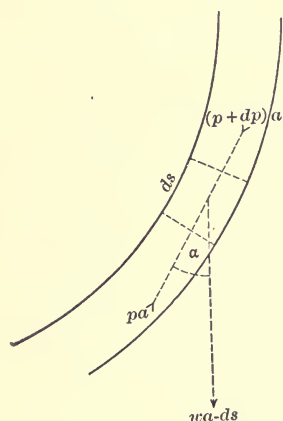
For a *sharp-crested* weir and *drowned* nappe $\frac{t}{H}$ increases with $\frac{h}{x}$, being .8 when $\frac{h}{x} = .4$, .855 when $\frac{h}{x} = 1$, and .87 when $\frac{h}{x} > 1$. As the down-stream level rises $\frac{t}{h}$ increases, exceeding .9 for the undulating condition, and necessarily tends to *unity* as the difference of level between the crest and the down-stream surface is greatly diminished.

In *beam* weirs with *free* nappes $\frac{t}{h} > .9$ for small values of $\frac{h}{c}$, and decreases as the head increases until the ratio becomes .855, when the nappe is on the point of separating from the sill.

In *beam* weirs with *drowned* nappes the variation of $\frac{t}{H_2}$ is somewhat complicated. The ratio diminishes until a minimum is reached, and

then increases and approximates to values which are the same as in the case of sharp-crested weirs.

In *wide-crested* weirs with *sloping faces* $\frac{t}{H_1}$ is very variable. Generally it increases as the down-stream slope diminishes, and diminishes with the up-stream slope. In weirs in which the crest is connected with the up-stream face by a curved surface $\frac{t}{H_2}$ may be less than .8, but the



determination of the nappe thickness is in such case much less accurate.

28. Bernoulli's Theorem.—A simple proof of this theorem is as follows:

Consider an indefinitely small element of a stream-line, of length ds and sectional area a .

Let $p, p + dp$ be the intensities of pressure at the ends.

" w be the specific weight of the fluid.

" α " " angle between the direction of motion of the element and the vertical.

" dz be the vertical projection of ds , so that $dz = ds \cdot \cos \alpha$.

Resolving in the direction of motion,

$$\begin{aligned} pa - (p + dp)a - wa \cdot ds \cos \alpha &= \text{accelerating force} \\ &= \text{mass} \times \text{acceleration} \\ &= \frac{w}{g} a \cdot ds \cdot \frac{dv}{dt} \\ &= \frac{w}{g} a \cdot v \cdot dt \cdot \frac{dv}{dt} \\ &= \frac{w}{g} av \cdot dv. \end{aligned}$$

$$\therefore -a \cdot dp - wa \cdot dz = -\frac{w}{g} av \cdot dv,$$

$$\text{or} \quad dz + \frac{dp}{w} + \frac{v \cdot dv}{g} = 0.$$

Integrating, $z + \int \frac{dp}{w} + \frac{v^2}{2g} = \text{a const.}$, is true for any fluid. If

the fluid is water, w is constant, and then $z + \frac{p}{w} + \frac{v^2}{2g} = \text{a const.}$

EXAMPLES.

(N. B. In the following examples $g = 32$ unless otherwise specified.)

1. T tons of water fall H feet per minute and are employed to turn turbines which transform into useful work one half of the total energy of the water. What is the H.P. of the turbines?

$$\text{Ans. } \frac{TH}{33} \checkmark$$

5/2

2. A turbine transforms into 9.72 H.P. of useful work the energy of the water falling $7\frac{1}{4}$ feet from a Thomson V-notch in which the water stands at a constant level ~~2~~³ ft. above the bottom of the notch. If the coefficient of discharge is .6, what is the efficiency of the turbine?

$$\text{Ans. } .8 \checkmark$$

3. A fall of 10 ft. supplies to a turbine 12 cu. ft. of water per sec. The turbine uses only 8 ft. of the fall, and the water leaves the turbine with a velocity of 8 ft. per sec. If 500 lbs.-ft. are lost in frictional resistance, etc., find the efficiency of the turbine.

$$\text{Ans. } .634 \checkmark$$

4. 10,000 50-volt incandescent and 250 450-watt arc lamps are to be supplied with power from a waterfall having an effective head of 40 ft., 20 miles distant. Losses between lamps and converting apparatus at receiving end of transmission, 5%; efficiency of converting apparatus, 92%; line losses, 10%; losses in generators and transformers between line and turbine shaft, 10%; efficiency of turbine, 85%. Required, necessary flow of water per hour.

$$\text{Ans. } 1,080,630 \text{ cu. ft. per hour.}$$

5. A frictionless pipe gradually contracts from a 6-in. diameter at A to a 3-in. diameter at B , the rise from A to B being 2 ft. If the delivery is 1 cu. ft. per second, find the difference of pressure between the two points A and B .

$$\text{Ans. } 504.6 \text{ lbs. per sq. ft.}$$

6. In a frictionless horizontal pipe discharging 10 cu. ft. of water per second, the diameter gradually changes from 4 in. at a point A to 6 in. at a point B . The pressure at the point B is 100 lbs. per square inch; find the pressure at the point A .

$$\text{Ans. } 4118 \text{ lbs. per sq. ft.}$$

7. A $\frac{1}{2}$ -in. horizontal pipe is gradually reduced in diameter to $\frac{1}{8}$ in. and then gradually expanded again to its mouth, where it is open to the atmosphere. Determine the maximum quantity of water which can be forced through the pipe (a) when the diameter of the mouth is $\frac{1}{2}$ in., (b) when the diameter is $\frac{3}{4}$ in. Also determine the corresponding velocities at the throat and the total heads (neglect friction, which, however, is very considerable).

$$\text{Ans. (a) } .24 \text{ cu. ft. per min. ; } 46.7 \text{ ft. per. sec.}$$

$$(b) .239 \text{ cu. ft. per min. ; } 46.66 \text{ ft. per sec.}$$

8. A short horizontal pipe ABC connecting two reservoirs gradually contracts in diameter from 1 in. at A to $\frac{1}{2}$ in. at B and then enlarges to 1 in. again at C . If the height of the water in the reservoir over C be 12 ins., determine the maximum flow through the pipe and sketch the curve of pressures. Also obtain an equation for this curve, assuming the rates of contraction and expansion of the pipe to be equal and uniform.

Ans. 4 cu. ft. per min.

9. In a diverging mouthpiece the diameter of the throat is .6 in., and the head of water over the axis is 30 ft. What is the discharge in gallons per minute when the vacuum at the throat is 18.3 ins. of mercury?

Ans. 42.

✓ 10. In a stream with still water 240 ft. above datum and flowing without friction, the velocity at a point 15 ft. above datum is 24 ft. per second. What is the pressure at this point?

Ans. 93.8
~~108.75~~ lbs. per sq. in.

✓ 11. A funnel-shaped mouthpiece leads from a reservoir into a 6-in. frictionless pipe, so that there is no contraction. The water flows with a velocity of 24 ft. per second. Find the pressure at a point in the pipe 10 ft. below the surface of the water in the reservoir.

Ans. 15.43 lbs. per sq. in.

12. A 3-in. pipe gradually expands to a bell-mouth; if the total head, H , be 40 ft., find the greatest diameter of the mouth at which it will run full when open to the atmosphere. Compare the discharge from this pipe with the discharge when the pipe is not expanded at the mouth.

Ans. 4.8 in.; discharge is 149.076 cu. ft. per minute with bell-mouth and 47.345 cu. ft. per minute without bell-mouth.

13. The pressure in a 12-in. pipe at A is 50 lbs. per sq. in.; the pipe then enlarges to a 15-in. pipe at B , the rise from A to B being 3 ft.; the discharge is 1100 cu. ft. per minute. Find the pressure at B ; also find the pressure at a point C , the rise from B to C being 6 ft.

Ans. 7142½ lbs. per sq. ft.; 6767½ lbs. per sq. ft.

14. One cubic foot of water per second flows steadily through a frictionless pipe. At a point A , 100 ft. above datum, the sectional area of the pipe is .125 sq. ft., and the pressure is 2500 lbs. per sq. ft. Find the total energy. At a point B in the datum-line the pressure is 1250 lbs. per sq. ft. and the sectional area is .0625 sq. ft. Find the loss of energy between A and B . Find the "loss in shock," if the sectional area at B abruptly changes (a) from .125 to .0625 sq. ft.; (b) from .0625 to .125 sq. ft.

Ans. 141 ft.-lbs.; 117 ft.-lbs.; 79 ft.-lbs. per cu. ft.; 62½ ft.-lbs. per cu. ft.

15. In a frictionless pipe the diameter gradually changes from 6 in. at a point A 20 ft. above datum to 3 in. at B 15 ft. above datum. The pressure at A is 20 lbs. per sq. in.; find the pressure at B , the delivery of the pipe being 2½ cu. ft. per sec.

Ans. 2.23 lbs. per sq. in.

16. A horizontal frictionless pipe gradually contracts to a throat of $\frac{1}{n}$ th of the area and then gradually enlarges again to a pipe of the same size. If V is the velocity of flow in the pipe, find the reduction of pressure at the throat.

$$\text{Ans. } \frac{WV^3}{2g}(n^2 - 1).$$

17. The pressure in a $3\frac{1}{2}$ -in. horizontal frictionless pipe is $62\frac{1}{2}$ lbs. per sq. in. above that of the atmosphere. The pipe is gradually reduced to a throat of *one fifth* of the area and discharges into the atmosphere. Find the velocity of efflux and the amount of the discharge in gallons per minute.

$$\text{Ans. } 97.98 \text{ ft. per sec. ; } 491.177 \text{ gals.}$$

18. A frictionless play-pipe gradually expands from a diam. of 1 in. at the base to a diam. of 3 in. at the mouth. There is a discharge of 33 cu. ft. per min. under a head of 183 feet. Find the coefficient of discharge, the force required to hold the nozzle, and the total H.P. developed.

$$\text{Ans. } .9265 ; 108.11 \text{ lbs. ; } 11.56 \text{ H.P.}$$

19. Find the discharge in cubic feet per minute under a head of 2 ft. through a horizontal frictionless pipe which gradually diminishes from a diam. of $\frac{3}{4}$ in. to a throat of $\frac{1}{4}$ in. diam., at which the pr. head = 6 ins., and then gradually enlarges to a pipe of same diameter as before.

$$\text{Ans. } .2017.$$

20. Find the head required to give 1 cu. ft. of water per second through an orifice of 2 square inches area, the coefficient of discharge being .625. ($g = 32$.)

$$\text{Ans. } 207.36 \text{ ft.}$$

21. The area of an orifice in a thin plate was 36.3 square centimetres, the discharge under a head of 3.396 metres was found to be .01825 cubic metre per second, and the velocity of flow at the contracted section, as determined by measurements of the axis of the jet, was 7.98 metres per second. Find the coefficients of velocity, contraction, discharge, and resistance. ($g = 9.81$.)

$$\text{Ans. } .977 ; .632 ; .616 ; .046.$$

22. The piston of a 12-in. cylinder containing salt-water is pressed down under a force of 3000 lbs. Find the velocity of efflux and the volume of discharge at the end of the cylinder through a well-rounded 1-in. orifice. Also find the power exerted, c_v being .977 and $c = .5343$.

$$\text{Ans. } 60.373 \text{ ft. per sec. ; } .176 \text{ cu. ft. per sec. ; } 1.166 \text{ H.P.}$$

23. In the condenser of a marine engine there is a back pressure of $26\frac{1}{2}$ in. of mercury; the injection orifices are 6 ft. below the sea-level. With what velocity will the injection-water enter the condenser? (Neglect resistance and take $g = 32.2$.)

$$\text{Ans. } 25.3 \text{ ft. per sec.}$$

24. Water in the feed-pipe of a steam-engine stands 12 ft. above the surface of the water in the boiler; the pressure per sq. in. of the steam is 20 lbs., of the atmosphere 15 lbs. Find the velocity with which the water enters the boiler, c_v being .97.

$$\text{Ans. } 5.376 \text{ ft. per sec.}$$

25. The injection orifice of a jet condenser is 5 ft. below sea-level

and vacuum = 27 in. of mercury. Find velocity of water entering condenser, supposing three fourths of the head lost by frictional resistance.

Ans. 23.86 ft. per sec.

26. The jet from an orifice of .008 sq. ft. area in the side of a tank and under a head of 16 ft. issues horizontally and falls 1 ft. vertically in a horizontal range of 7.68 feet. The delivery is 60 gallons per minute. Find the coefficients of velocity, discharge, contraction, and resistance.

Ans. .96; .625; .65; .085.

27. The jet from a circular sharp-edge orifice $\frac{1}{2}$ in. in diam. under a head of 18 ft., strikes a point at a distance from the orifice of 5 ft. measured horizontally and 4.665 ft. measured vertically. The discharge is 98.987 gallons in 569.218 seconds. Find the coefficients of discharge, velocity, contraction, and resistance.

Ans. .6009; .945; .635; .1196.

28. A sluice 3 ft. square and with a head of 12 ft. over the centre has, from the thickness of the frame, the contraction suppressed on all sides when fully open; when partially open, the contraction exists on the upper edge, i.e., against the bottom of the gate, which is formed of a thin sheet of metal. Find the discharge in cubic feet when opened 1 ft., 2 ft., and also when fully open.

Ans. 57.22; 113.38; 173.51.

29. A vessel containing water is placed on scales and weighed. How will the weight be affected by opening a small orifice in the bottom of the vessel?

30. Water is supplied by a scoop to a locomotive tender at 7 feet above trough. Find lowest speed of train at which the operation is possible.

Ans. 14.44 miles per hour.

Also find the velocity of delivery when train travels at 40 miles per hour, assuming half the head lost by frictional resistance. ($c_v = 1$.)

Ans. 35.68 ft. per second.

31. The head in a prismatic vessel at the instant of opening an orifice was 6 ft. and at closing it had decreased to 5 ft. Determine the mean constant head h at which, in the same time, the orifice would discharge the same volume of water.

Ans. 5.488 ft.

32. A cylindrical vessel 5.747 in. in diameter has an orifice of .2 in. diam. at the bottom; the surface sinks from 16 in. to 12 in. in 53 seconds. Find the coefficient of discharge.

Ans. .6.

33. A prismatic basin with a horizontal sectional area of 9 sq. ft. has an orifice of .9 sq. in. at the bottom; it is filled to a depth of 6 ft. above the centre of the orifice. Find the time required for the surface to sink 2 ft., $3\frac{1}{2}$ ft., 5 ft.

Ans. 258.9 sec.; 500.16 sec.; 834.8 sec.

34. The water in a cylindrical cistern of 144 sq. in. sectional area is 16 ft. deep. Upon opening an orifice of 1 sq. in. in the bottom the water fell 7 ft. in 1 minute. Find the coefficient of discharge. The coefficient of contraction being .625, find the coefficients of velocity and resistance.

Ans. .6; .96; 0.85.

35. How long will it take to fill a paraboloidal vessel up to the level of the outside surface through a hole in the bottom 2 feet under water? ($g = 32$ and $c = .625$.)

Ans. $\frac{176\sqrt{2}}{105} \frac{B}{A}$, B being the parameter of the parabola and A the sectional area of the orifice.

36. How long will it take to fill a spherical vessel of radius r up to the level of the outside surface through a hole of area A at the lowest point and 2 ft. under water, c being .625?

Ans. $\frac{22}{35A} (7.54r - 6.53)$.

37. A 100-gallon tank is 100 feet above ground and is filled by a $1\frac{1}{2}$ -inch pipe connected with an accumulator having a 3-ft. cylr. piston loaded with 50 tons. If the mean lift of the piston is 10 ft. and if $\frac{9}{10}$ of the head is lost in frictional resistance, how long will it take to fill the tank?

Ans. 14.49 secs.

38. A bucket of water in a balance discharges 4 lbs., of water per minute through an orifice in its base at 45° to the vertical, and is kept constantly full by a vertical stream which issues from an orifice 8 ft. above the surface with a velocity of 30 ft. per sec. Show that the bucket must be counterpoised by about .066 lb. more than its weight.

39. The water in a vessel 9 ft. in height and 2 ft. in diameter is 8 ft. deep. In what time would one half of the water flow away through an orifice in the bottom 1 inch in diameter? If the orifice is closed and the vessel is made to rotate about its axis at the rate of $76\frac{4}{11}$ revolutions per minute, to what height will the water rise on the vessel's surface? If the orifice is opened, find velocity of efflux when the surface at the axis is 3 ft. above the orifice. Also find the difference of pressure-head in a horizontal plane 6 inches from the axis.

Ans. 190.77 secs.; to the top; 16 ft. per. sec.; 3 ins.

✓ 40. A cylindrical vessel, 10 ft. high and 1 ft. in diameter, is half full of water. Find the number of revolutions per minute which the vessel must make so that the water may just reach the top, the axis of revolution being (1) coincident with the axis of the vessel, (2) a generating line of the vessel.

Ans. (1) 483; (2) $241\frac{1}{2}$.

41. A vessel full of water weighs 350 lbs. and is raised vertically by means of a weight of 450 lbs. Find the velocity of efflux through an orifice in the bottom, the head being 4 ft. and $g = 32.2$.

Ans. 17.02 ft. per sec.

42. A vessel full of water makes 100 revols. per min. Find the velocity of efflux through an orifice 2 ft. below the surface of the water at the centre, the diam. of the vessel being 3 ft. and $c_v = 1$.

Ans. 33.4 ft. per sec.

✓ What will be the velocity if the vessel is at rest?

Ans. 11.3 ft. per sec.

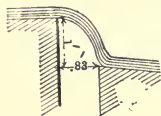


FIG. 77.

43. Show that when the water flowing over has a depth greater than .3874 ft. it is carried completely over the longitudinal opening, .83 ft. in width. At what depth does *all* the water flow in? *Ans.* .221 ft.

44. A square box 2 ft. in length and 2 ft. across a diagonal is placed with a diagonal vertical and filled with water. How long will it take for the whole of the water to flow out through a hole at the bottom of .02 sq. ft. area? ($c = .625$) *Ans.* 97.52 secs.

45. A pyramid 2 ft. high, on a square base, is inverted and filled with water. Find the time in which the water will all run out through a hole of .02 sq. ft. at the apex. A side of the base is 1 ft. in length. ($c = .625$) *Ans.* 5.656 sec.

46. Find the discharge under a head of 25 ft. through a thin-lipped square orifice of 1 sq. in. sectional area, (a) when it has a border on one side, (b) when it has a border on two sides.

Ans. (a) .3576 cu. ft. per sec.; (b) .3706 cu. ft. per sec.

47. A vessel in the form of a paraboloid of revolution has a depth of 16 in. and a diam. of 12 in. at the top. At the bottom is an orifice of 1 sq. in. sectional area. If water flows into the vessel at the rate of $2\frac{1}{2}$ cubic feet per minute, to what level will the water ultimately rise? How long will it take to rise (a) 11 in., (b) 11.9 in., (c) 11.99 in., (d) 12 in. above the orifice? If the supply is now stopped, how long (e) will it take to empty the vessel?

Ans. 12 inches; (a) 49.17 sec.; (b) 124.2 sec.; (c) 202 sec.; (d) an infinite length of time; (e) 11.3 sec.

48. If the vessel in Example 47 is a sphere 1 ft. in diameter, to what height will the water rise? How long will it take for the water to rise (a) 11 in., (b) 12 in. above the orifice? How long (c) will it take to empty the vessel? *Ans.* 12 inches; (a) 67.16 sec.; (b) 81.46 sec.; (c) 24.13 sec.

49. In a vortical motion two circular filaments of radii r_1 , r_2 , of velocities v_1 , v_2 , and of equal weight W are made to change place. Show that a stable vortex is produced if $\frac{v^2}{r} = \text{const.}$; and if $r_2 > r_1$, show that the surfaces of equal pressure are cones.

50. Sometimes the crest of a dam is raised by floating a stick L into the position L_1 , where it is supported against the verticals. The stick then falls of itself into position L_2 and rests on the crest. Explain the reason of this.

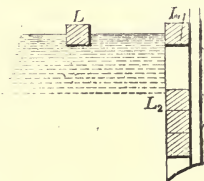


FIG. 78.

51. A 6-in. pipe discharges 8000 gals. per hour into a 9-in. pipe. Find the loss of head at the junction.

Ans. 1.58 ft.

52. Prove that for a Borda's mouthpiece running full the coefficient of discharge is $\frac{1}{\sqrt{2}}$.

53. Find the discharge in pounds per minute through a Borda's mouthpiece 1 in. in diameter, the lip being 12 in. below the water-surface, (a) when the jet springs clear from the edge, (b) when the mouth-piece runs full. *Ans.* (a) 81.845; (b) 115.74.

54. The surface of the water in a tank is kept at the same level; obtain the discharge at 60 in. below the surface (a) through a circular orifice 1 sq. in. in area, (b) through a cylindrical ajutage of the same sectional area fitted to the outside, (c) through the same ajutage fitted to the inside, and determine the mechanical effect of the efflux in each case.

Ans. (a) 4.85 lbs. per sec.; 20.536 ft.-lbs. per sec.

(b) 6.366 " " " ; 21.404 " " "

(c) 5.49 " " " ; 13.725 " " " if running full.

3.69 " " " ; 16.638 " " " if jet springs clear.

55. Water is discharged under a head of 64 feet through a short cylindrical mouthpiece 12 in. in diameter. Find (a) the loss of head due to shock, (b) the volume of discharge in cubic feet per second, (c) the energy of the issuing jet. ($g = 32$.)

Ans. (a) 20.736 ft.; (b) 41.23 cub. ft.; (c) 201.64 H.P.

56. If a bell-mouth is substituted for the mouthpiece in the preceding question, find the discharge and the mechanical effect of the jet.

Ans. 49.28 cub. ft. per sec.; 344.2 H.P.

57. Compare the energies of a jet issuing under an effective head of 100 ft. through (1) a 12-in. cylindrical ajutage, (2) a 12-in. divergent ajutage, (3) a 12-in. convergent ajutage, the angle of convergence being 21° . Draw the plane of charge in each case.

Ans. (1) 393.8 H.P.; (2) 672.28 H.P.; (3) 552.58 H.P.

58. Find the discharge through a rectangular opening 36 in. wide and 10 in. deep in the vertical face of a dam, the upper edge of the opening being 10 ft. below the water-surface.

Ans. 40.2 cub. ft. per sec.

59. A centrifugal pump has a wheel of 2 ft. outside and 1 ft. inside diam., and also a large whirlpool chamber. Draw to scale a curve showing the pressure at different points in the wheel and whirlpool chamber when the water fills the pump but flows very slowly towards the point of discharge. Take 1 atm. as the pr. at the inlet surface.

60. A submerged sluice in the vertical face of a reservoir is 30 ft. wide. The effective head over the sluice is 18 inches. How high must the sluice be raised to give a delivery of 45,000 gal. per minute? ($c = .6$.)

Ans. 8.164 ins.

61. The sill of a sluice in the vertical face of a reservoir is clear above the tail-race; the head of water above the sill is 5 feet. If the sluice is 24 ft. wide, what must be the opening to give 93,750 gals. per min.? ($c = .6$.)

Ans. 12.3 ins.

62. A sluice in the vertical side of a reservoir is partially submerged.

the surface of the tail-race water being 6 ins. above the sill. The surface on the upstream side is $2\frac{1}{2}$ ft. above the sill. If the sluice is 18 ft. wide, what must be the total opening of the sluice to give 15,637 $\frac{1}{2}$ tons (of 2000 lbs.) per hour? *Ans.* 1.203 ft., c being .6.

63. Find the discharge in cub. ft. per sec. through a sharp-edge orifice, 6 ins. square, in a vertical plate, the centre of the orifice being 15 ins. below the water-surface, (a) if the velocity of approach is 1 ft. per second, (b) if the channel of approach is 3 ft. wide by 2 ft. deep.

Ans. (a) 1.3478; (b) 1.34.

64. A reservoir half an acre in area with sides nearly vertical, so that it may be considered prismatic, receives a stream yielding 9 cub. ft. per second, and discharges through a sluice 4 ft. wide, which is raised 2 ft. Calculate the time required to lower the surface 5 ft., the head over the centre of the sluice when opened being 10 ft. *Ans.* 1079 secs.

65. Show that the energy of a jet issuing through a large rectangular orifice of breadth B is $125B(H_2^{\frac{5}{2}} - H_1^{\frac{5}{2}})$, H_1 , H_2 being the depths below the water-surface of the upper and lower edges of the orifice, and the coefficient of discharge being .625.

66. A reservoir at full water has a depth of 40 ft. over the centre of the discharging-sluice, which is rectangular and 24 in. wide by 18 in. deep. Find the discharge in cubic feet per second at that depth, and also when the water has fallen to 30, 20, and 10 ft., respectively; find the mechanical effect of the efflux in each case, c being .625.

Ans. 94.8 cu. ft.; 82.1 cu. ft.; 67 cu. ft.; 47.4 cu. ft.; 431.2 H.P.; 280 H.P.; 152.5 H.P.; 53.95 H.P.

67. Require the head necessary to give 7.8 cu. ft. per second through an orifice 36 sq. in. in sectional area, c being .625. *Ans.* 38.9 ft.

68. The upper and lower edges of a vertical rectangular orifice are 6 and 10 ft. below the surface of the water in a cistern, respectively; the width of the orifice is 1 ft. Find the discharge through it.

Ans. 56.42 cu. ft. per sec.

69. The two sluices each 4 ft. wide by 2 ft. deep in a lock-gate are submerged one half their depth. The constant head of water above the axis of the sluice is 12 ft. Find the discharge through the sluice, the velocity of approach being 4 ft. per second, c being .625.

Ans. 16,626.2 cu. ft. per min.

70. Find the flow through a square opening, one diagonal being vertical and 12 in. in length, the upper extremity of the diagonal being in the surface of the water, and c being .625. *Ans.* 1.724 cu. ft. per sec.

71. To find the quantity of water conveyed away by a canal 3 ft. wide, a board with an orifice 2 ft. wide and 1 ft. deep is placed across the canal and dams it back until it attains a height of $2\frac{1}{4}$ ft. above the bottom and $1\frac{1}{4}$ ft. above the lower edge of the orifice. Find the discharge in cubic feet per second, c being .625.

Ans. 17.59, or 20.21 if orifice is drowned.

72. Six thousand gallons of water per minute are forced through a line of piping ABC and are discharged into the atmosphere at C , which is 6 ft. vertically above A . The pipe AB is 6 in. in diameter and 12 ft. in length; the pipe BC is 12 in. in diameter and 12 ft. in length. Disregarding friction, find the "loss in shock" and draw the plane of charge.

Ans. Loss of head in shock = 58.3 ft.

73. What quantity of water flows through the vertical aperture of a dam, its width being 36 in. and its depth 10 in.; the upper edge of the aperture is 16 ft. below the surface.

Ans. 50.65 cu. ft. per sec.

74. 264 cu. ft. of water are discharged through an orifice of 5 sq. ins. in 3 min. 10 sec. Find the mean velocity of efflux.

Ans. 64 ft. per. sec.

75. One of the locks on the Lachine Canal has a superficial area of about 12,150 sq. ft., and the difference of level between the surfaces of the water in the lock and in the upper reach is 9 ft. Each leaf of the gates is supplied with one sluice, and the water is levelled up in 2 min. 48 secs. Determine the proper area of the sluice-opening. (Centre of sluice 20 ft. below surface of upper reach and $c = .625$.)

Ans. Area of one sluice = 43.39 sq. ft.

76. The horizontal section of a lock-chamber may be assumed a rectangle, the length being 360 ft. When the chamber is full, the surface width between the side walls, which have each a batter of 1 in 12, is 45 ft. How long will it take to empty the lock through two sluices in the gates, each 8 ft. by 2 ft., the height of the water above the centre of the sluices being 13 ft. in the lock and 4 ft. in the canal on the downstream side.

Ans. 600.75 sec., c being .625.

77. Water approaches a rectangular opening 2 ft. wide with a velocity of 4 ft. per second. At the opening the head of water over the lower edge = 13 ft., and over the surface of the tail-race = 12 ft.; the discharge through the opening is 70 cu. ft. per second. Find the height of the opening, c being .625.

Ans. 1.022 ft.

78. The water in a regulating-chamber is 8 ft. below the level of the water in the canal and 8 ft. above the centre of the discharging-sluice. Determine the rise in the canal which will increase the discharge by 10 per cent.

Ans. 1.68 ft.

The horizontal sectional area of the chamber is constant and equal to 400 sq. ft.; in what time will the water in the chamber rise to the level of that in the canal, if the discharging-sluice is closed; the sluice between the canal and chamber being 3 sq. ft. in area?

Ans. 150.83 sec.

79. A lock on the Lachine Canal is 270 ft. long by 45 ft. wide and has a lift of $8\frac{3}{4}$ ft.; there are two sluices in each leaf, each $8\frac{1}{4}$ ft. wide by $2\frac{1}{2}$ ft. deep; the head over the horizontal centre line of the sluices is 19 ft. Find the time required to fill the lock.

Ans. 163.5 sec.

80. The locks on the Montgomeryshire Canal are 81 ft long and $7\frac{3}{4}$ ft wide; at one of the locks the lift is 7 ft.; a 24-in. pipe leads the water

from the upper level and discharges below the surface of the lower level into the lock-chamber; the mouth of the pipe is square, 2 ft. in the side, and gradually changes into a circular pipe 2 ft. in diameter. Find time of filling the lock. ($c = 1$.) *Ans.* 132.11 sec.

81. A canal lock is 115.1 ft. long and 30.44 ft. wide; the vertical depth from centre of sluice to lower reach is 1.0763 ft., the charge being 6.3945 ft.; the area of the two sluices is 2×6.766 sq. ft. Find the time of filling up to centre of sluices. ($c = .625$ for the sluice, but is reduced to .548 when both are opened.) Also, find time of filling up to level of upper reach from centre of sluice-doors. *Ans.* 25 sec.; 298 sec.

82. How many gallons of water will flow through a 90° notch in 24 hours if the depth of the water is 27 ins. for the first 8 hours, 12 ins. for the second 8 hours, and 3 ins. for the third 8 hours, c being .6? *Ans.* 3,974,400.

83. Show that in a channel of V section an increment of 10 per cent in the depth will produce a corresponding increment of 5 per cent in the velocity of flow and of 25 per cent in the discharge.

84. The angle of a triangular notch is 90° . How high must the water rise in the notch so that the discharge may be 1000 gallons per minute? *Ans.* 12 ins. very nearly.

85. A reservoir, rectangular in plan and 10,000 sq. ft. in area, has in one side a 90° triangular notch 2 ft. deep. If the reservoir is full, in what time will the level sink 6 ins.? *Ans.* 496.87 secs.

86. How long will it take to lower by 3 ft. the surface of a reservoir of 640,000 sq. ft. area through a 90° V notch 4 ft. deep? *Ans.* 40.50 hrs., c being .6.

87. Find the discharge in cubic feet per second through a 90° notch when the depth of water in the notch is 4 ft., c being .617. *Ans.* 84.24.

88. A pond whose area is 12,000 sq. ft. has an overfall outlet 36 in. wide, which at the commencement of the discharge has a head of 2.8 ft. Find the time required to lower the surface 12 in. *Ans.* 354.58 sec.

89. How much water will flow through a rectangular notch 24 in. wide, the surface of still water being 8 in. above the crest of the notch? (Take into account side contraction.) *Ans.* 3.383 cu. ft. per sec.

90. A weir passes 6 cubic feet per second, and the head over the crest is 8 inches. Find the length of the weir, c being .625. *Ans.* 3.3068 ft.

91. A weir 400 ft. long, with a 9-in. depth of water on it, discharges through a lower weir 500 ft. long. Find the depth of water on the latter. *Ans.* .6457 ft.

92. A weir is 545 ft. long; how high will the water rise over it when it rises .68 ft. upon an upper weir 750 ft. long? *Ans.* .8413 ft.

93. What should be the height of a drowned weir 400 ft. long, to deepen the water on the up-stream side by 50 per cent, the section of

The stream being 400 ft. \times 8 ft., and the velocity of approach 3 ft. per second ? *Ans.* 7.084 ft.

94. The depth of water on the crest of a rectangular notch 5 ft. long is 2 feet. Find the discharge when the notch has (a) two end contractions, (b) one end contraction, (c) no end contraction, *c* in each case being $\frac{5}{8}$.

Ans. (a) 43.369 cu. ft. per sec.; (b) 45.254 cu. ft. per sec.; (c) 47.14 cu. ft. per sec.

95. Show that upon a weir 10 ft. long with 12 ins. depth of water flowing over, an error of $\frac{1}{1000}$ of a foot in measuring the head will cause an error of 3 cu. ft. per minute in the discharge, and an error of $\frac{1}{100}$ of a foot in measuring the length of the weir will cause an error of 2 cu. ft. in the discharge.

96. In the weir at Killaloe the total length is 1100 ft., of which 779 ft. from the east abutment is level, while the remainder slopes 1 in 214, giving a total rise at the west abutment of 1.5 ft. Calculate the total discharge over the weir when the depth of water on the level part is 1.8 ft., which gives .3 ft. on highest part of weir. (Divide slope into 8 lengths of 40 ft. each, and assume them severally level, with a head equal to the arithmetic mean of the head at the beginning and end of each length.) *Ans.* 7496 cu. ft. per sec.

97. A watercourse is to be augmented by the streams and springs above its level. The latter are severally dammed up at suitable places and a narrow board is provided in which an opening 12 in. long by 6 in. deep is cut for an overfall; it was surmised that this would be sufficient for the largest streams; another piece attached to the former would reduce the length to 6 in. for smaller streams. Calculate the delivery by the following streams :

In No. 1 stream with the 12-in. notch, depth over crest = .37 ft.

" No. 2 " " " 6-in. " " " " = .41 ft.

" No. 3 " " " 12-in. " " " " = .29 ft.

" No. 4 " " " 6-in. " " " " = .19 ft.

(Take into account the side contractions.)

Ans. No. 1, .696 cu. ft.; No. 2, .3658 cu. ft.; No. 3, .4904 cu. ft.;

No. 4, .1275 cu. ft.

98. A rectangular notch has two complete end contractions and the length of the crest is *three* times the depth of the water on the crest. What must be the length of the crest to give a minimum discharge of 18,750 gals. per minute, *c* being $\frac{5}{8}$? *Ans.* 5.87 ft.

99. A stream 30 ft. wide, 3 ft. deep, discharges 310 cu. ft. per second; a weir 2 ft. deep is built across the stream. Find increased depth of latter, (a) neglecting velocity of approach, (b) taking velocity of approach into account. *Ans.* (a) 1.26 ft. to 1.265 ft.; (b) 1.19 ft.

100. In a stream 50 ft. wide and 4 ft. deep water flows at the rate of 100 ft. per minute; find the height of a weir which will increase the depth

to 6 ft., (1) neglecting velocity of approach, (2) taking velocity of approach into account.

Ans. (1) 4.4126 ft.; (2) 4.4305 ft.

101. A stream 50 ft. wide and 4 ft. deep has a velocity of 3 ft. per second; find the height of the weir which will double the depth, (1) neglecting velocity of approach, (2) taking velocity of approach into account.

Ans. (1) 5.651 ft.; (2) 5.6862 ft.

102. A stream 80 ft. wide by 4 ft. deep discharges across a vertical section at the rate of 640 cu. ft. per second; a weir is built in the stream, increasing its depth to 6 ft. Find the height of the weir.

Ans. 4.233 ft.

103. Salmon-gaps are constructed in a weir; they are each 10 ft. wide and their crests are 18 in. below the weir crest. Calculate the discharge down three of these gaps, the water on the level part of the weir being 8 in. deep.

Ans. 238.15 cu. ft. per sec.

104. A channel of rectangular section and 20 ft. wide conveys 3,600,000 gallons per hour, the depth of the stream being 8 ft. A dam 2 ft. high is built across the channel. Find the "height of swell" (a) disregarding the velocity of approach, (b) taking the velocity of approach into account.

Ans. (a) .07 ft.; (b) .0545 ft.

105. The water in a flume 8 ft. wide is 3 ft. deep and is supplied from a sluice 6 ft. wide at the rate of 27,000 gals. per minute. If the coefficient of contraction is unity and if 10 per cent is allowed for fractional loss, find the difference of level between the water-surfaces above the sluice and in the flume when the sluice opening is (a) 1 ft., (b) 2 ft.

Ans. (a) 2.32 ft.; (b) .31 ft.

106. A stream of rectangular section 24 ft. wide delivers 145 cu. ft. per second. The edge of a drowned weir is 15 ins. below the surface of the water on the down-stream side. Determine the difference of level between the surfaces of the water on the up- and down-stream sides, the velocity of approach being 2 ft. per second.

Ans. 7.9 ins.

CHAPTER II.

FLUID FRICTION AND PIPE FLOW.

I. Fluid Friction.—The term fluid friction is applied to the resistance to motion which is developed when a fluid flows over a solid surface, and is due to the viscosity of the fluid. This resistance is necessarily accompanied by a loss of energy caused by the production of eddies along the surface, and similar to the loss which occurs at an abrupt change of section, or at an angle in a pipe or channel.

Froude's experiments on the resistance to the edgewise motion of planks in a fluid mass, the planks being $\frac{3}{16}$ in. thick, 19 in. deep, and 1 to 50 ft. long, each plank having a fine cutwater and run, are summarized in the following table:

Nature of Surface Covering.	Length of Surface in Feet.											
	2 Feet.			8 Feet.			20 Feet.			50 Feet.		
	A	B	C	A	B	C	A	B	C	A	B	C
Varnish.....	2.00	.41	.390	1.85	.325	.264	1.85	.278	.240	1.83	.250	.226
Paraffine.....		.38	.370	1.94	.314	.260	1.93	.271	.237			
Tinfoil.....	2.16	.30	.295	1.99	.278	.263	1.90	.262	.244	1.83	.246	.232
Calico.....	1.93	.87	.725	1.92	.626	.504	1.89	.531	.447	1.87	.474	.423
Fine sand.....	2.00	.81	.690	2.00	.583	.450	2.00	.480	.384	2.00	.405	.337
Medium sand....	2.00	.90	.730	2.00	.625	.488	2.00	.534	.465	2.00	.488	.456
Coarse sand....	2.00	1.10	.880	2.00	.714	.520	2.00	.588	.490			

Columns A give the power of the speed (v) to which the resistance is approximately proportional.

Columns B give the mean resistance, in pounds per square

foot, of the whole surface of a board of the lengths stated in the table.

Columns C give the resistance, in pounds, of a square foot of surface at the distance sternward from the cutwater stated in the heading, each plank having a standard speed of 10 ft. per second. The resistance at other speeds can be easily calculated.

An examination of the table shows that the mean resistance per square foot diminishes as the length of the plank increases. This may be explained by the supposition that the friction in the forward portion of the plank develops a force which drags the water along with the surface, so that the relative velocity of flow over the rear portion is diminished. Again, the decrease of the mean resistance per square foot is .132 lb. when the length of a varnished plank is increased from 2 to 20 ft., while it is only .028 lb. when the length increases from 20 to 50 ft. Hence for greater lengths than 50 ft. the decrease of resistance may be disregarded without much, if any, practical effect.

Thus, generally speaking, these experiments indicate that the mean resistance is proportional to the n th power of the relative velocity, n varying from 1.83 to 2.16, and its average value being very nearly 2.

Colonel Beaufoy, as a result of experiments at Deptford, also assumed the mean resistance to be proportional to the n th power of the relative velocity, the value of n in three series of observations being 1.66, 1.71, and 1.9.

The frictional resistance is evidently proportional to some function of the velocity, $F(v)$, which should vanish when v is nil, as when the surface is level, and should increase with v .

Coulomb assumed the function $F(v)$ to be of the form $av + bv^2$, a and b being coefficients to be determined by experiment. Experiment shows that when b does not exceed 5 ft. per minute the resistance is directly proportional to the velocity, but that it is more nearly proportional to the square

of the velocity when the velocity exceeds 30 ft. per minute;
or,

$$F(v) = av \text{ when } v \leq 5 \text{ ft. per minute,}$$

and

$$F(v) = bv^2 \text{ when } v > 30 \text{ ft. per minute.}$$

Again, observations on the flow of water in town mains indicate that no difference of resistance is developed under widely varying pressures, and this independence of pressure is also verified by Coulomb's experiment showing that, if a disc is oscillated in water, there is no apparent change in the rate of decrease of the oscillations, whether the water is under atmospheric pressure or not.

From the preceding and other similar experiments the following general laws of fluid friction have been formulated:

(1) The frictional resistance is independent of the pressure between the fluid and the surface over which it flows.

(2) The frictional resistance is proportional to the area of the surface.

(3) The frictional resistance is proportional to some function, usually the square, of the velocity.

To these three laws may be added a fourth, viz.:

(4) The frictional resistance is proportional to the density and viscosity of the fluid.

A fifth law, viz., that "the frictional resistance is independent of the nature of the surface against which the fluid flows," has been sometimes enunciated, and at very low velocities the law is approximately true. At high velocities, however, such as are common in engineering practice, the resistance has been shown by experiment, and especially by the experiments carried out by Darcy, to be very largely influenced by the nature of the surface.



Let p be the frictional resistance in pounds per square foot of surface at a velocity of 1 ft. per second.

Let A be the area of the surface in square feet.

Let v be the relative velocity of the surface and the water in which it is immersed.

Let R be the total frictional resistance.

Then from the laws of fluid friction

$$R = p \cdot Av^2.$$

Take $f = \frac{2g}{w} p$, w being the specific weight of the fluid.

Then

$$R = fwA \frac{v^2}{2g}.$$

The coefficient f is approximately constant for any given surface, and is termed the coefficient of fluid friction. The power absorbed by the frictional resistance

$$= pAv^2 \times v = pAv^3 = fwA \frac{v^3}{2g}.$$

TABLE GIVING THE AVERAGE VALUES OF f IN THE CASE OF LARGE SURFACES MOVING IN AN INDEFINITELY LARGE MASS OF WATER.

Surface.	Coefficient of Friction (f).
New well-painted iron plate.....	.00489
Painted and planed plank.....	.0035
Surface of iron ships.....	.00362
Varnished surface.....	.00258
Fine sand surface.....	.00418
Coarse sand surface.....	.00503

Ex. The wetted surface of a vessel moving at 8 knots per hour is 7500 sq. ft., and the resistance is .4 lbs. per sq. ft. at a speed of 10 ft. per second. Find the surface-resistance and the horse-power required to propel the vessel.

The resistance in lbs. per sq. ft. at 1 ft. per. sec. = $\frac{.4}{10^2} = \frac{4}{1000}$.

Therefore the *total skin-resistance*

$$= \frac{4}{1000} \cdot 7500 \left(\frac{8 \times 6086}{60 \times 60} \right)^2 = 5487.3 \text{ lbs.}$$

$$\text{The horse-power} = 5487.3 \times \frac{8 \times 6086}{3600} \times \frac{1}{550} = 134.93.$$

2. Surface Friction of Pipes.—Assuming that the laws of fluid friction already enunciated hold good when water flows through a pipe, it has been shown by numerous experiments that the coefficient of friction f lies between the limits .005 and .01, its average value under ordinary conditions being about .0075. No single value of f is applicable to very different cases. Indeed, f depends not only upon the condition of the surface, but also upon the diameter of the pipe and the velocity of the water. Some authorities have expressed its value by a relation of the form

$$\frac{f}{g} = a + \frac{b}{v},$$

a and b being constants whose values are to be determined by experiment.

The following table gives some of the best numerical results obtained for a and b :

Authority.	a	b
Prony.....	.00021230	.00003466
D'Aubuisson.....	.0002090	.000037608
Eytelwein.....	.00017059	.00004441

In pipes of small diameter in which the velocity of flow is less than 4 ins. per second the term a may be disregarded so that

$$\frac{f}{g} = \frac{b}{v}.$$

In ordinary practice and when the pipes have been in use

for some time the velocity usually exceeds 4 ins. per second, and the term $\frac{b}{v}$ may then be disregarded, so that

$$\frac{f}{g} = a.$$

Darcy's and other more recent experiments show that a and b are not constant, but are more correctly expressed as functions of the diameter. In Darcy's experiments the pipes were laid very nearly horizontal and the head could be varied at will by the opening or closing of valves.

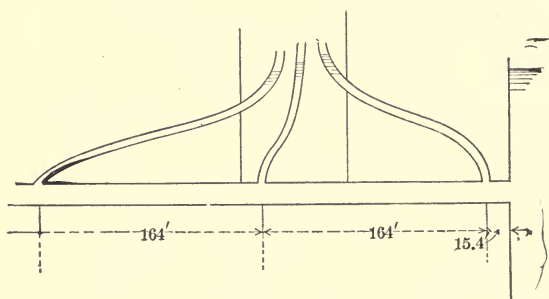


FIG. 79.

Piezometers were inserted at intervals of 164 ft. (50 m.), commencing at 15.4 ft. (4.7 m.) from the inlet, i.e., at a point where the pipe was running full and the flow was steady. The upper ends of the piezometers terminated on a vertical plank so placed as to allow the water-levels in them to be observed and compared. In any two consecutive piezometers the difference of level, which is of course constant, represents the frictional loss of head in a 164-ft. length of pipe. From the results of these experiments Darcy made the following deductions:

(a) *The frictional resistance depends upon the material and condition of the pipe.*

For example, the resistance to flow is much less in a glass than in an iron pipe, and is *approximately twice* as great in

pipes which have become incrustated with use as in new clean pipes. It must be remembered, however, that although numerous experiments have been made with new pipes, there have been comparatively few experiments with old pipes. Thus, in pipes in which the velocity of flow exceeds 4 ins. per second, Darcy considered it more correct to express $a \left(= \frac{f}{g} \right)$ in the form

$$\frac{f}{g} = a = \alpha + \frac{\beta}{d},$$

d being the diameter of the pipe, and α and β coefficients to be determined by experiment. The following values for α and β are given by Darcy:

	α	β
For drawn wrought-iron or smooth		
cast-iron pipes....	.0001545	.000012973
For pipes with surface covered by		
light incrustations.....	.0003093	.00002598

Without sensibly altering the values of these coefficients they can be put into the following simple form:

$$\frac{f}{g} = a = \frac{\mu}{g} \left(1 + \frac{1}{12d} \right),$$

d being the diameter in feet, and μ being .005 or .01 according as the pipes are clean or have become slightly incrustated.

(b) *The coefficient b is not constant, but varies slightly both with the diameter and the velocity, its value diminishing as d or v increases.*

In practice it is assumed that b is constant and the error involved has the advantage of giving to the pipe a larger sectional area than is actually required for a given discharge. Thus allowance is partially made for the incrustations with which the surface gradually becomes covered.

Darcy proposed to include all cases in the more general form

$$\frac{f}{g} = \alpha + \frac{\beta}{d} + \left(\alpha' + \frac{\beta'}{d} \right) \frac{1}{v},$$

in which, for new and smooth iron pipes,

$$\begin{aligned} \alpha &= .00001350 & \beta &= .000012402 \\ \alpha' &= .000031635 & \beta' &= .00000016186 \end{aligned}$$

This value for $\frac{f}{g}$ is rarely if ever used.

TABLE GIVING DARCY'S VALUES OF f FOR VELOCITIES EXCEEDING 4 IN. PER SECOND.

Diam. of Pipe in Inches.	Value of f .		Diam. of Pipe in Inches.	Value of f .		Diam. of Pipe in Inches.	Value of f .	
	New Pipes.	Incrusted Pipes.		New Pipes.	Incrusted Pipes.		New Pipes.	Incrusted Pipes.
2	.0075	.0150	9	.00556	.01111	27	.00519	.01037
3	.00667	.01333	12	.00542	.01083	30	.00517	.01033
4	.00625	.0125	15	.00533	.01067	36	.00514	.01028
5	.0060	.012	18	.00528	.01056	42	.00512	.01024
6	.00583	.01167	21	.00524	.01048	48	.00510	.01021
7	.00571	.01143	24	.00521	.01042	54	.00509	.01019
8	.00563	.01125						

Weisbach gives the formula

$$f = .0036 - \frac{.00429}{\sqrt{v}}.$$

Poiseuille's experiments indicate that the surface friction in capillary tubes is directly proportional to the velocity, but in pipes, in ordinary practice, the frictional resistance is certainly more nearly proportional to the square of the velocity, and must be largely due to eddies which are the more readily formed as the viscosity diminishes. This viscosity, again,

increases as the temperature falls, and the surface friction is diminished by about 1 per cent for every rise of 5° F. in the temperature. The resistance to the motion of a body in water, or to the flow of water along a surface, is evidently of two kinds, the one due to surface contact, the other to the formation of eddies. Hele Shaw's experiments clearly show the effect of surface contact upon stream-line motion and the manner in which the motion is modified by the presence of obstacles (Trans. Naval Architects, 1897-98), while the two kinds of resistance are plainly demonstrated by the interesting experiments of Osborne Reynolds. The water flows through

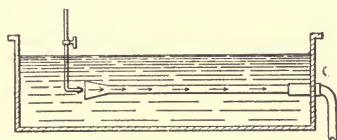


FIG. 80.

a glass pipe AB having a trumpet-shaped mouth A . A glass tube CD with a funnel E terminates in a pipette F , the axis of the pipette being in line with the axis of the pipe. The tube is filled with an aniline dye which is allowed to escape through the pipette in a thin thread-like stream, the discharge being governed by a small cock. So long as the velocity of flow in the pipe does not exceed a certain value, which Reynolds calls the *critical* velocity, the aniline thread is unbroken, so that the motion of the water is undisturbed and must be in parallel lines. As soon as the critical velocity is exceeded the colored thread is broken up, becoming sinuous in character, and the parallel stream-line motion is completely destroyed within a very short distance from the mouth of the pipe.

According to Reynolds the critical velocity (v_c), in metres per sec., is given by the formula

$$v_c d = \mu P,$$

μ being $\frac{1}{43.79}$ (?) for capillary tubes and $\frac{1}{278}$ for ordinary pipes, while

$$\frac{1}{P} = 1 + .0336t + .000221t^2,$$

t being the temperature in degrees centigrade.

It has been shown by H. T. Barnes, D.Sc., in his experiments on the specific heat of water, that, if water be heated while flowing through a tube at velocities less than the critical velocity, the temperature distribution in the column is not uniform. If the heat be applied electrically, by means of a wire threaded through the flow-tube, the hot water flows along the wire, leaving the walls of the tube almost entirely unheated. If the heat be applied to the walls of the tube, the colder water passes through the centre of the tube unheated, leaving a cloak of hot water along the sides. In neither case is there any tendency to mix as long as stream-line flow is maintained.

A new method for determining the critical velocity of a fluid, based on the above experiments, has been recently worked out by Drs. Barnes and Coker in the McGill hydraulic laboratory. In this method, a sensitive mercury thermometer is placed exactly in the centre of a column of water as it emerges from the tube under examination, with the bulb just beyond the end. The walls of the tube are maintained at a constant temperature, slightly above that of the water flowing through, but for stream-line flow the temperature indicated by the thermometer will be that of the water in the head supplying the constant flow. The arrival of the critical velocity, at which stream-line flow becomes eddying and sinuous, is at once shown by a sudden small increase in the reading of the thermometer, and is due to the mixture of the water-film next the surface with the colder water flowing through the body of the pipe. The point is very sharply defined, and the method is in many cases far more applicable and convenient than the usual color-band test.

The experiments now in progress in the hydraulic laboratory by Barnes and Coker are being made, both by the thermal and color-band methods, under the most favorable conditions for securing the perfectly steady conditions necessary for maintaining stream-line flow. The results so far obtained show that the effect of temperature is very marked in altering the point of instability of flow, and that this variation accords at least approximately with the formula quoted by Osborne Reynolds and taken from Poiseuille's experiments. The effect of pressure has been studied over a limited range, and it has been shown that water flowing under a high head has greater stability, which means that there is a definite increase in the velocity at which stream-line motion breaks down. Indeed, under the present arrangements, it has been possible to maintain stream-line motion to very much higher velocities than is possible in experiments carried out with the apparatus used by Reynolds.

3. Resistance of Ships.—The motion of a ship through water causes the production of waves and eddies, and the total resistance to the movement of a ship is made up of a frictional resistance, a wave-making resistance, and an eddy-making resistance. Although there is no theory by which the resistance at a given speed of a ship of definite design can be absolutely determined, Froude's experiments render it possible to make certain inferences and furnish some useful data.

According to Froude, the frictional resistance is sensibly the same as that of a rectangular surface moving with the same speed, of the same length as the ship in the direction of motion, and of an area equal to the immersed surface of the ship. Experiments seem to indicate that as the speed increases, the frictional resistance of well-designed ships with clean bottoms is from 90 to 60 per cent of the total resistance, and that the percentage is greater when the bottoms become foul.

The wave-making resistance is especially affected by the form and proportions of the ship, depending, for a given

length, upon the proportions of the entrance, middle body, and run. For every ship there is a limit of speed below which the resistance is approximately proportional to the square of the speed, being chiefly due to friction, and beyond which it increases more rapidly than as the square.

The eddy-resistance in the case of well-formed ships should not exceed about 10 per cent of the total resistance, and is often much less.

Froude's law of resistance may be enunciated as follows:

Let l_1 , l_2 be the lengths of a ship and its model.

Let Δ_1 , Δ_2 be the displacements of a ship and its model.

Let R_1 , R_2 be the resistances of a ship and its model at the speeds v_1 and v_2 .

Then, if

$$\frac{v_1}{v_2} = \frac{l_1^{\frac{1}{2}}}{l_2^{\frac{1}{2}}} = \frac{\Delta_1^{\frac{1}{6}}}{\Delta_2^{\frac{1}{6}}},$$

the resistances are in the ratio of

$$\frac{R_1}{R_2} = \frac{\Delta_1}{\Delta_2} = \frac{l_1^3}{l_2^3} = \left(\frac{v_1}{v_2}\right)^6.$$

Hence, too, the H.P., and therefore also the coal consumption per hour, is proportional to Rv , that is, to

$$\Delta^{\frac{7}{6}} \quad \text{or} \quad l^{\frac{7}{2}} \quad \text{or} \quad v^7,$$

and the coal consumption per mile is proportional to

$$\Delta \quad \text{or} \quad l^3 \quad \text{or} \quad v^6.$$

Again, R is proportional to l^3 ;

that is, to $l \times l^2$;

that is, to $v^2 \times \Delta^{\frac{2}{3}}$;

and it is sometimes convenient to express the resistance in pounds in the form

$$R = k \cdot v^2 \Delta^{\frac{2}{3}},$$

v being the speed in knots, Δ the displacement in tons, and k a coefficient depending upon the type of ship and varying from .55 to .85 when the bottom is clean.

Ex. If the New York, with a displacement of 10,000 tons and requiring 20,000 H. P. for a speed of 20 knots, is taken as the model for a new steamer which is to have a speed of 21 knots, then

$$\text{new steamer's displacement} = 10,000 \left(\frac{21}{20} \right)^6 = 13,400, \text{ approximately,}$$

$$\text{H. P. of new steamer} = 20,000 \left(\frac{21}{20} \right)^7 = 28,000, \text{ approximately.}$$

4. Pipe-flow Assumptions.—In the ordinary theory of the flow of water in a pipe it is assumed that the water consists of thin plane layers perpendicular to the axis of the pipe, that each layer is driven through the pipe by the action of gravity and by the difference of pressure on its plane faces, and that the liquid molecules in any layer at any given moment will also be found in a plane layer after any interval of time. In such motion the internal work done in deforming a layer may be generally disregarded.

It is further assumed that there is no variation of velocity over the surface of a layer, and this is equivalent to saying that each liquid molecule in a cross-section has the same mean velocity.

The disagreement of these assumptions with the results of recent experimental researches will be referred to in a subsequent article.

5. Steady Motion in a Pipe of Uniform Section.—Since the motion is to be steady, the same volume Q cu. ft. of water will always arrive at any given cross-section of A sq. ft. with the same mean velocity v ft. per second. Then

$$Q = Av.$$

But since the pipe is of constant diameter, A is constant, and hence also v is constant, so that the mean velocity is the same throughout the whole length of the pipe.

Consider an elementary mass of the fluid $AABB$, bounded by the pipe and by the two cross-sections AA , BB . Let dl be the length AB of the element, the length l ft. of the pipe being measured along the axis from any origin O .

Let z , $z + dz$ be the elevations in feet above a datum line of the centres of pressure in the cross-sections AA , BB , respectively.

Let p , $p + dp$ be the intensities of the pressures on these cross-sections in pounds per square foot.

Let P be the perimeter of the pipe.

Let w be the specific weight of the water in pounds per cubic foot.

Work Done by Gravity.—In one second wQ lbs. of water are transferred from AA to BB , falling through a vertical distance of dz ft. Thus the work done by gravity per second

$$= -wQ \cdot dz,$$

a positive quantity if dz is negative, and *vice versa*.

Work Done by Pressure.—The total pressure on AA parallel to the axis $= pA$; the total pressure on BB parallel to the axis $= (p + dp)A$.

Therefore the total resultant pressure parallel to the axis in the direction of motion $= -A \cdot dp$, and the work done per second on the volume Q by this pressure $= -Q \cdot dp$.

NOTE.—The work done by the pressure at the pipe surface is nil, as its direction is at right angles to the line of motion.

Work Absorbed by Frictional Resistance.—From the laws of fluid friction this work per second is evidently

$$= -P \cdot dl \cdot F(v) \times v = -\frac{P}{A} \cdot Q \cdot F(v) \cdot dl,$$

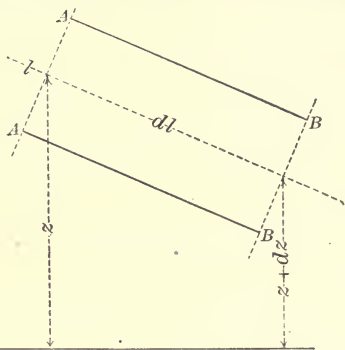


FIG. 81.

the sign being negative as the work is done against a resistance.

Since the motion is steady, the work done by the external forces must be equivalent to the work absorbed by the frictional resistance, and hence

$$-wQ \cdot dz - Q \cdot dp - \frac{P}{A} Q \cdot F(v) \cdot dl = 0,$$

or

$$dz + \frac{dp}{w} + \frac{P}{A} \cdot \frac{F(v)}{w} \cdot dl = 0.$$

Integrating,

$$z + \frac{p}{w} + \frac{P}{A} \cdot \frac{F(v)}{w} \cdot l = \text{a constant} = H,$$

so that H ft.-lbs. per pound of fluid is the uniformly distributed total constant energy.

$\frac{A}{P}$ is called the hydraulic mean radius of a pipe and will be denoted by m .

Take

$$\frac{F(v)}{w} = f \frac{v^2}{2g},$$

the value adopted in ordinary practice, f being the coefficient of friction. Then

$$z + \frac{p}{w} + \frac{fl}{m} \frac{v^2}{2g} = H.$$

Let z_1 , A_1 , p_1 be the elevation above datum, the area of the cross-section, and the intensity of the pressure at any point X on the axis of the pipe distant l_1 from the origin (Fig. 82).

Let z_2 , A_2 , p_2 be the elevation above datum, the area of the cross-section, and the intensity of the pressure at any other point Y on the axis distant l_2 from the origin (Fig. 82).

Then, from the equation just deduced,

$$z_1 + \frac{p_1}{w} + \frac{fl_1}{m} \frac{v^2}{2g} = H = z_2 + \frac{p_2}{w} + \frac{fl_2}{m} \frac{v^2}{2g}.$$

Hence

$$\left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right) = \frac{f}{m} \frac{v^2}{2g} (l_2 - l_1) = \frac{fL}{m} \frac{v^2}{2g},$$

L being the length $l_2 - l_1$ of the pipe between the two points X and Y .

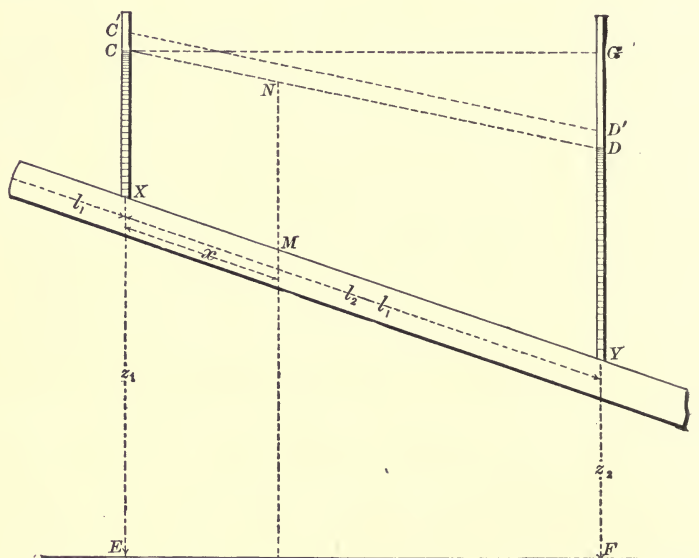


FIG. 82.

Let vertical tubes (pressure-columns) be inserted in the pipe at X and at Y . The water will rise in these tubes to the levels C and D , and evidently

$$p_1 = w \cdot CX + p_0,$$

$$p_2 = w \cdot DY + p_0,$$

p_0 being the intensity of the atmospheric pressure.

Hence, if CX and DY are produced to meet the datum line in E and F ,

$$z_1 + \frac{p_1}{w} = z_1 + CX + \frac{p_0}{w} = CE + \frac{p_0}{w}$$

and

$$z_2 + \frac{p_2}{w} = z_2 + DY + \frac{p_0}{w} = DF + \frac{p_0}{w}.$$

Therefore

$$\left(z_1 + \frac{p_1}{w}\right) - \left(z_2 + \frac{p_2}{w}\right) = CE - DF = DG = \frac{fL}{m} \frac{v^2}{2g},$$

G being the point in which the horizontal through C meets FD produced.

DG is called the "virtual fall" of the pipe, being the fall of level in the pressure-columns; and since there would be no fall of level if the friction were nil, DG is said to be the head lost in friction in the distance XY .

Denote this head by h ; then

$$h = \frac{fL}{m} \frac{v^2}{2g},$$

and therefore

$$\frac{h}{L} = \frac{f}{m} \frac{v^2}{2g}.$$

This ratio $\frac{h}{L}$ is designated the virtual slope of the pipe, and is the head lost in friction per unit of length. It will be denoted by i , so that

$$\frac{h}{L} = i = \frac{f}{m} \frac{v^2}{2g}.$$

If the section of the pipe is a circle of diameter d , or a square with a side of length d , then

$$m = \frac{A}{P} = \frac{d}{4}, \quad \frac{V}{2}$$

$$\frac{12 V}{42}$$

and

$$\frac{h}{L} = i = \frac{4f}{2g} \frac{v^2}{r} = \alpha \frac{v^2}{r},$$

where $\alpha = \frac{f}{g}$ and r is the radius.

6. Influence of the Pipe's Position and Inclination on the Flow.—In Fig. 82 join CD . Now since the fall of level (h) is proportional to L , the free surface in any other column between X and Y must also be on the line CD . Thus the pressure p' at any intermediate point M distant x ($=XM$) from X is given by

$$\frac{p'}{w} = MN + \frac{p_0}{w} = CX + \frac{x}{L}(DY - CX) + \frac{p_0}{w}.$$

Hence, at every point of a pipe laid below CD , the fluid pressure (p') exceeds the atmospheric pressure (p_0) by an amount $w \cdot MN$, so that if holes are made in such a pipe, the water will flow out and there will be no tendency on the part of the air to flow in. In pipes so placed vertical bends may be introduced, care being taken to provide for the removal of the air which may collect in the upper parts of the bends.

If the line of the pipe coincides with CD , i.e., with the virtual slope or line of free surface level, $MN = 0$, and the fluid pressure is equal to that of the atmosphere. If holes are now made in the pipe, it can easily be shown by experiment that there will be neither any tendency on the part of the water to flow out nor on the part of the air to flow in.

Next take $CC' = DD' = \frac{p_0}{w}$, and join $C'D'$.

If the pipe is placed in any position between CD and $C'D'$, MN becomes negative, and the fluid pressure in the pipe is less than that of the atmosphere. If holes are made in this pipe, there will be no tendency on the part of the water to flow out, but the air will flow in. Thus, if a pipe rises above the line

of virtual slope, there is a danger of air accumulating in the pipe and impeding, or perhaps wholly stopping, the flow. No vertical bends should be introduced, as the air is easily set free and would collect in the upper parts of the bends, with the effect of impeding the flow and of acting detrimentally upon the water itself, which the liberation of the air renders less wholesome. If the line of pipe coincides with $C'D'$, then the fluid pressure is nil.

Finally, if the pipe at any point rises above $C'D'$, the pressure becomes negative, which is impossible. In fact, the continuity of flow is destroyed, and the pipe will no longer run full bore. Air will be disengaged and will rise and collect at the point in question, so that in order to prevent the flow being wholly impeded, it will be necessary to introduce an air-chamber at this point from which the air can be removed when required.

NOTE.—In the preceding it has been assumed that the pipe is straight. If the pipe is curved, so also is the line of virtual slope. In ordinary practice, however, the vertical changes of level in a pipe at different points are small as compared with the length of the pipe, and distances measured along the pipe are sensibly proportional to distances measured along the horizontal projection of the pipe. Hence the line of virtual slope may be assumed to be a straight line without error of practical importance.

7. Formulæ of Darcy, Hagen, Thrupp, Reynolds, etc.—

Darcy arranged the results of his experiments in a table drawn up as follows:

Diameter.	Section.	Velocities in m./sec.								
		.10		.12		.13		.14		to 3 m./sec.
<i>D</i>	<i>A</i>	$\frac{h}{L}$	<i>Q</i>	$\frac{h}{L}$	<i>Q</i>	$\frac{h}{L}$	<i>Q</i>	$\frac{h}{L}$	<i>Q</i>	

The first column gives the several diameters.

The second column gives the corresponding sectional areas.

The remaining columns give the several velocities of flow from 4 ins. (.1 m.) up to 10 ft. (3 m.) per second, and each velocity column is subdivided into two columns, the one giving the loss of head $\left(\frac{h}{L}\right)$ per unit of length, and the other giving the discharge (Q).

An examination of the table of Darcy's results shows that *approximately* the loss h is directly proportional to the length L of pipe under consideration and to the square of the velocity, v , and is inversely proportional to the diameter d .

Therefore

$$h \propto \frac{L}{d} v^2 = \frac{4fL}{d} \frac{v^2}{2g} = \alpha \frac{L v^2}{r},$$

where
$$\alpha = \frac{f}{g} = \frac{\mu}{g} \left(1 + \frac{1}{12d}\right), \quad (\text{p. 127.})$$

In Hagen's formula, viz.,

$$\frac{h}{L} = \frac{a v^n}{d^x},$$

the values of a , n , and x vary with the velocity, the diameter, and with the roughness of the surface. The results obtained by this formula are in accord with the results of Pearsall's experiments with pipes in good condition and of diameters varying from .9 ft. to 4 ft., when

$$a = .0004, \quad n = 1.87, \quad \text{and} \quad x = 1.4,$$

but the agreement is not so close if the pipe surface is very smooth.

If the pipes are rough, the approximate values of the indices are

$$a = .0007, \quad n = 2, \quad \text{and} \quad x = 1.1,$$

but these values must necessarily vary with every different class of pipe.

Various modifications of Hagen's formula have been proposed, and perhaps one of the best is that contained in a paper by Thrupp, read before the Society of Engineers (London) in 1887. It may be written

$$\frac{1}{i} = \text{cosec. of slope angle} = \frac{L}{h} = \left(\frac{m^x}{c^y} \right)^n,$$

$x + y\sqrt{\frac{z-m}{m}}$ being substituted for x when m is small. The values of n , c , x , y , and z , for a pipe or channel, are given by the following table:

Surface.	n	c	x	y	z
Wrought-iron pipes.....	1.80	0.004787	0.65	0.018	0.07
Riveted sheet-iron pipes....	1.825	0.005674	0.677		
New cast-iron pipes.....	1.85	0.005347	0.67		
	2.00	0.006752	0.63		
Lead pipes.....	1.75	0.005224	0.62		
Pure cement rendering ...	1.74	0.004000	0.67		
	1.95	0.006429	0.61		
Brickwork (smooth).....	2.00	0.007746	0.61		
" (rough).....	2.00	0.008845	0.625	0.01224	0.50
Unplaned plank.....	2.00	0.008451	0.615	0.03349	0.50
Small gravel in cement....	2.00	0.01181	0.66	0.03938	0.60
Large " " "	2.00	0.01415	0.705	0.07590	1.00
Hammer-dressed masonry.	2.00	0.01117	0.66	0.07825	1.00
Earth (no vegetation).....	2.00	0.01536	0.72		
Rough stony earth.....	2.00	0.02144	0.78		

Osborne Reynolds has propounded a simple law of resistance embracing the results of Poiseuille and Darcy, and taking into account the effects of viscosity, temperature, etc. This law may be expressed in the form (the units being a foot and a second)

$$i = \text{the slope} = \frac{h}{L} = \frac{B^n}{AP^{n-2}} \frac{v^n}{d^{3-n}},$$

in which

$$A = 1.917 \times 10, B = 36.8, \text{ and } \frac{1}{P} = 1 + .0336t + .000221t^2,$$

t being the temperature in degrees centigrade. Approximately, the index n is 1 if the critical velocity is not exceeded, and 1.7 to 2 for values of v greater than the critical velocity. According to Unwin the index of d is not exactly $3 - n$ and should be determined independently. For a rough surface $n = 2$, for a smooth cast-iron pipe $n = 1.9$, and for a lead pipe $n = 1.723$ —a limitation which is analogous to that found by Froude in his experiments upon surface friction.

It may be noted that the sum of the exponents of v and d is constant and equal to 3.

In a paper read before the Royal Society of New South Wales, 1897, Knibbs investigates the effects of temperature and records the results of a number of experiments, but the formula he deduces is too complicated to be of much practical value and requires further verification.

Fournie has also studied temperature effect and has suggested a formula, but his results are not complete (*Annales des Ponts et Chaussées*, 1898).

Again, a simple empirical law connecting v , m , and i may be expressed in the form

$$v = cm^{\frac{2}{3}}i^{\frac{1}{2}},$$

in which c is a coefficient whose value is to be determined by experiment. Taking $c = \frac{g}{wf} = \frac{32.16}{62.42 \times f} = \frac{.513}{f}$, then, if $3n = f$, this formula may be written

$$v = \frac{1.54}{n} m^{\frac{2}{3}} i^{\frac{1}{2}}.$$

For values of n from .008 to .018 the results are practically the same as those obtained by substituting the same values for n in Kutter's more complicated formula (Chap. III); but while the

two formulæ closely agree in ordinary cases, they both fail in extreme cases.

The formula is also equally applicable to open channels (Chap. III), m being the mean hydraulic depth; but Tutton has found that when m is small, and especially in the case of open channels, it is preferable to use the modified expression

$$v = \left(\frac{1.54}{n} - \frac{2}{m} \right) m^{\frac{2}{3}} i^{\frac{1}{2}}.$$

Lampe's well-known formula for iron pipes is

$$v = 203.3 m^{\frac{2}{3}} i^{\frac{5}{9}},$$

while Foss gives for the same case

$$v = 191 m^{\frac{8}{11}} i^{\frac{6}{11}}.$$

In 1867, M. Lévy in his *Théorie d'un Courant Liquide*, the units being a *metre and second*, gave:

$$\text{for new cast-iron pipes} \quad v = 36.4 \{ri(1 + \sqrt{r})\}^{\frac{1}{2}};$$

$$\text{" cast-iron pipes in service } \quad v = 20.5 \{ri(1 + 3\sqrt{r})\}^{\frac{1}{2}}.$$

To these Vallot added in 1888:

$$\text{for cleaned cast-iron pipes} \quad v = 32.5 \{ri(1 + \sqrt{r})\}^{\frac{1}{2}}.$$

The corresponding formulæ, with a *foot and second* as units, are:

$$v = 93.24 \{mi(1 + .7809 \sqrt{m})\}^{\frac{1}{2}};$$

$$v = 52.51 \{mi(1 + 2.3427 \sqrt{m})\}^{\frac{1}{2}};$$

$$v = 83.24 \{mi(1 + .7809 \sqrt{m})\}^{\frac{1}{2}}.$$

Vallot also modified the expression for *pipes in service*, and deduced

$$v = 64.788 m^{\frac{2}{3}} i^{\frac{1}{2}} \text{ in metric units,}$$

or

$$v = 96.27 m^{\frac{2}{3}} i^{\frac{1}{2}}, \text{ a foot and second being the units.}$$

Manning, in 1890, gave the formula

$$v = \frac{1.486}{n} m^{\frac{2}{3}} i^{\frac{1}{2}},$$

n being the same as in Kutter's formula, Chap. III.

Flamant, in 1892, deduced the expression

$$v = cm^{\frac{5}{7}} i^{\frac{4}{7}},$$

and gave the following values for c :

For tin pipe.	$c = 284.5$
“ lead “	$c = 272.7$
“ glass “	$c = 262.1$
“ wrought-iron and asphalted pipe.	$c = 257.3$
“ new cast-iron and tarred pipe.	$c = 232.5$
“ lightly incrustated iron pipes in service..	$c = 205.4$

8. Graphical Representation of the formula $v = cm^x i^y$.—

The preceding formulæ are special applications of the general expression

$$v = cm^x i^y,$$

in which the coefficients c , x and y for any series of experiments can be graphically determined in the following manner:

Taking logarithms,

$$\log v = \log c + x \log m + y \log i;$$

and if i_1 is a particular value of i corresponding to a value v_1 of v ,

$$\log v_1 = \log c + x \log m + y \log i_1.$$

Then

$$\log v - \log v_1 = y (\log i - \log i_1),$$

and is the equation to a straight line, the rectangular coordinates being the logarithms of v and of i . Selecting any set of experiments and plotting the corresponding values of $\log v$

and $\log i$, a series of parallel straight lines, inclined at a constant angle $\tan^{-1} \gamma$, is obtained. For all the velocities corresponding to $\log i = 0$ or $i = 1$, i.e., at the intersections of these lines with the axis of " $\log v$," the general expression becomes

$$v = cm^x.$$

Taking logarithms again,

$$\log v = \log c + x \log m,$$

and if v_1 is the value of v corresponding to a particular value m_1 of m ,

$$\log v_1 = \log c + x \log m_1.$$

Therefore

$$\log v - \log v_1 = x(\log m - \log m_1),$$

and is the equation to a straight line, the rectangular coordinates being the logarithms of v and of m .

Plotting the different values of $\log m$ corresponding to the particular values of $\log v$ in question, a series of parallel straight lines, inclined at a constant angle $\tan^{-1} x$, is obtained. When $\log m = 0$, or $m = 1$, i.e., at the intersections of these lines with the axis of " $\log v$," the general expression becomes

$$v = c.$$

Therefore

$$\log v = \log c,$$

and the coefficient c can be at once obtained from the diagram, as it is the value of $\log v$ corresponding to $m = 1$ and $i = 1$.

In 1896, Tutton completed an admirable collaboration of the most important sets of experiments on pipe-flow, more than 1000 in number, and varying widely in diameter and kind of pipe.

9. **Diagrams Showing Results of Experiments.**—By means of the method just described, Tutton has plotted (Figs. 83–89), representing graphically a very large number of experiments on pipe-flow as follows.

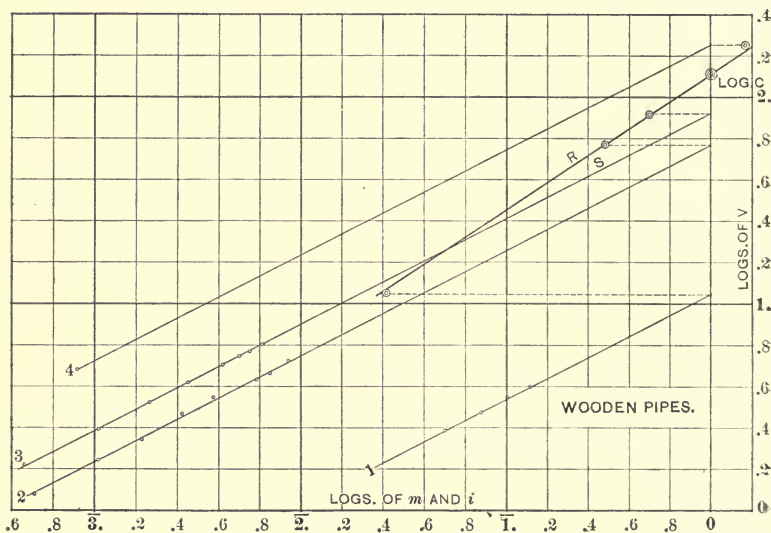


FIG. 83.—Flow through Wooden Pipes.

- | | |
|-------------------------------------|-------------|
| 1. Hamilton Smith. Series 10..... | $m = .0263$ |
| 2. Darcy and Bazin. " 52..... | $m = .303$ |
| 3. Darcy and Bazin. " 51..... | $m = .505$ |
| 4. Clarke, Moon Island conduit..... | $m = 1.50$ |

4 series, 22 experiments.

$$\text{Formula: } v = 129m^{.66}l^{.51}.$$

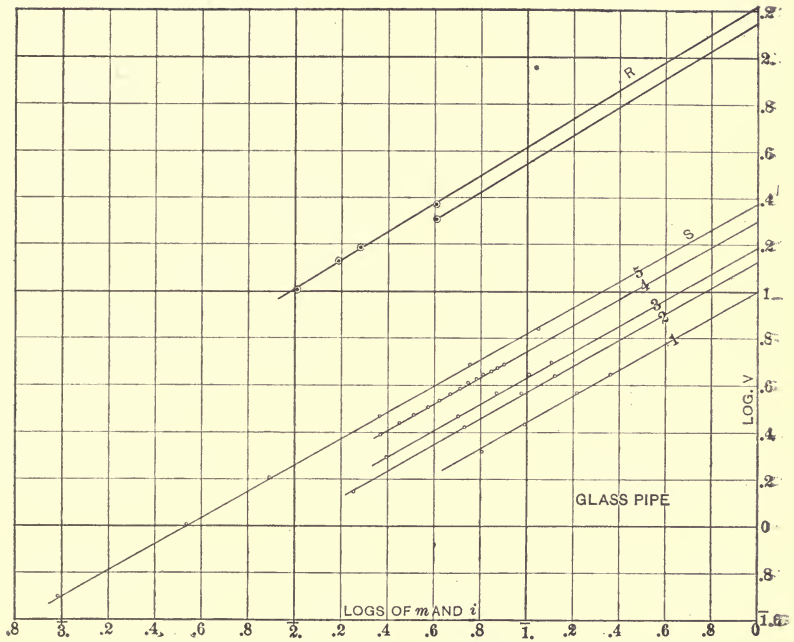


FIG. 84.—Flow through Glass Pipes.

5. Darcy.	Series II.....	$m = .04075$
4. Darcy.	" 11A.....	$m = .04132$
3. Smith.	" 7.....	$m = .0191$
2. Smith.	" 8.....	$m = .01555$
1. Smith.	" 9.....	$m = .01045$

5 series, 32 experiments.

Formula: $v = 141 \text{ to } 169 m^{.61} d^{.56}$.

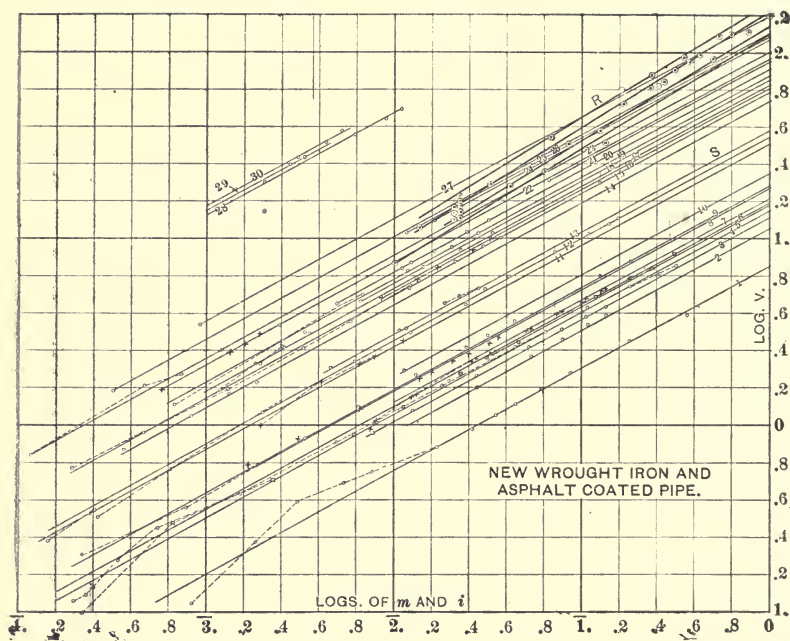


FIG. 85.—Flow through New Wrought-iron and Asphalt-coated Pipes.

1. Darcy. Series 1.... $m=.01$	16. { Smith,	
2. Smith. " 6.... $m=.01307$	17. { North Bloomfield } $m=.22775$	
3. Smith. " 5.... $m=.021325$	18. { Iben, Bonn pipe } $m=.251$	
4. Darcy. " 2.... $m=.02182$	19. { Smith,	
5. Smith. " 4.... $m=.0219$	20. { North Bloomfield } $m=.264$	
6. Darcy " 7.... $m=.02198$	21. Darcy. Series 10 .. $m=.23375$	
7. Smith. " 1, 2. $m=.0219$	22. { Smith,	
7. Smith. " 3.... $m=.0218$	23. { North Bloomfield } $m=.3075$	
8. { Ehmann, }	24. Smith, Texas Creek. $m=.354$	
8. { Hahnwald }	25. Lampe..... $m=.34325$	
9. Darcy. Series 3.... $m=.0324$	26. Tubbs, Rochester... $m=.50$	
10. { Crozet,	27. Iben, Sternchanze .. $m=.4167$	
10. { Blue Ridge siphon }	28. Smith, Humbug pipe $m=.5385$	
11. Couplet..... $m=.1332$	29. { Smith,	
11. Iben, Deseniss St... $m=.08375$	30. { Cherokee pipe } .. $m=.6075$	
12. Darcy. Series 8.... $m=.06775$	31. Tubbs, Rochester... $m=.75$	
13. Iben, Schoen St.... $m=.12475$	32. Gale $m=1.0$	
13. Iben. Series 5a .. $m=.12475$	33. Rowland, High Heads $m=.0208$	
14. Ehmann, Stuttgart.. $m=.1655$	34. " " " $m=.0208$	
15. Darcy. Series 9.... $m=.16075$	35. " " " $m=.0208$	

36 series, 195 experiments.

Formula: For asphalt-coated $v = 139 \text{ to } 188 m^{.62} d^{.55}$.For new wrought-iron $v = 127 \text{ to } 165 m^{.62} d^{.55}$.

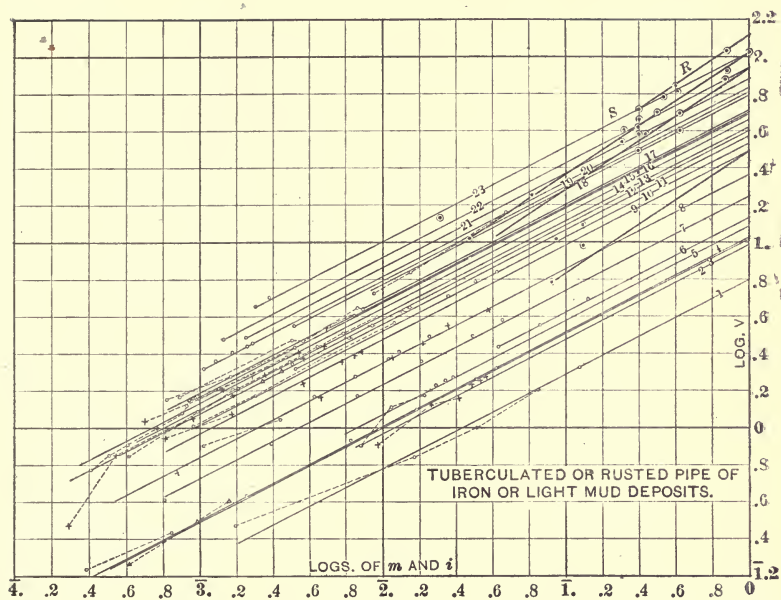


FIG. 86.—Flow through Tuberculated or Rusty Pipe of Iron or Light Mud Deposits.

1. Iben, Koppel St., 19 years old.....	$m = .08375$
2. Iben, Schulweg, 19 years old.....	$m = .1247$
3. Couplet.....	$m = .0888$
4. Darcy. Series 12.....	$m = .02945$
5. Iben, Schulweg, 13 years old.....	$m = .1247$
6. Fanning, rusted pipe.	$m = .020835$
7. Darcy. Series 14.....	$m = .0652$
8. Iben. " 15a, 22 years.....	$m = .2502 +$
8. Iben, Strohhaus, 22 years.....	$m = .2502$
9. Iben, Carolinen, 15 years.....	$m = .2502$
10. Darcy. Series 19.....	$m = .1995$
11. Couplet.....	$m = .266$
12. Iben, Rotherbaum.....	$m = .25$
12. Ehmann.....	$m = .2075 +$
12. Iben, Heidenkampsweg, 25 years.....	$m = .4167$
13. Iben, Hamm St.....	$m = .25$
14. Iben, Glacis Chaussée.....	$m = .250$
14. Duncan.....	$m = .25 +$
15. Bailey.....	$m = .4167$

16. Leslie.....	$m = .3125$
17. Simpson. Series 3.....	$m = .25$
18. Leslie.....	$m = .3333$
19. Couplet.....	$m = .4$
20. Simpson.....	$m = .3957$
21. Greene.....	$m = .75$
22. McElroy, Brooklyn main.....	$m = .75$
23. Sherrerd, Pequannock main.....	$m = .75$
23. Sherrerd, " ".....	$m = 1.0$

30 series, 132 experiments.

Formula: For light tuberculations $v = 87$ to $132 m^{.66} i^{.51}$.

For heavy tuberculations $v = 31$ to $80 m^{.66} i^{.51}$.

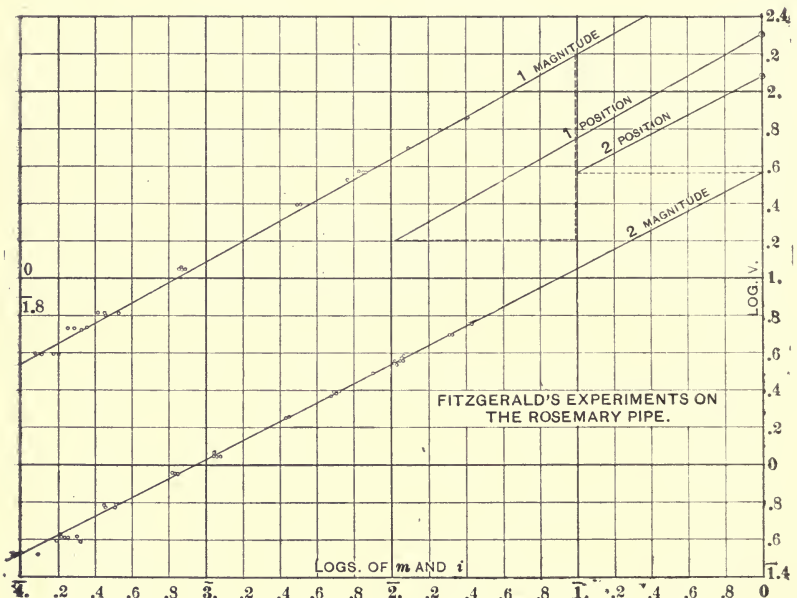


FIG. 87.—FitzGerald's Experiments on the Rosemary Pipe.

1. North pipe. Cleaned, asphalted..... $m = 1.00$
 2. Both pipes. Tuberculated..... $m = 1.00$
- 4 series, 57 experiments.

Formula: Cleaned, asphalted, $v = 199 m^{.62} i^{.55}$.

Tuberculated, $v = 117 m^{.66} i^{.51}$.

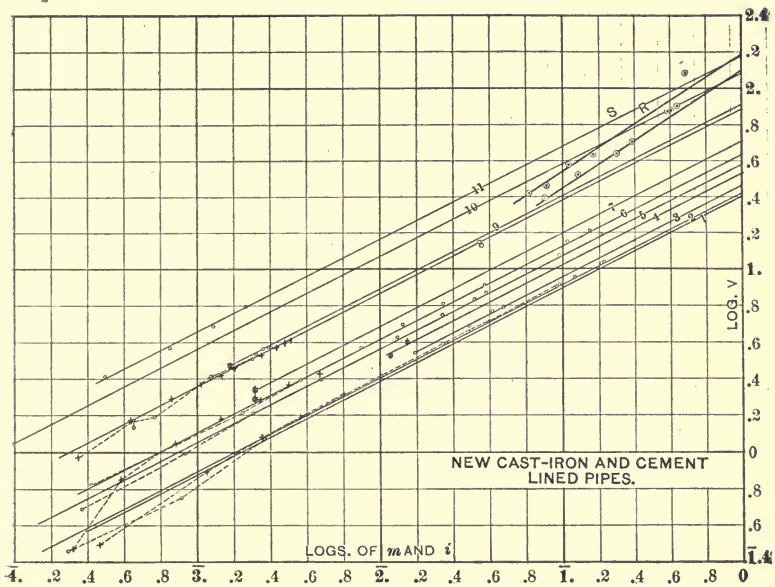


FIG. 88.—Flow through New Cast-iron and Cement-lined Pipes.

1. Ehmann, Neckar St.....	$m = .0827 +$
2. Darcy. Series 16.....	$m = .067175$
3. Iben, Wenden St.....	$m = .0835$
4. Iben, Haller St.....	$m = .1245$
5. Darcy. Series 17.....	$m = .11237$
6. Darcy. " 18.....	$m = .1542$
6. Ehmann, Stuttgart.....	$m = .20725 +$
7. Russell, St. Louis.....	$m = .25$
8. Darcy. Series 22.....	$m = .4101$
8. Fanning, cement-lined.....	$m = .4167 +$
9. Friend, Seville.....	$m = .4375$
10. Woods, Newton (doubtful).....	$m = .5$
11. Stearns, Rosemary pipe.....	$m = 1.0$

13 series, 79 experiments.

Formula : $v = 126 \text{ to } 158 m^{.66} d^{.51}$.

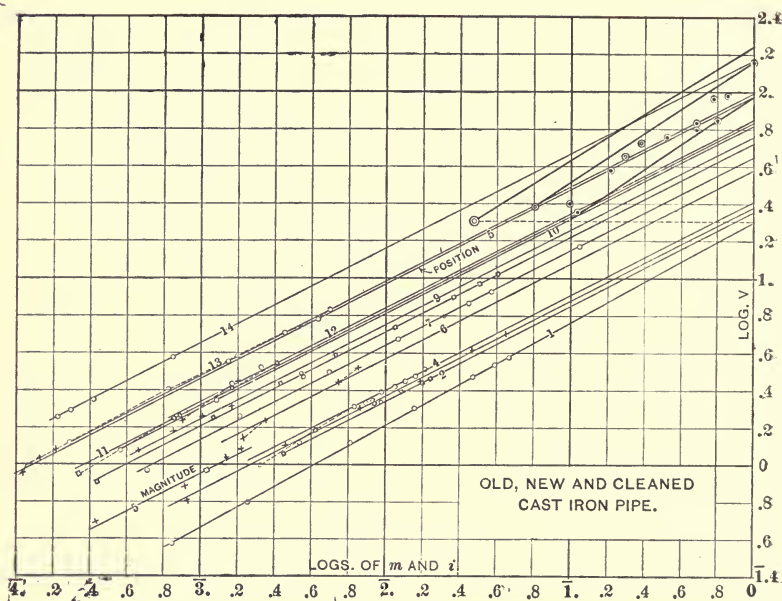


FIG. 89.—Flow through Old, New, and Cleaned Cast-iron Pipes.

- | | |
|--|--------------|
| 1. Darcy. Series 13..... | $m = .02985$ |
| 2. Meunier, Torcy, 28-30 years old..... | $m = .1107$ |
| 3. Darcy. Series 15..... | $m = .0657$ |
| 4. Meunier, Nogent sur Seine, new..... | $m = .1035$ |
| 5. Coffin, Taunton main, $2\frac{1}{2}$ years old..... | $m = .625$ |
| 6. Meunier, Charenton, 2 years old..... | $m = .1640$ |
| 7. Darcy. Series 20..... | $m = .2007$ |
| 8. Darcy. " 21..... | $m = .2436$ |
| 9. Forbes, Brookline main, 8 years old..... | $m = .3333$ |
| 10. Humblot, 1 series, 10 years old..... | $m = .4921$ |
| 11. Humblot, Bercy..... | $m = .4921$ |
| 12. Humblot, 3 series, 6, 7, and 12 years old ... | $m = .6562$ |
| 13. Meunier, Canal de l'Oise, 1 year old.... | $m = .7382$ |
| 14. Bruce, Blane Valley, new | $m = 1.0$ |
- 16 series, 80 experiments.

Formula: $v = 96 \text{ to } 148 m^{.66} i^{.51}$.

10. Values of c , x , and y in the Formula $v = cm^x i^y$.—Tutton found (see Reynolds' formula) that, in the general formula $v = cm^x i^y$,

$$x + y = \text{a constant} = 1.17,$$

and therefore

$$v = cm^{1.17-y} i^y.$$

The values of c and y he has tabulated as follows:

	c	y	
For tin pipe.	189	.58	
For lead pipe.	168	.58	Older experiments give $c = 189$.
For brass, zinc, and glass pipe	165	.56	In one set of glass experiments $c = 141$.
For wrought-iron pipe	160	.55	c varies from 127 to 165, approximating to the higher number.
For wood-stave pipe.	125	.51	
For new cast-iron or tarred pipe	130	.51	In tarred pipes c varies from 115 to 152, the values being about the same as in cast-iron pipes of same size. Benzinger gives for a 60-in. cast-iron pipe $c = 129$.
For pipe in service.	104	.51	Generally c is about 105. In the Rosemary pipe $c = 117$.
For tuberculated pipe.	30 to 80	.51	
For lap-riveted pipe.	115	.51	c varies from 125 to 135 for new to 110 to 114 for pipe in service.
For rubber and leather hose..	160	.51	
For wrought-iron pipe asphalt-coated	170	.55	In some cases $c = 140$, and in the 48-in. pipe $c = 199$.
For large brick conduits.	129	.52	Unobstructed by shafts.
For large brick conduits.	91	.52	Fullerton Avenue conduit of Chicago water-supply.
For large brick conduits.	110	.52	Chicago land tunnel.

The values of c in this table are *mean* values and necessarily vary with age and roughness.

Throughout the analysis of these experiments the total head was diminished by the loss of head at entrance, and in the cases in which this loss had not been found by means of piezometers it has been calculated from $\frac{1}{c_c} \frac{v^2}{2g}$, c_c being the coefficient of contraction.

Assuming for y the approximate value .5, Tutton's formula becomes

$$v' = cm^{.67} i^{.5} = cm^{\frac{2}{3}} i^{\frac{1}{2}}.$$

Ex. 1. The head over the sharp-edge entrance into a pipe, 1000 ft. long and passing 1 cu. ft. of water per sec., is 9 ft. Find the diameter, taking $f = .0055$.

$$9 = \frac{v^2}{64} \left(\frac{1}{2} + 1 + \frac{4 \times .0055 \times 1000}{d^5} \right) = \frac{49}{16 \cdot 121 d^4} \left(1.5 + \frac{22}{d} \right).$$

For a *first* approximation, disregarding the first term on the right-hand side, which is small as compared with the second term,

$$9 = \frac{49}{16 \cdot 121} \frac{22}{d^5} = \frac{49}{88} \frac{1}{d^5},$$

and

$$d = .57319 \text{ ft.}$$

For a *second* approximation

$$9 = \frac{49}{16 \cdot 121 d^4} \left(1.5 + \frac{22}{.57319} \right),$$

or

$$d^4 = \frac{49}{9 \cdot 16 \cdot 121} \times 39.8817,$$

and

$$d = .5787 \text{ ft.}$$

Ex. 2. The effective height of the grade line above the entrance into a clean iron 3-in. branch, 1000 ft. long, is 20 ft. 5 ins. How many people will the branch supply with 20 gallons of water per head per day of 24 hours?

$$f = .005 \left(1 + \frac{1}{12 \times \frac{1}{4}} \right) = \frac{1}{150},$$

$$20 \frac{5}{12} = \frac{4 \times \frac{1}{150} \times 1000}{\frac{1}{4}} \frac{v^2}{64} = \frac{5}{3} v^2,$$

and $v = 3\frac{1}{2}$ ft per sec.

The delivery in cu. ft. per sec.

$$= \frac{22}{7} \frac{1}{4} \left(\frac{1}{4} \right)^2 3\frac{1}{2} = \frac{11}{64}.$$

The delivery in gallons per day

$$= \frac{11}{64} \times 6\frac{1}{4} \times 60 \times 60 \times 24 = 92,812\frac{1}{2},$$

and the number of people served per day = $\frac{92812\frac{1}{2}}{20} = 4640\frac{5}{8},$

or 4640.

Ex. 3. Find the proper diameter of a rough pipe to give 60,000,000 of gallons every 24 hours, the slope of the pipe being 1 in 800.

$$\frac{22}{7} \frac{d^2}{4} v = \frac{60,000,000}{6\frac{1}{4} \cdot 60 \cdot 60 \cdot 24},$$

or $d^2 v = \frac{14000}{99}.$

Using Hagen's formula, viz., $\frac{h}{L} = \frac{av^n}{dx} = \frac{1}{800},$

and taking $a = .0007, \quad n = 2, \quad \text{and} \quad x = 1.1,$

$$\frac{1}{800} = \frac{.0007}{d^{1.1}} v^2 \pm \frac{.0007}{d^{1.1}} \left(\frac{14000}{99d^2} \right)^2,$$

or $d^{5.1} = 800 \times .0007 \left(\frac{14000}{99} \right)^2.$

Therefore $d = 6.22$ ft.

Ex. 4. What should be the slope of a 24-in wooden-stave pipe to give 5,940,000 gallons per day?

$$\frac{22}{7} \frac{(2)^2}{4} \cdot v = \frac{5,940,000}{6\frac{1}{4} \cdot 24 \cdot 60 \cdot 60} = 11,$$

and $v = 3\frac{1}{2}$ ft. per sec.

Take the formula

$$v = cm^{1.17-y} i^y.$$

By the Table,

$$c = 125 \quad \text{and} \quad y = .51.$$

Therefore

$$3\frac{1}{2} = 125 \left(\frac{2}{4} \right)^{.66} i^{.51},$$

and

$$i = .002212, \text{ or about } 22 \text{ in } 10,000$$

11. Transmission of Energy by Hydraulic Pressure.—

Let Q cu. ft. of water per second be driven through a pipe of diameter d ft. and length L ft. under a total head of H ft. Also let n per cent of the total head be absorbed in overcoming the frictional resistance in the pipe. Then

$$\begin{aligned}\text{the head expended in useful work} &= H - h \\ &= H \left(1 - \frac{n}{100} \right),\end{aligned}$$

$$\text{and the efficiency} = \frac{H - h}{H} = 1 - \frac{n}{100}$$

Again,

$$\frac{nH}{100} = h = \frac{4fL}{d} \frac{v^2}{2g} = \frac{fLQ^2}{\pi^2 d^5}.$$

Since $Q = \frac{\pi d^2}{4} v$, and g is assumed to be 32,

$$Q = \frac{\pi}{10} \sqrt{\frac{nHd^5}{fL}},$$

and the total available work in foot-pounds per second

$$= wQH = \frac{275}{14} \sqrt{\frac{nH^3d^5}{fL}}.$$

If N is the number of horse-power delivered at the end of the pipe,

$$N = \frac{wQH}{550} \left(1 - \frac{n}{100} \right) = \frac{1}{28} \left(1 - \frac{n}{100} \right) \sqrt{\frac{nH^3d^5}{fL}},$$

an equation giving the distance L to which N horse-power can be transmitted with a loss of n per cent of the total head.

Again,

$$\text{the efficiency} = 1 - \frac{h}{H} = 1 - \frac{2fL}{gH} \frac{v^2}{d} = 1 - \frac{2fLw}{g} \frac{v^2}{pd},$$

p ($= wH$) being the pressure corresponding to the head H . The efficiency diminishes as v increases and therefore, so far as efficiency is concerned, it is advantageous to transmit energy at a low speed. Again, the efficiency is constant if $\frac{v^2}{pd}$ is constant.

Assuming this to be the case, take $v^2 = c^2 \cdot pd$. Then the total energy transmitted $= wQH = w \frac{\pi d^2}{4} vH$

$$= \frac{\pi c}{4} p^{\frac{3}{2}} d^{\frac{5}{2}}.$$

If it be also assumed that the thickness t of the pipe-metal is so small that the formula

$$pd = 2f't$$

holds true, f' being the circumferential stress induced in the metal, then

$$\begin{aligned} \text{the energy transmitted} &= \frac{\pi c}{4} p^{\frac{3}{2}} d^{\frac{5}{2}} \\ &= \frac{\pi c f' t d}{2} \sqrt{pd} \\ &= \frac{cf'V}{2} \sqrt{pd}, \end{aligned}$$

V being the volume of the pipe per unit of length.

Hence, for a given volume V of metal and a constant efficiency, the energy transmitted is a maximum when pd is a maximum.

If p is increased beyond a certain limit, the ratio $\frac{t}{d}$ is no longer small and the thickness t will have a greater value than that given by the equation $pd = 2f't$. Then the cost of the pipe will also increase. On the other hand, if d is increased,

the ratio $\frac{t}{d}$, and therefore also the pressure p , will remain small, and thus the cost of the pipe will not increase. Hence it is more economical to employ large pipes and low pressures than small pipes and high pressures.

The demand for hydraulic power in large cities has led to the laying down of networks of mains through which water is conveyed under pressure and is distributed to the consumer for various industrial purposes. Since the loss of head due to frictional resistance is approximately proportional to the square of the velocity, and since also the momentum of the moving fluid must not be so great as to make excessive shocks possible, high velocities cannot be allowed in the mains or in the machines operated by the pressure-water except for very short distances. Thus, the velocity of flow in the mains is limited to 6 ft. per second, and rarely exceeds 8 ft. per second in the machines. In London the average rate is 4 ft. per second. Again, the *quantity* of power conveyed by a single main cannot be great. Hence the hydraulic distribution of power, in which the pressure of water is directly utilized, is especially adapted for machines with slow-moving rams, which are intermittent in action and which work only for short intervals of time, as, for example, in lifting and pressing operations and when a great effort is to be exerted through a short distance. In London the pressure in the mains is 750 lbs. per sq. in., but in the more recent distributions in Manchester and Glasgow the pressure is 1100 lbs. per sq. in. The working stress in the cast-iron mains, the largest in use being $7\frac{1}{2}$ ins. in diameter, is 2800 lbs. per sq. in., and they are generally tested to 2500 lbs. per sq. in. before laying and to about 1000 lbs. per sq. in. after laying. The thickness t in inches of cast-iron main of d ins. diameter under a water-pressure of p lbs. per sq. in. may be determined by the formula

$$t = .00078pd + .25 \text{ in.}$$

Another formula gives $t = .0024pd + .75$ in., p being the pressure in atmospheres.

With suitable joints, and drawn tubes of steel with a tenacity of 15,000 lbs. per sq. in., the hydraulic system of distribution could be greatly extended.

Again, for an hydraulic pipe or press

$$f = \frac{p_0 r_0^2 - p_1 r_1^2}{r_0^2 - r_1^2} + \frac{p_0 - p_1}{r^2} \frac{r_0^2 r_1^2}{r_0^2 - r_1^2},$$

where p_0, p_1 are the intensities of pressure at the outer and inner surfaces;

f is the intensity of stress at the radius r ;

r_0, r_1 are the radii of the outer and inner surfaces.

(See Appendix, Bovey's "Theory of Structures.")

EX. 1. An accumulator supplies a pressure of 700 lbs. per sq. in. What length of 8-in. pipe will deliver 200 H.P. of useful energy with a loss of 20 per cent?

250 H.P. enter the pipe. Therefore, if Q is the delivery in cu. ft. of water per sec.,

$$250 = \frac{144 \cdot 700}{550} Q,$$

$$\text{or} \quad \frac{275}{252} = Q = \frac{22}{7} \cdot \frac{1}{4} \left(\frac{2}{3} \right)^2 v,$$

$$\text{and} \quad v = \frac{25}{8} \text{ ft. per sec.}$$

$$\text{Take } f = .005 \left(1 + \frac{1}{12 \times \frac{2}{3}} \right) = \frac{9}{1600}, \text{ for a clean iron pipe.}$$

Then

$$\frac{20}{100} \times 250 = \text{loss} = 62\frac{1}{2} \cdot \frac{275}{252} \cdot \frac{4 \times \frac{9}{1600} \times L}{\frac{2}{3}} \frac{(\frac{25}{8})^2}{64} \cdot \frac{1}{550},$$

L being the length of the pipe.

$$\text{Therefore} \quad L = 78,293.7 \text{ ft.} = 14.8 \text{ miles.}$$

EX. 2. The efficiency of an engine is .6; it burns 2 lbs. of coal per hour per H.P., and works 16 hours a day for 300 days in the year. The

cost of the engine is \$12 per H.P., and the cost of the coal \$3 per ton. An amount of 4500 gallons of water per minute is to be raised a vertical height of 200 ft. What must be the minimum diam., D , of the pipe, assuming that the cost of the piping is \$ D per lineal foot, and that $f = .0064$?

Let h feet be the frictional loss of head.

$$\text{Then, since } \frac{4500}{60 \times 6\frac{1}{4}} = \frac{22}{7} \frac{D^2}{4} v = 12,$$

$$\text{or } vD^2 = \frac{168}{11},$$

$$h = \frac{4 \times .0064 \times 200}{D} \frac{1}{64} \left(\frac{168}{11} \right)^2 = \frac{2}{25} \left(\frac{168}{11} \right)^2 \frac{1}{D^5}.$$

Again, let N be the number of H.P.

$$\begin{aligned} \text{Then } N &= \frac{5}{3} \frac{12 \cdot 62\frac{1}{2}}{550} (200 + h) \\ &= \frac{25}{11} \left\{ 200 + \frac{2}{25} \left(\frac{168}{11} \right)^2 \frac{1}{D^5} \right\}. \end{aligned}$$

$$\text{Cost of coal capitalized at } 5\% = N \cdot \frac{2 \cdot 16 \cdot 300 \cdot 3}{2000} \cdot \frac{100}{5} = \$288N.$$

$$\text{Cost of engine} = \$12N.$$

$$\text{Cost of piping} = \$200D.$$

$$\text{Total prime cost} = 300N + 200D$$

$$= 300 \times \frac{25}{11} \left\{ 200 + \left(\frac{168}{11} \right)^2 \frac{1}{D^5} \right\} + 200D,$$

which must be a minimum.

$$\text{Therefore } 0 = -300 \times \frac{25}{11} \left(\frac{168}{11} \right)^2 \frac{5}{D^6} + 200,$$

$$\text{and } D = 3.98 \text{ ft.}$$

$$\text{Hence, also, } h = \frac{2}{25} \left(\frac{168}{11} \right)^2 \frac{1}{(3.98)^5} = .01868 \text{ ft.}$$

$$\text{No. of H.P.} = N = 454.59.$$

$$\text{Capital cost} = \$137,173.$$

12. Pressure Due to Shock.—Water flows through a line of piping with a velocity of v ft. per second, and at a certain point the motion is suddenly arrested by the closing of a valve, developing a sudden increase in the pressure at the valve of

f lbs. per sq. in. Water being slightly compressible—losing $\frac{1}{70}$ of its bulk under a pressure of 2 tons (of 2240 lbs.) per square inch—a compression-wave starts from the valve and moves backwards throughout the whole length L ft. of the moving column of water. The water still enters the pipe for the period of t seconds, during which the compression continues.

Let a ft. be the sectional area of the water-column;

“ x ft. be the diminution in the length L of the water column;

“ K be the modulus of cubic elasticity of water = 300,000 lbs. per sq. in.

Then

$$\frac{x}{L} = \frac{f}{K}$$

$$x = vt,$$

and $144a$ ft. = momentum of the fluid mass = $\frac{w}{g} aLv$.

Hence

$$f = \frac{1}{144} \frac{wL}{g} \frac{v}{t} = \frac{1}{144} \frac{wL}{g} \frac{L}{t^2} x = \frac{1}{144} \frac{wL^2}{g} \frac{f}{K},$$

and

$$\frac{L}{t} = \text{velocity of the wave-propagation} = t \sqrt{\frac{gK}{w}}.$$

Substituting the values of g , K , and w , the velocity of wave-propagation is found to be about 4720 ft. per second, which is also the velocity of sound in water.

Ex. A volume of water 50 ft. in length, flowing through a pipe with a velocity of 24 ft. per sec., is quickly and uniformly stopped in *one tenth* of a second by closing a stop-valve. Find the increase of pressure per sq. in. in the pipe near the valve.

$$\text{The pres. per sq. in.} = \frac{62\frac{1}{2}}{32} \cdot \frac{1}{144} \cdot \frac{50 \cdot 24}{.1} = 162.76 \text{ lbs.}$$

13. Flow in a Pipe of Uniform Section and of Length L , connecting two Reservoirs at Different Levels.—Let z ft. be the difference of level between the water-surface in the two reservoirs.

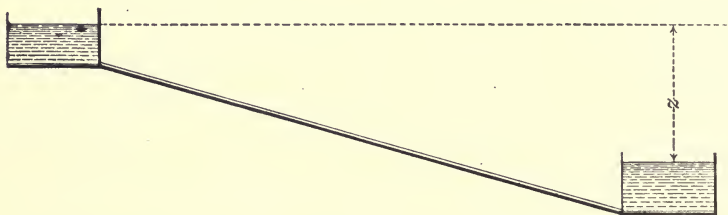


FIG. 90.

The work done per second is evidently equal to the work done by the fall of wQ lbs. of water through the vertical distance z , and is expended—

- (1) In producing the velocity of flow v ft. per second, which requires a head of z_1 ft. and an expenditure of wQz_1 ft.-lbs. of work per second;
- (2) In overcoming the resistance at the entrance from the upper reservoir into the pipe, which requires a head of z_2 ft. and an expenditure of wQz_2 ft.-lbs. of work per second;
- (3) In overcoming the frictional resistance, which requires a head of z_3 ft. and an expenditure of wQz_3 ft.-lbs. of work per second. Thus

$$wQz = wQz_1 + wQz_2 + wQz_3,$$

or

$$z = z_1 + z_2 + z_3.$$

Now $z_1 = \frac{v^2}{2g}$ ft., and the corresponding energy wQz_1 is ultimately wasted in producing eddy motions, etc., in the lower reservoir.

z_2 may be expressed in the form $n\frac{v^2}{2g}$ ft., n being a coeffi-

cient whose value varies with the nature of the construction of the entrance into the pipe. If the pipe-entrance is bell-mouth in form, $n = .01$ or $.02$, but if it is cylindrical, $n = .49$. Finally,

$$z_3 = \frac{L}{m} \frac{F(v)}{w} \text{ ft.} = \frac{4fL}{d} \frac{v^2}{2g} \text{ ft.,}$$

taking $\frac{F(v)}{w} = f \frac{v^2}{2g}$, as is usual in practice. Hence

$$\begin{aligned} z &= \frac{v^2}{2g} \left(1 + n + \frac{4fL}{d} \right) \\ &= \frac{Q}{4\pi^2 d^4} \left(1 + n + \frac{4fL}{d} \right), \end{aligned}$$

since $Q = \frac{\pi d^2}{4} v$, and g is assumed to be 32.

For given values of Q and z a first approximate value of d may be obtained from the last equation by neglecting the term

$\frac{Q^2}{4\pi^2 d^4} (1 + n)$. Call this value d_1 , and substitute it for the d

in the term $\frac{4fL}{d}$ within the brackets. A second approximation

may now be made by deducing d from the formula

$$z = \frac{Q^2}{4\pi^2 d^4} \left(1 + n + \frac{4fL}{d_1} \right),$$

and the operation may be again repeated if desired.

Generally speaking, $1 + n$ is usually very small as compared with $\frac{4fL}{d}$, and may be disregarded without error of practical importance.

The formula then becomes

$$z = \frac{4fL}{d} \frac{v^2}{2g},$$

which is known as Chezy's formula for long pipes.

The term $1 + n$ need only be taken into account in the case of short pipes and high velocities.

Ex. The difference of level between the water-surfaces of two reservoirs, connected by a 24-in. pipe $6\frac{1}{4}$ miles in length, is $172\frac{1}{4}$ ft. The pipe, having been in use for some time, has its inside surface coated with a deposit, and no special provision is made to diminish the resistance at the upper end. Determine the discharge into the lower reservoir in gallons per hour.

Take
$$f = .01 \left(1 + \frac{1}{12 \times 2} \right) = \frac{1}{96}.$$

Then
$$172\frac{1}{4} = \frac{v^2}{64} \left(\frac{1}{2} + 1 + \frac{4 \times \frac{1}{96} \times 6\frac{1}{4} \times 5280}{2} \right) = \frac{v^2}{64} \times 689,$$

and
$$v = 4 \text{ ft. per sec.}$$

Therefore the discharge in cu. ft. per hour

$$= \frac{22}{7} \cdot \frac{2^2}{4} \cdot 4 \cdot 60 \cdot 60 = 45,257\frac{1}{7},$$

and the discharge in gallons per hour

$$= 45,257\frac{1}{7} \times 6\frac{1}{4} = 282,857\frac{1}{7}.$$

14. Losses of Head due to Abrupt Changes of Section, Elbows, Valves, etc.—When the velocity, or the direction of motion of a mass of water flowing through a pipe, is abruptly changed, the water is broken up into eddies or irregular motions which are soon destroyed by viscosity, the corresponding energy being wasted.

CASE I. *Loss due to a sudden contraction.* (Art. 17, Chap. I.)

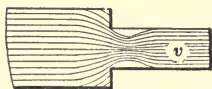


FIG. 91.

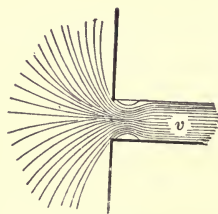


FIG. 92.

(a) Let water flow from a pipe (Fig. 91), or from a reservoir (Fig. 92) into a pipe of sectional area A .

Let c_c be the coefficient of contraction.

Then the area of the contracted section $= c_c A$, and

$$\begin{aligned}\text{the loss of head} &= \frac{1}{2g} \left(\frac{v}{c_c} - v \right)^2 \\ &= \frac{v^2}{2g} \left(\frac{1}{c_c} - 1 \right)^2 \\ &= m \frac{v^2}{2g},\end{aligned}$$

where $m = \left(\frac{1}{c_c} - 1 \right)^2$.

The value of m has not been determined with any great degree of accuracy; but if $c_c = .64$, then $m = .316$. The value of c_c is sometimes obtained from the formula

$$\frac{1}{c_c} = 1.3 \sqrt{2.618 - 1.618 \frac{a^2}{A^2}}.$$

When the water enters a cylindrical (not bell-mouthed) pipe from a large reservoir, the value of m is about .505.

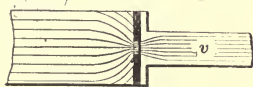


FIG. 93.

(b) Let the water flow across the abrupt change of section through a central orifice in a diaphragm placed as in Fig. 93.

Let a be the area of the orifice.

Then $c_c a$ is the area of the contracted section, and

$$\text{the loss of head} = \left(\frac{A}{c_c a} - 1 \right)^2 \frac{v^2}{2g} = m \frac{v^2}{2g},$$

where $m = \left(\frac{A}{c_c a} - 1 \right)^2$.

According to Weisbach,

if $\frac{a}{A} =$.1	.2	.3	.4	.5
$c_c =$.616	.614	.612	.610	.607
$m =$	231.7	50.99	19.78	9.612	5.256
if $\frac{a}{A} =$.6	.7	.8	.9	1.00
$c_c =$.605	.603	.601	.598	.596
$m =$	3.077	1.876	1.169	.734	.48

(c) A diaphragm with a central orifice of area a , placed in a cylindrical pipe of sectional area A as in Fig. 94.

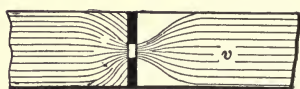


FIG. 94.

The "contracted area" of the water $= c_c a$ and

$$\begin{aligned} \text{the loss of head} &= \frac{1}{2g} \left(\frac{vA}{c_c a} - v \right)^2 = \frac{v^2}{2g} \left(\frac{A}{c_c a} - 1 \right)^2 \\ &= m \frac{v^2}{2g}, \end{aligned}$$

$$\text{where } m = \left(\frac{A}{c_c a} - 1 \right)^2.$$

Generally m must be determined by experiment, but Weisbach gives the following results:

if $\frac{a}{A} =$.1	.2	.3	.4	.5
$c_c =$.624	.632	.643	.659	.681
$m =$	225.9	47.77	30.83	7.801	3.753
if $\frac{a}{A} =$.6	.7	.8	.9	1.00
$c_c =$.712	.755	.813	.892	1.00
$m =$	1.796	.797	.29	.06	00

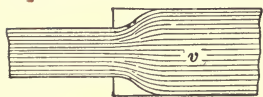
CASE II. *Loss due to a Sudden Enlargement.* (Fig. 95.)

FIG. 95.

Let A_1 = external area of small pipe." A_2 = " " " large "

Then

$$\begin{aligned} \text{loss of head} &= \frac{1}{2g} \left(\frac{vA_2}{A_1} - v \right)^2 = \frac{v^2}{2g} \left(\frac{A_2}{A_1} - 1 \right)^2 \\ &= m \frac{v^2}{2g}, \end{aligned}$$

where $m = \left(\frac{A_2}{A_1} - 1 \right)^2$.

NOTE.—The losses of head in Case I (a) and in Case II may be avoided by substituting a gradual and regular change of section for the abrupt changes.

CASE III. *Loss of Head due to Elbows.* (Fig. 96.)—The loss of head due to the disturbance caused by an elbow is expressed by Weisbach in the form

$$m \frac{v^2}{2g},$$

where $m = .9457 \sin^2 \frac{\phi}{2} + 2.047 \sin^4 \frac{\phi}{2}$,

ϕ being the elbow angle.

Weisbach deduced this formula from the results of experiments with pipes 1.2 in. in diameter.

The velocity v_1 with which the water flows along the length AB may be resolved into a component v with which the water flows along BC and a component u at right angles to the direction of v . The component u and therefore the corresponding head, viz., $\frac{u^2}{2g}$, is wasted. The component u evidently diminishes with the angle ϕ and becomes nil when a

gradually and continuously curved bend is substituted for the elbow.

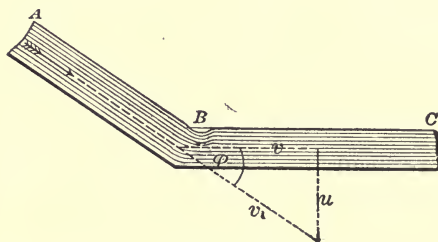


FIG. 96.

CASE IV. Weisbach gives the following empirical formula for the loss of head at a bend in a pipe, ϕ being the angle of curvature:

$$h_b = m_b \frac{v^2}{2g} \frac{\phi^\circ}{180^\circ},$$

where $m_b = .131 + 1.847 \left(\frac{d}{2\rho} \right)^{\frac{7}{2}}$

for a circular pipe of diameter d , ρ being the radius of curvature of the bend, and

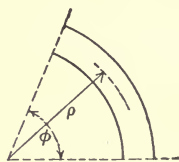


FIG. 97.

$$m = .124 + 3.104 \left(\frac{s}{2\rho} \right)^{\frac{1}{2}}$$

for a pipe of rectangular section, s being the length of a side of the section parallel to the radius of curvature (ρ) of the bend.

According to Navier,

$$h_b = (.0128 + .0186R) \frac{L}{R} \frac{v^2}{2g},$$

R being the radius and L the length of the bend measured along the axis.

As a result of recent experiments by Gardner S. Williams and others (Proc. Am. Soc. C. E., May, 1901) it is claimed that, down to a limit of $2\frac{1}{2}$ diameters, curves of short radius offer less resistance to flow than do curves of longer radius, which is contrary to the ordinary hypothesis.

CASE V. *Valves, Cocks, Sluices, etc.*—The loss of head in each of the cases represented by the several figures may be traced to a contraction of the stream similar to the contraction which occurs in the case of an abrupt change of section. The loss may be expressed in the form $m \frac{v^2}{2g}$, and the following tables give the results obtained by Weisbach:

(a) *Sluice in Pipe of Rectangular Section.* (Fig. 98.)

Area of pipe = a ; area of sluice = s .



$\frac{s}{a} =$	1	.9	.8	.7	.6	.5	.4	.3	.2	.1
$m =$.00	.09	.39	.95	2.08	4.02	8.12	17.8	44.5	193

FIG. 98.

(b) *Sluice in Cylindrical Pipe.* (Fig. 99.)

s = ratio of height of opening to diameter of pipe.

$s =$	1	.875	.75	.625	.5	.375	.25	.125
$m =$.00	.07	.26	.81	2.06	5.52	17.00	97.8

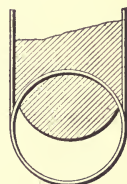


FIG. 99.

(c) *Cock in Cylindrical Pipe* (Fig. 100).

s = ratio of cross-sections;

θ = angle through which cock is turned.

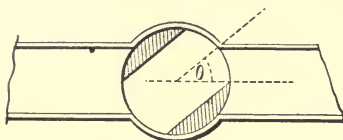


FIG. 100.

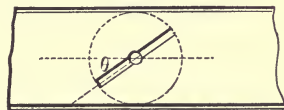


FIG. 101.

If $\theta =$	5°	10°	15°	20°	25°	30°	35°
$s =$.926	.85	.772	.692	.613	.535	.458
$m =$.05	.29	.75	1.56	3.1	5.47	9.68
If $f =$	40°	45°	50°	55°	60°	65°	82°
$s =$.385	.315	.25	.19	.137	.091	.00
$m =$	17.3	31.2	52.6	106	206	486	∞

(d) *Throttle-valve in Cylindrical Pipe* (Fig. 101).

θ = angle through which valve is turned.

If $\theta = 5^\circ \quad 10^\circ \quad 15^\circ \quad 20^\circ \quad 25^\circ \quad 30^\circ \quad 35^\circ \quad 40^\circ$
 $m = .24 \quad .52 \quad .90 \quad 1.54 \quad 2.51 \quad 3.91 \quad 6.22 \quad 10.8$

If $\theta = 45^\circ \quad 50^\circ \quad 55^\circ \quad 60^\circ \quad 65^\circ \quad 70^\circ \quad 90^\circ$
 $m = 18.7 \quad 32.6 \quad 58.8 \quad 118 \quad 256 \quad 751 \quad \infty$

CASE VI. The fall of free surface-level, or loss of head, due to sudden changes of section, frictional resistance, etc., may be graphically represented as in Fig. 102.

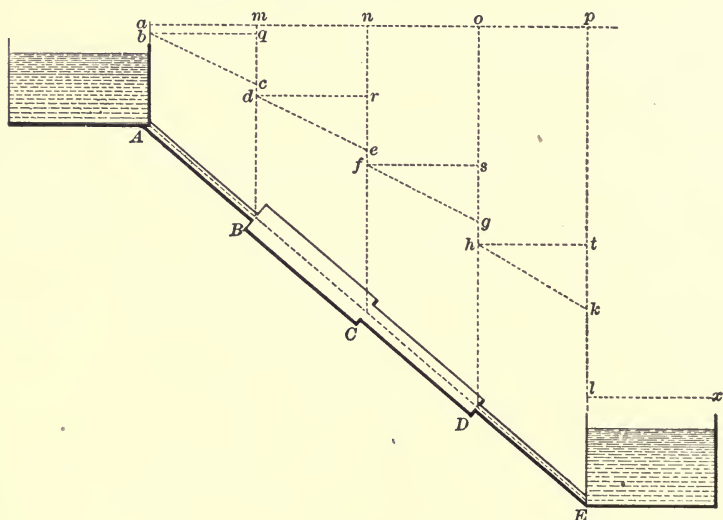


FIG. 102.

Let a length of piping AE connect two reservoirs, and let h be the difference of surface-level of the water in the reservoirs.

Let L_1, r_1 be length and radius of portion AB of pipe.

" L_2, r_2 " " " " " " BC " "

" L_3, r_3 " " " " " " CD " "

" L_4, r_4 " " " " " " DE " "

" u_1, u_2, u_3, u_4 be the velocities of flow in AB, BC, CD, DE , respectively.

The reservoir opens abruptly into the pipe at A .

There is an abrupt change at B from a pipe of radius r_1 to one of radius r_2 .

There is an abrupt change at C from a pipe of radius r_2 to one of radius r_3 .

At D the water flows through an orifice of area A in a diaphragm. At E the velocity of the water as it enters the lower reservoir is immediately dissipated in eddies or vortices.

Draw the horizontal plane $amnop$ at a distance from the water-surface in the upper reservoir equal to the head due to atmospheric pressure.

Draw vertical lines at A, B, C, D, E . Take

$$ab = \text{loss of head at the entrance } A = .49 \frac{u_1^2}{2g};$$

$$qc = \text{ " " " due to friction from } A \text{ to } B = \frac{2f u_1^2}{r_1} L_1;$$

$$cd = \text{ " " " due to change of section at } B = \left(\frac{r_2^2}{r_1^2} - 1 \right) \frac{u_1^2}{2g};$$

$$re = \text{ " " " due to friction from } B \text{ to } C = \frac{2f u_2^2}{r_2} L_2;$$

$$ef = \text{ " " " due to change of section at } C = .316 \frac{u_3^2}{2g};$$

$$sg = \text{ " " " due to friction from } C \text{ to } D = \frac{2f}{r_3} \cdot \frac{u_3^2}{2g} L_3;$$

$$gh = \text{ " " " due to change of section at } D = \left(\frac{\pi r_3^2}{cA} - 1 \right) \frac{u_3^2}{2g};$$

$$tk = \text{ " " " due to friction from } D \text{ to } E = \frac{2f u_4^2}{r_4} L_4;$$

$$kl = \text{ " " " corresponding to } u = \frac{u_4^2}{2g}.$$

Through l draw a horizontal plane lx . This plane must evidently be at a distance from the water-surface in the lower reservoir equal to the pressure-head due to the atmosphere.

Then the *total* loss of head = lp

$$\begin{aligned}
 &= ab + cd + ef + gh + kl + qc + re + sg + tk, \\
 &= .49 \frac{u_1^2}{2g} + \left(\frac{r_2^2}{r_1^2} - 1 \right)^2 \frac{u_2^2}{2g} + .316 \frac{u_3^2}{2g} + \left(\frac{\pi r_3^2}{cA} - 1 \right)^2 \frac{u_4^2}{2g} + \frac{u_4^2}{2g} \\
 &\quad + \frac{2f}{r_1} \frac{u_1^2}{2g} L_1 + \frac{2f}{r_2} \frac{u_2^2}{2g} L_2 + \frac{2f}{r_3} \frac{u_3^2}{2g} L_3 + \frac{2f}{r_4} \frac{u_4^2}{2g} L_4 \\
 &= \frac{u_1^2}{2g} \left\{ .49 + \left(\frac{r_2^2}{r_1^2} - 1 \right)^2 \frac{r_1^4}{r_2^4} + .316 \frac{r_1^4}{r_3^4} + \left(\frac{\pi r_3^2}{cA} - 1 \right)^2 \frac{r_1^4}{r_4^4} + \frac{r_1^4}{r_4^4} \right\} \\
 &\quad + \frac{f}{g} u_1^2 \left\{ \frac{L_1}{r_1} + \frac{L_2}{r_2} \frac{r_1^4}{r_2^4} + \frac{L_3}{r_3} \frac{r_1^4}{r_3^4} + \frac{L_4}{r_4} \frac{r_1^4}{r_4^4} \right\} \\
 &= \frac{Q^2}{2\pi^2 g} \left\{ .49 + \left(\frac{r_2^2}{r_1^2} - 1 \right)^2 \frac{1}{r_2^4} + .316 \frac{1}{r_3^4} + \left(\frac{\pi r_3^2}{cA} - 1 \right)^2 \frac{1}{r_4^4} + \frac{1}{r_4^4} \right\} \\
 &\quad + \frac{fQ^2}{\pi^2 g} \left\{ \frac{L_1}{r_1} + \frac{L_2}{r_2} + \frac{L_3}{r_3} + \frac{L_4}{r_4} \right\}.
 \end{aligned}$$

The broken line $abcdefghkl$ is the hydraulic gradient.

Ex. A clean 6-in. pipe, 400 ft. long, containing a 60° bend with a 12-in. radius, a 90° bend with a 72-in. radius, and a 120° bend with a 48-in. radius, discharges 1 cu. ft. of water per sec. into a clean 12-in. pipe, 200 ft. long, which again discharges into a clean 4-in. pipe, 500 ft. long, containing four sharp knees, viz., one of 60° , one of 90° , one of 120° , and one of 150° . Find the total head wasted at the pipe entrance, at the bends, knees, sudden changes of section, and in the straight lengths.

Let v_1, v_2, v_3 be the velocities of flow in the first, second, and third lengths, respectively. Then

$$\frac{22}{7} \frac{1}{4} \left(\frac{1}{2} \right)^2 v_1 = 1 = \frac{22}{7} \frac{1}{4} (1^2) v_2 = \frac{22}{7} \frac{1}{4} \left(\frac{1}{3} \right)^2 v_3$$

and

$$v_1 = \frac{56}{11} \text{ ft. per sec.}, \quad v_2 = \frac{14}{11} \text{ ft. per sec.}, \quad v_3 = \frac{126}{11} \text{ ft. per sec.}$$

$$\text{Head wasted at pipe entrance} = \frac{1}{2} \left(\frac{56}{11} \right)^2 \frac{1}{64} = .20332 \text{ ft.}$$

$$\text{The head wasted at a bend} = m_b \frac{\phi}{180^\circ} \frac{v^2}{2g},$$

$$\text{where } m_b = .131 + 1.847 \left(\frac{d}{2\rho} \right)^{\frac{7}{2}}.$$

$$\text{For } \frac{d}{2\rho} = \frac{6}{26} = \frac{1}{4}, \quad m_b = .14544;$$

$$\text{" } \frac{d}{2\rho} = \frac{6}{144} = \frac{1}{24}, \quad m_b = .13102727;$$

$$\text{" } \frac{d}{2\rho} = \frac{6}{96} = \frac{1}{6}, \quad m_b = .131113.$$

Hence

$$\text{head wasted at } 60^\circ \text{ bend} = .14544 \times \frac{60}{180} \times \frac{1}{64} \times \left(\frac{56}{11} \right)^2 = .019632 \text{ ft.,}$$

$$\text{" } 90^\circ \text{ " } = .130273 \times \frac{90}{180} \times \frac{1}{64} \times \left(\frac{56}{11} \right)^2 = .0265303 \text{ ft.,}$$

$$\text{" } 120^\circ \text{ " } = .131113 \times \frac{120}{180} \times \frac{1}{64} \times \left(\frac{56}{11} \right)^2 = .035396 \text{ ft.,}$$

and the head wasted in bends = .081558 ft.

$$\text{The head wasted at a knee} = m_k \frac{v^2}{2g},$$

$$\text{where } m_k = .9457 \sin^2 \frac{\phi}{2} + 2.047 \sin^4 \frac{\phi}{2}.$$

$$\text{For a } 60^\circ \text{ knee} \dots \phi = 120^\circ, \quad m_k = 1.8607$$

$$\text{" } 90^\circ \text{ " } \dots \phi = 90^\circ, \quad m_k = .9846$$

$$\text{" } 120^\circ \text{ " } \dots \phi = 60^\circ, \quad m_k = .36436$$

$$\text{" } 150^\circ \text{ " } \dots \phi = 30^\circ, \quad m_k = .07254$$

Then

$$\text{head wasted at } 60^\circ \text{ knee} = 1.8607 \times \frac{1}{64} \left(\frac{126}{11} \right)^2 = 3.81463 \text{ ft.,}$$

$$\text{" } 90^\circ \text{ " } = .9846 \times \frac{1}{64} \left(\frac{126}{11} \right)^2 = 2.01853 \text{ "}$$

$$\text{" } 120^\circ \text{ " } = .36436 \times \frac{1}{64} \left(\frac{126}{11} \right)^2 = .74697 \text{ "}$$

$$\text{" } 150^\circ \text{ " } = .07254 \times \frac{1}{64} \left(\frac{126}{11} \right)^2 = .14871 \text{ "}$$

and the head wasted in knees = 6.72884 ft.

Head wasted at junction

$$\text{between 6-in. and 12-in. pipes} = \frac{1}{64} \left(\frac{56}{11} - \frac{14}{11} \right)^2 = .22778 \text{ ft.,}$$

$$\text{" } 12\text{-in. and } 4\text{-in. pipes} = \frac{.316 \left(\frac{126}{11} \right)^2}{64} = .64783 \text{ ft.,}$$

and the head wasted at sudden changes of section = .87561 ft.

For straight lengths

$$\text{take } f = .005 \left(1 + \frac{1}{12 \times \frac{1}{2}} \right) = \frac{.035}{6} \text{ for 6-in. pipe,}$$

$$\text{" } f = .005 \left(1 + \frac{1}{12 \times 1} \right) = \frac{.065}{12} \text{ " 12-in. "}$$

$$\text{" } f = .005 \left(1 + \frac{1}{12 \times \frac{1}{3}} \right) = \frac{.025}{4} \text{ " 4-in. "}$$

Then head wasted

$$\text{in 1st length} = \frac{4 \times \frac{.035}{6} 400}{\frac{1}{2}} \frac{1}{64} \left(\frac{56}{11} \right)^2 = 7.5592 \text{ ft.,}$$

$$\text{" 2d " } = \frac{4 \times \frac{.065}{12} 200}{1} \frac{1}{64} \left(\frac{14}{11} \right)^2 = 7.01929 \text{ ft.,}$$

$$\text{" 3d " } = \frac{4 \times \frac{.025}{4} 500}{\frac{1}{3}} \frac{1}{64} \left(\frac{126}{11} \right)^2 = 76.8788 \text{ ft.,}$$

and the frictional loss of head = 91.14329 ft.

Hence the total head wasted

$$= .081558 + 6.72884 + .87561 + 91.45729 = 99.1433 \text{ ft.}$$

16. Nozzles.—Let a pipe AB , of length l and diameter d , lead from a reservoir h ft. above the end B , Fig. 103.

First. Let the pipe be open to the atmosphere at B . Then

$$h = \text{head to overcome resistance to entrance at } A \left(= n \frac{v^2}{2g} \right)$$

$$+ \text{head to overcome resistance due to bends, etc.} \left(= m \frac{v^2}{2g} \right)$$

$$+ \text{head to overcome frictional resistance} \left(= \frac{4fl}{d} \frac{v^2}{2g} \right)$$

$$\begin{aligned}
 & + \text{head corresponding to the velocity } v \text{ in the pipe and} \\
 & \text{at the outlet } \left(= \frac{v^2}{2g} \right) \\
 & = \frac{v^2}{2g} \left(n + m + \frac{4fl}{d} \right) + \frac{v^2}{2g}.
 \end{aligned}$$

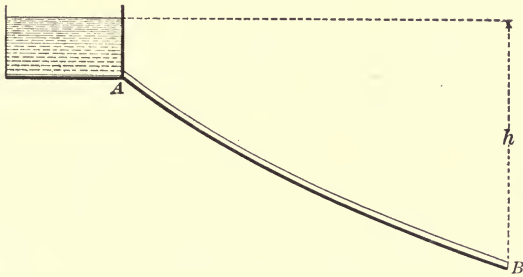


FIG. 103.

Hence the height to which the water is capable of rising at B

$$= \frac{v^2}{2g} = h - \frac{v^2}{2g} \left(n + m + \frac{4fl}{d} \right),$$

or, again, is

$$= \frac{h}{1 + n + m + 4f \frac{l}{d}}.$$

Second. Let a nozzle be fitted on the pipe at B.

Let V be the velocity with which the water leaves the nozzle.

Let D be the diameter of the nozzle-outlet.

This diameter is very small as compared with the diameter d of the pipe. But

$$\frac{\pi D^2}{4} V = \frac{\pi d^2}{4} v,$$

and therefore

$$V = \frac{d^2}{D^2} v,$$

so that V is very large as compared with v .

Also,

$$\begin{aligned}
 h &= \text{head to overcome the resistance to entrance at } A \\
 &+ \text{head to overcome the resistance due to bends, etc.} \\
 &+ \text{head to overcome the frictional resistance in pipe} \\
 &+ \text{head to overcome the frictional resistance in nozzle} \\
 &\quad \left(= m' \frac{V^2}{2g} \right) \\
 &+ \text{head corresponding to the velocity } V \text{ with which the} \\
 &\quad \text{water leaves the nozzle } \left(= \frac{V^2}{2g} \right) \\
 &= \frac{v^2}{2g} \left(n + m + \frac{4fl}{d} \right) + m' \frac{V^2}{2g} + \frac{V^2}{2g},
 \end{aligned}$$

and the height to which the water is now capable of rising at B is

$$\begin{aligned}
 \frac{V^2}{2g} &= h - \frac{v^2}{2g} \left(n + m + \frac{4fl}{d} \right) - m' \frac{V^2}{2g} \\
 &= \frac{h}{1 + m' + \frac{D^4}{d^4} \left(n + m + \frac{4fl}{d} \right)}.
 \end{aligned}$$

Let $\frac{p_n}{w}$, $= h_n$, be the pressure-head at the entrance to the nozzle. Then the effective head at the same point

$$= h_n + \frac{v^2}{2g} = (1 + m') \frac{V^2}{2g}.$$

Hence

$$\frac{V^2}{2g} = \frac{h_n}{1 + m' + \frac{D^4}{d^4}}.$$

It will be observed that the delivery from the nozzle is less than that from the pipe before the nozzle was attached, but that the velocity-head at the nozzle-outlet is enormously increased. The actual height to which the water rises on leaving a nozzle is less than the calculated height, owing to

air-resistance and to the impact of particles of water as they fall back.

The force required to hold the nozzle is evidently

$$\frac{wQ}{g}V = \frac{w\pi D^2}{g} \frac{V^2}{4}.$$

If the water flowing through a pipe, or hose, of length l ft., with a velocity of v ft. per second, is quickly and uniformly shut off by a stop-valve in t sec., the pressure in the pipe near the valve is increased by an amount $\frac{wlv}{gt}$ lbs. per square foot.

Of two forms of nozzle in general use, the one (Fig. 105) is a surface of revolution with a section which gradually diminishes to the outlet, while the other (Fig. 104) is a frustum

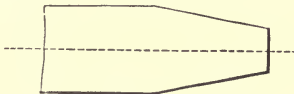


FIG. 104.

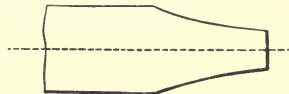


FIG. 105.

of a cone, having a diaphragm with a small circular orifice at the outlet. Denoting the former by A and the latter by B , the following table gives the results of Ellis's experiments:

Pressure in lbs. per sq. in.	Head in feet.	Height of jet from 1-inch Nozzle.		Height of jet from 1½-inch Nozzle.		Height of jet from 1½-inch Nozzle.	
		A	B	A	B	A	B
10	23	22	22	22	22	23	22
20	46	43	42	43	43	43	43
30	69	62	61	63	62	63	63
40	92	79	78	81	79	82	80
50	115	94	92	97	94	99	95
60	138	108	104	112	108	115	110
70	161	121	115	125	121	129	123
80	184	131	124	137	131	142	135
90	207	140	132	148	141	154	146
100	230	148	136	157	149	164	155

The coefficients of discharge for smooth cone nozzles are, very approximately, .983 for a $\frac{3}{4}$ -in., .982 for a $\frac{7}{8}$ -in., .972 for a 1-in., .976 for a 1½-in., and .971 for a 1¾-in. nozzle.

Freeman proposed the $1\frac{1}{8}$ -in. nozzle shown by Fig. 106

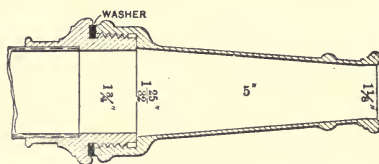


FIG. 106.

as a standard with a coefficient of discharge = .977. The coefficient of discharge for a square ring nozzle is about .74.

FREEMAN'S TABLE SHOWING COMPARATIVE FRICTIONAL LOSS IN VARIOUS KINDS OF HOSE.

The comparison is made on the basis of a flow of 240 gals. per min., which is about the quantity discharged by a $1\frac{1}{8}$ -in. nozzle under a pressure of 40 lbs. per square inch at base of play-pipe.

Character of Hose.	Diameter of Couplings.	Average Internal Diameter of Hose.	Per cent Increase of Length under Average Pressure of 50 Pounds.	Observed Loss of Pressure in 100 Feet in Pounds per Square Inch.	Per Cent corresponding to Diameter of Hose to be Added or Deducted to give Friction.	Loss of Pressure in 100 Ft. Corrected for $2\frac{1}{2}$ -inch Diameter.	Mean Velocity in Feet per Second.
$2\frac{1}{2}$ " solid rubber hose, extra heavy, smooth and free from ridges	2.52"	2.65"	$\frac{3}{4}$	10.0	+34	13.4	13.96
$2\frac{1}{2}$ " solid rubber hose lighter than preceding and not so carefully made	2.53	2.60	$1\frac{1}{2}$	11.5	+22	14.0	14.50
$2\frac{1}{2}$ " woven cotton hose, rubber-lined, regular heavy fire-department hose.....	2.53	2.47	4	15.0	- 6	14.1	16.07
$2\frac{1}{2}$ " woven cotton hose, rubber-lined, lighter than preceding, but of about the same smoothness of interior	2.47	2.49	5	14.5	- 2	14.2	15.81
$2\frac{1}{2}$ " knit cotton hose, rubber-lined. A medium-weight hose.....	2.50	2.68	$3\frac{1}{2}$	11.3	+42	16.0	13.65
$2\frac{1}{2}$ " knit cotton hose, rubber-lined. Interior medium smooth.....	2.50	2.50	$4\frac{1}{2}$	16.8	0	16.8	15.69
$2\frac{1}{2}$ " knit cotton hose, rubber-lined. A regular fire-department hose.....	2.51	2.60	$1\frac{1}{2}$	13.9	+22	17.0	14.50
$2\frac{1}{2}$ " knit cotton hose, rubber-lined. Inside rather rough.....	2.51	2.62	3	14.4	+27	18.3	14.28
$2\frac{1}{2}$ " knit cotton hose, rubber-lined. About same as preceding, but a little heavier.....	2.51	2.60	$2\frac{1}{2}$	13.5	+44	19.4	13.55
$2\frac{1}{2}$ " leather hose.....	2.50	2.80	$2\frac{1}{2}$	12.2	+76	21.5	12.51
$2\frac{1}{2}$ " woven cotton, rubber-lined, mill hose. Medium thin rubber lining.....	2.48	2.53	5	24.1	+ 6	25.5	15.31
$2\frac{1}{2}$ " unlined linen hose.....	2.50	2.60	$2\frac{1}{2}$	27.2	+22	33.2	14.50
$2\frac{1}{2}$ " woven cotton, rubber-lined hose.....	2.07	2.12	$4\frac{1}{2}$	33.2	-56	14.6	21.81
$2\frac{1}{2}$ " linen hose with $2\frac{1}{2}$ " couplings.....	1.95	2.30		49.5	-34	32.7	18.53

Third. If an engine, working against a pressure of p_c lbs. per square foot, pumps Q cu. ft. of water per second through a nozzle at the end of a hose l ft. in length, then

$$\text{the pumping H.P. of the engine} = \frac{Qp_c}{550}.$$

The total head at the engine end of the hose = the head corresponding to the pressure p in the hose + the head required to produce the velocity of flow v

$$= \frac{p}{w} + \frac{v^2}{2g},$$

and this head is expended in overcoming the frictional resistance of the hose (all other resistances are disregarded) and in producing the velocity of flow V at the outlet. Hence

$$\frac{p_c}{w} = \frac{p}{w} + \frac{v^2}{2g} = \frac{4fl}{d} \frac{v^2}{2g} + \frac{V^2}{2g},$$

and therefore

$$\begin{aligned} \frac{p}{w} &= \frac{4fl}{d} \frac{v^2}{2g} + \frac{V^2}{2g} - \frac{v^2}{2g}, \\ &= \frac{8Q^2}{g\pi^2} \left(\frac{1}{D^4} - \frac{1}{d^4} + \frac{4fl}{d^5} \right), \end{aligned}$$

$$\text{since} \quad Q = \frac{\pi d^2}{4} v = \frac{\pi D^2}{4} V.$$

The pumping H.P.

$$= \frac{8wQ^3}{550g\pi^2} \left(\frac{1}{D^4} + \frac{4fl}{d^5} \right).$$

17. Motor Driven by Water from a Pipe.—Let the nozzle in the preceding article be replaced by a cylinder having its piston driven by the water from the pipe.

Let u = the velocity of the piston per second.

Let p_m = unit pressure at the end of the pipe, i.e., in the cylinder.

Let d_m = diameter of cylinder.

Then

$$\text{velocity of flow in pipe} = \left(\frac{d_m}{d}\right)^2 u.$$

Hence

$$h = \left(\frac{d_m}{d}\right)^4 \frac{u^2}{2g} + \frac{4fl}{d} \left(\frac{d_m}{d}\right)^4 \frac{u^2}{2g} + \frac{p_m}{w},$$

other losses of head being disregarded.

Ex. A $3\frac{1}{2}$ -in. clean pipe, 525 ft. long, leads from a reservoir with a water surface 300 ft. above datum to a point A , 187 $\frac{1}{2}$ ft. above datum. Find (a) the height to which the water is capable of rising at A (1) if the pipe is open to the atmosphere; (2) if it terminates in a 1-in. nozzle. What (b) force is required to hold the nozzle? If the pipe is used to supply pressure to a water-engine with a 28-in. cylinder, determine (c) the maximum power which can be developed and the corresponding velocity of flow in the pipe. In the latter case, what (d) is the total pressure on the piston? Take into account the resistance at the pipe entrance and assume $f = .005$.

Let v and V be velocities of flow in pipe and from nozzle, respectively.

(a) 1. $300 - 187\frac{1}{2} = 112\frac{1}{2}$ = total effective head

$$= \frac{v^2}{2g} \left(.5 + 1 + \frac{4 \times .005 \times 525}{\frac{3\frac{1}{2}}{12}} \right) = \frac{v^2}{2g} \cdot 37\frac{1}{2},$$

and $\frac{v^2}{2g} = 3$ ft. = height to which water can rise.

$$2. v = \frac{V}{(3\frac{1}{2})^2}; \text{ and}$$

$$112\frac{1}{2} = \frac{V^2}{2g} + \frac{V^2}{2g} \left(\frac{1}{3\frac{1}{2}} \right)^4 \left(.5 + \frac{4 \times .005 \times 525}{\frac{3\frac{1}{2}}{12}} \right) = \frac{V^2}{2g} \frac{2985}{2401}.$$

Therefore

$$\frac{V^2}{2g} = 112\frac{1}{2} \times \frac{2401}{2985} = 90.49 \text{ ft.} = \text{height to which water can rise.}$$

(b) Force = momentum

$$= \frac{62\frac{1}{2}}{32} \cdot \frac{22}{7} \cdot \frac{1}{4} \left(\frac{1}{12} \right)^2 V^2 = \frac{125}{64} \cdot \frac{11}{14} \cdot \frac{1}{144} \cdot 64 \cdot \frac{225}{2} \cdot \frac{2401}{2985} = 61.8 \text{ lbs.}$$

(c) Let p be the pressure in pounds per sq. ft. at A . Then

$$112\frac{1}{2} = \frac{p}{62\frac{1}{2}} + \frac{v^2}{2g} \cdot 37\frac{1}{2},$$

or

$$p = 62\frac{1}{2} \cdot 37\frac{1}{2} \left(3 - \frac{v^2}{2g} \right).$$

Hence

$$\begin{aligned} \text{the H.P.} &= \frac{pQ}{550} = \frac{62\frac{1}{2} \cdot 37\frac{1}{2}}{550} \left(3 - \frac{v^2}{2g} \right) \frac{22}{7} \cdot \frac{1}{4} \left(\frac{3\frac{1}{2}}{12} \right)^2 v \\ &= \frac{875}{3072} \left(3v - \frac{v^3}{64} \right), \end{aligned}$$

which is a max. when $3 - \frac{3v^2}{64} = 0$, or $v = 8$ ft. per sec., and the max. H.P. = $4\frac{19\frac{1}{2}}{19\frac{1}{2}} = 4.557$.

Also $p = 62\frac{1}{2} \cdot 75 = 4687\frac{1}{2}$ lbs. per sq. ft., and total pres. on piston = $4687\frac{1}{2} \times \frac{22}{7} \cdot \frac{1}{4} \cdot \left(\frac{28}{12} \right)^2 \frac{1}{2000} = 10\frac{5}{19\frac{1}{2}}$ tons.

18. Siphons.—A siphon is a bent tube, $ABCD$, Fig. 107, and is often employed to convey water from one reservoir to another at a lower level.

Let h_1 , h_2 , respectively, be the differences of level between the top of the siphon and the entrance A and outlet D to the siphon. Then, so long as the height h_1 does not exceed the head of water ($= 32.8$ ft.) which measures

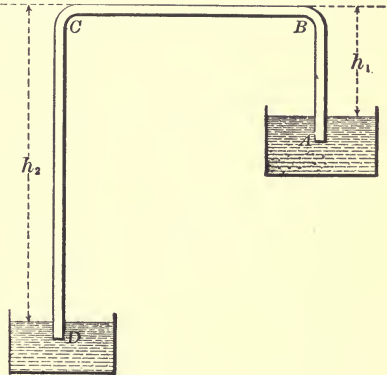


FIG. 107.

the atmospheric pressure, the water will flow along the tube in the direction of the arrow, with a velocity v given by the equation

$$h_2 - h_1 = \frac{4fl}{d} \frac{v^2}{2g},$$

l being the length of the tube $ABCD$, and all resistances, except that due to frictional resistance, being disregarded.

If $h_1 > 32.8$ ft., each of the branches AB and DC becomes a water-barometer, and the siphon will no longer work.

Even when the siphon does work, an arrangement must be made for withdrawing the air which will always collect at the upper part of the siphon.

19. Inverted Siphons.—The existence of a cutting or a valley sometimes renders it necessary to convey the water from a course AB to a course DE by means of an inverted siphon BCD of length l .

Let u be the velocity of flow in AB , and h_1 the height of B above a datum line.

Let v be the velocity of flow in the siphon, and h_2 the height of D above datum.

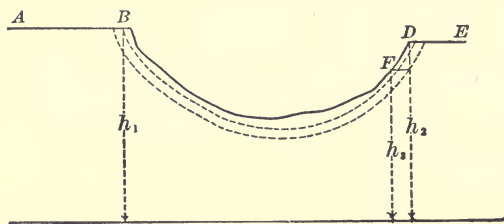


FIG. 108.

Then

$$\begin{aligned}
 h_1 - h_2 &= \text{loss of head at } B \\
 &\quad + \text{frictional loss of head in siphon} \\
 &\quad + \text{loss of head at } D \\
 &= \frac{u^2}{2g} + \frac{4fl}{d} \frac{v^2}{2g} + \frac{v^2}{2g} \\
 &= \frac{4fl}{d} \frac{v^2}{2g}, \text{ approximately,}
 \end{aligned}$$

assuming the entrance and outlet to the siphon formed in such a manner as to considerably reduce the losses $\frac{u^2}{2g}$ and $\frac{v^2}{2g}$, and

to allow of these losses being disregarded without practical error. Find, by chaining along the ground, the length of the siphon from B up to a point F not far from D . Call this length l_1 , and let h_3 be the height above datum of F , obtained with a level. Generally speaking, DF is nearly always of uniform slope. Call the slope α . Then,

$$DF = (h_2 - h_3) \operatorname{cosec} \alpha.$$

But

$$\begin{aligned} \frac{4fl}{d} \frac{v^2}{2g} &= \frac{4f}{d} \frac{v^2}{2g} (l_1 + DF) = h_1 - h_2 = h_1 - h_3 - (h_2 - h_3) \\ &= h_1 - h_3 - DF \cdot \sin \alpha, \end{aligned}$$

an equation from which DF can be found, as $h_1 - h_3$ can be determined by means of a level.

20. Air in a Pipe.—The effect of an air-bubble in a pipe $ABCD$ may be discussed as follows:

Let the air occupy the portion BC of a pipe.

Let the surface of the water in the reservoir supplying the pipe be h_1 ft. vertically above E , and h_2 ft. above D .

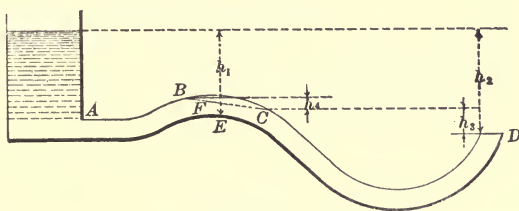


FIG. 109.

Also, let h_3 be the difference of level between C and D , h_4 the difference of level between B and C , and t the thickness of the water-layer EF .

Let H designate the head equivalent to the elastic resistance of the air in BC . Then, approximately,

$$h_1 + \frac{p_0}{w} - H - t = \frac{4fl_1}{d} \frac{v^2}{2g}$$

and

$$H + h_3 - \frac{p_0}{w} = \frac{4fl_2}{d} \frac{v^2}{2g},$$

l_1 being the length of the portion of pipe from A to E , and l_2 the length from E to D .

Adding the two equations,

$$h_1 + h_3 - t = \frac{4fv^2}{d} \frac{1}{2g} (l_1 + l_2) = \frac{4fl}{d} \frac{v^2}{2g},$$

l being total length of pipe.

But $h_1 - t + h_4 = h_2 - h_3$, very nearly. Hence

$$h_2 - h_4 = \frac{4fl}{d} \frac{v^2}{2g},$$

an equation showing the variation of v with a variation in the height h_4 of the space occupied by the air.

NOTE.— H of course varies with the temperature.

21. Flow of Water in a Pipe of Varying Diameter.—The variation in the diameter is supposed to be so gradual that the fluid filaments may still be assumed to flow in sensible parallel lines.

Consider a thin slice of the moving fluid, bounded by the transverse sections AB , CD , distant s and $s + ds$, respectively, from an origin on the axis of the pipe.

Let p be the mean intensity of pressure, A the water area, P the wetted perimeter for the section AB .

Let these symbols become $p + dp$, $A + dA$, $P + dP$, respectively, for the section CD .

Let z be the height of the C. of G. of the section AB above datum.

Let $z + dz$ be the height of the C. of G. of the section CD above datum.

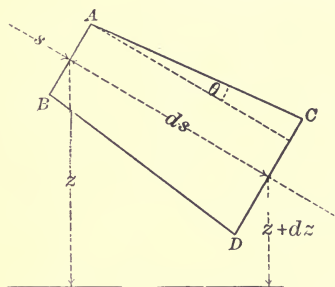


FIG. 110.

Let $u, u + du$ be the velocities of flow across the sections AB, CD , respectively.

Then

The rate of increase of momentum of the slice $ABCD$ in the direction of the axis $\left. \vphantom{\begin{matrix} \text{The rate of increase of} \\ \text{momentum of the slice} \\ \text{ABCD in the direction} \\ \text{of the axis} \end{matrix}} \right\} = \left\{ \begin{array}{l} \text{momentum generated by} \\ \text{the effective forces acting} \\ \text{upon the slice in the same} \\ \text{direction.} \end{array} \right.$

The acceleration in time $dt = \frac{w}{g} Au \cdot dt \frac{du}{dt} = \frac{w}{g} Au \cdot du$.

The total pressure on $AB = p \cdot A$, and acts along the axis.

The total pressure on $CD = (p + dp)(A + dA)$, and acts along the axis.

The total normal pressure on the surface $ACBD$ of the pipe

$$= 2\pi \left(r + \frac{dr}{2} \right) \left(p + \frac{dp}{2} \right) AC = 2\pi r p \cdot AC, \text{ very nearly.}$$

The component of this pressure along the axis

$$\begin{aligned} &= 2\pi r p AC \cdot \sin \theta \\ &= 2\pi p r \cdot dr, \text{ nearly,} \end{aligned}$$

θ being the angle between AC and the axis.

Thus the *total resultant pressure along the axis*

$$\begin{aligned} &= pA - (p + dp)(A + dA) + 2\pi p r \cdot dr \\ &= -p \cdot dA - A \cdot dp + 2\pi p r \cdot dr \\ &= -A \cdot dp, \end{aligned}$$

since $A = \pi r^2$, and therefore $dA = 2\pi r \cdot dr$.

The component of *the weight* of the slice along the axis

$$= \left(A + \frac{dA}{2} \right) ds \cdot w \sin i = - \left(A + \frac{dA}{2} \right) w \cdot dz = -wA \cdot dz.$$

The *frictional resistance* $= P \cdot AC \cdot F(u) = P \cdot ds \cdot F(u)$, very nearly. Hence

$$\frac{wAu \cdot du}{g} = -A \cdot dp - wA \cdot dz - P \cdot ds \cdot F(u),$$

and therefore

$$dz + \frac{dp}{w} + \frac{u \cdot du}{g} + \frac{P}{A} \frac{F(u)}{w} ds = 0.$$

Integrating,

$$z + \frac{p}{w} + \frac{u^2}{2g} + \int \frac{P}{A} \frac{F(u)}{w} ds = \text{a constant.}$$

$$\text{Take } \frac{F(u)}{w} = f \frac{u^2}{2g} = \frac{f}{2g} \frac{Q^2}{\pi^2 r^4}.$$

Then

$$z + \frac{p}{w} + \frac{u^2}{2g} + \int \frac{f}{g} \frac{Q^2}{\pi^2 r^5} ds = \text{a constant.}$$

The integration can be effected as soon as the relation between r and s is fixed.

Example.—Take $r = a + bs$, and assume f and Q to be constant. Then

$$z + \frac{p}{w} + \frac{u^2}{2g} + \frac{1}{b} \frac{fQ^2}{g\pi^2} \int \frac{dr}{r^5} = \text{a constant,}$$

and therefore

$$z + \frac{p}{w} + \frac{u^2}{2g} + \frac{1}{4b} \frac{fQ^2}{g\pi^2} \frac{1}{r^4} = \text{a constant.}$$

22. Equivalent Uniform Main.—A water-main usually consists of a series of lengths of different diameters.

As a first approximation the smaller losses of head due to changes of section, etc., may be disregarded, and the calculations may be further simplified by substituting for the several lengths a single pipe of uniform diameter giving the same frictional loss of head. Such a pipe is called an equivalent main.

Let l_1, l_2, l_3 be the successive lengths of the main.

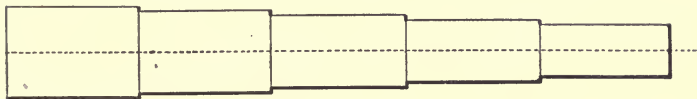


FIG. III.

Let d_1, d_2, d_3 be the diameters of these lengths.

Let v_1, v_2, v_3 be the velocities of flow in these lengths.

Let h_1, h_2, h_3 be the frictional losses of head in these lengths.

Let L, d, v, h be the corresponding quantities for the equivalent uniform main.

Then

$$h = h_1 + h_2 + h_3 + \dots,$$

and therefore

$$\frac{4f}{d} \frac{v^2}{2g} L = \frac{4f}{d_1} \frac{v_1^2}{2g} l_1 + \frac{4f}{d_2} \frac{v_2^2}{2g} l_2 + \frac{4f}{d_3} \frac{v_3^2}{2g} l_3 + \dots$$

Hence

$$L \frac{v^2}{d} = l_1 \frac{v_1^2}{d_1} + l_2 \frac{v_2^2}{d_2} + l_3 \frac{v_3^2}{d_3} + \dots,$$

where it is assumed that f is the same for the several lengths of the main and also for the equivalent pipe.

But

$$\frac{\pi d^2}{4} v = Q = \frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2 = \text{etc.}$$

Hence

$$\frac{L}{d^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \text{etc.},$$

an equation giving the diameter d of an equivalent pipe having the same total frictional loss of head.

Ex. What must be the diameter of a uniform pipe which may be substituted for a line of piping consisting of an 800-ft. length of 12-in. pipe and a 200-ft. length of 6-in. pipe?

$$\frac{800 + 200}{d^5} = \frac{800}{1^5} + \frac{200}{(\frac{1}{2})^5} = 7200,$$

or

$$d^5 = \frac{5}{36},$$

and therefore $d = .6738$ ft., or about 8 ins.

23. Branch Main of Uniform Diameter.—In a branch main AB of length L and diameter d , receiving its supply at A ,—

Let Q_w be the way-service, i.e., the amount of water given up to the service-pipes on each side.

Let Q_e be the end-service, i.e., the amount of water discharged at the end B .

Then it may be assumed, and it is approximately true, that the way-service per lineal foot, viz., $\frac{Q_w}{L}$, is constant.

Thus the amount of water consumed in way-service in a length AC of the main, where $BC = s$, is

$$\frac{Q_w}{L}(L - s),$$

while the total amount of water flowing across the section of the pipe at C

$$= Q_e + \frac{Q_w}{L}s = \frac{\pi d^2}{4}v,$$

v being the velocity of flow at C .

Now dh , the frictional loss of head at C for an elementary length ds of the pipe, is given by the equation

$$\begin{aligned} dh &= \frac{4f}{d} \frac{v^2}{2g} \cdot ds \\ &= \frac{f}{\pi^2 d^5} \left(Q_e + \frac{Q_w}{L}s \right) ds, \end{aligned}$$

if $g = 32$.

Integrating, the total loss of head is

$$h = \frac{fL}{\pi^2 d^5} \left(Q_e^2 + Q_e Q_w + \frac{Q_w^2}{3} \right).$$

SPECIAL CASES.

CASE I. Let Q_e' be the total discharge for the same frictional loss of head, h , when the whole of the way-service is stopped. Then

$$\frac{fL}{\pi^2 d^5} Q_e'^2 = h = \frac{fL}{\pi^2 d^5} \left(Q_e^2 + Q_e Q_w + \frac{Q_w^2}{3} \right),$$

and therefore

$$Q_e'^2 = Q_e^2 + Q_e Q_w + \frac{Q_w^2}{3}.$$

Hence

$$Q_e'^2 > \left(Q_e + \frac{Q_w}{2} \right)^2 \quad \text{and} \quad < \left(Q_e + \frac{Q_w}{\sqrt{3}} \right)^2,$$

and Q_e' lies between $Q_e + \frac{Q_w}{2}$ and $Q_e + \frac{Q_w}{\sqrt{3}}$, its mean value being $Q_e + .55 Q_w$.

CASE II. If there is no end-service, all the water having been absorbed in way-service, $Q_e = 0$, and therefore $Q_e' = \frac{Q_w}{\sqrt{3}}$ and

$$h = \frac{1}{3} \frac{fL Q_w^2}{\pi^2 d^5}.$$

CASE III. If $Q_e = 0$,

$$dh = \frac{f Q_w^2}{\pi^2 d^5 L^2} s^2 ds = \text{elementary frictional loss of head.}$$

Integrating between 0 and s ,

$$h = \frac{1}{3} \frac{f Q_w^2 s^2}{\pi^2 d^5 L^2},$$

and the vertical slope, or line of free pressure, becomes a cubical parabola.

CASE IV. Let the main receive its supply at A from a reservoir X in which the surface of the water is h_1 above datum, and let it discharge at the end B into a reservoir Y with its surface h_2 above datum, Fig. 114.

Since $(Q_e')^2 = Q_e^2 + Q_e Q_w + \frac{Q_w^2}{3}$, therefore

$$Q_e = -\frac{Q_w}{2} + \sqrt{(Q_e')^2 - \frac{Q_w^2}{12}}.$$

If $Q_w = \sqrt{3Q_e'}$, $Q_e = 0$; and if $Q_w > \sqrt{3Q_e'}$, then the reservoir Y will furnish a portion of the way-service.

Suppose that X gives the supply for the distance $AO (= l_1)$ and that Y supplies $BO (= l_2)$.

Let z be the height above datum of the surface in a pressure column inserted at O .

Then, neglecting the loss of head at entrance,

$$\begin{aligned} h_1 - z &= \left(h_1 + \frac{p_0}{w} \right) - \left(z + \frac{p_0}{w} \right) \\ &= \text{loss of head between } A \text{ and } O = \frac{1}{3} \frac{f Q_w^2 l_1^3}{\pi^2 d^5 L^2}, \end{aligned}$$

and

$$\begin{aligned} h_2 - z &= \left(h_2 + \frac{p_0}{w} \right) - \left(z + \frac{p_0}{w} \right) \\ &= \text{loss of head between } B \text{ and } O = \frac{1}{3} \frac{f Q_w^2 l_2^3}{\pi^2 d^5 L^2}. \end{aligned}$$

Also, $l_1 + l_2 = L$.

24. Three Reservoirs at Different Levels connected by a Branched Pipe.—Let a pipe DO of length l_1 ft. and radius r_1 ft., leading from a reservoir A in which the water stands h_1 ft. above datum, divide at O into two branches, the one, OE , of length l_2 ft. and radius r_2 ft., leading to a reservoir B in which the water stands h_2 ft. above datum, the other, OF , of length l_3 ft. and radius r_3 ft., leading to a reservoir C in which the water stands h_3 ft. above datum.

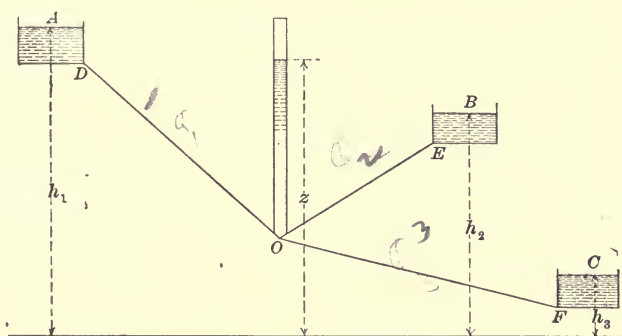


FIG. 112.

Let v_1 , v_2 , v_3 be the velocities of flow in DO , OE , OF , respectively.

Let Q_1 , Q_2 , Q_3 be the quantities of flow in DO , OE , OF , respectively.

Let z be the height above datum to which the water will rise in a tube inserted at the junction.

Two problems will be considered, and all losses of head excepting those due to frictional resistance will be disregarded.

PROBLEM I. Given h_1 , h_2 , h_3 ; r_1 , r_2 , r_3 ; to find Q_1 , Q_2 , Q_3 ; v_1 , v_2 , v_3 , and z . Taking $\frac{f}{g} = \alpha$,

For the pipe DO , $\frac{h_1 - z}{l_1} = \alpha \frac{v_1^2}{r_1} \dots (1)$ and $Q_1 = \pi r_1^2 v_1 \dots (2)$

For the pipe OE , $\frac{\pm z \mp h_2}{l_2} = \alpha \frac{v_2^2}{r_2} \dots (3)$ “ $Q_2 = \pi r_2^2 v_2 \dots (4)$

“ “ “ OF , $\frac{z - h_3}{l_3} = \alpha \frac{v_3^2}{r_3} \dots (5)$ “ “ $Q_3 = \pi r_3^2 v_3 \dots (6)$

Also, $Q_1 = \pm Q_2 + Q_3 \dots (7)$

From these seven equations the seven required quantities can be found.

In equations (3) and (7) the upper or lower signs are to be taken according as the flow is from O towards E or from E towards O .

This may be easily determined as follows:

Assume $z = h_2$, and then find v_1 and v_3 by means of equations (1) and (5), and hence Q_1 and Q_3 by means of equations (2) and (6). If it is found that $Q_1 > Q_3$, then the flow is from O to E , and equations (3) and (7) become

$$\frac{z - h_2}{l_2} = \alpha \frac{v_2^2}{r_2} \quad \text{and} \quad Q_1 = Q_2 + Q_3;$$

while if $Q_1 < Q_3$, the flow is from E to O , and the equations are

$$\frac{h_2 - z}{l_2} = \alpha \frac{v_2^2}{r_2} \quad \text{and} \quad Q_1 + Q_2 = Q_3.$$

Note.—It is assumed that $\alpha \left(= \frac{f}{g} \right)$ is the same for each pipe.

SPECIAL CASE. (Fig. 113.)—Suppose the pipe OE closed at E .

Also, let $r_1 = r_2 = r_3 = r$, and let V be the velocity of flow from A to C .

The “plane of charge” for the reservoir A is a horizontal plane MQ distant $\frac{p_0}{w}$ from the water-surface, p_0 being the atmospheric pressure.

The "plane of charge" for the reservoir C is a horizontal plane TS distant $\frac{p_0}{w}$ from the water-surface.

In the vertical line VTQ , take $TN = \frac{V^2}{2g}$ and join MN . Then, neglecting the loss of head at entrance, MN is the "line of charge," or hydraulic gradient, for the pipe DF , and is approximately a straight line.

Let the "plane of charge" KK for the reservoir B , distant $\frac{p_0}{w}$ from the water-surface, meet MN in G .

If the junction O is vertically below G , there is no head

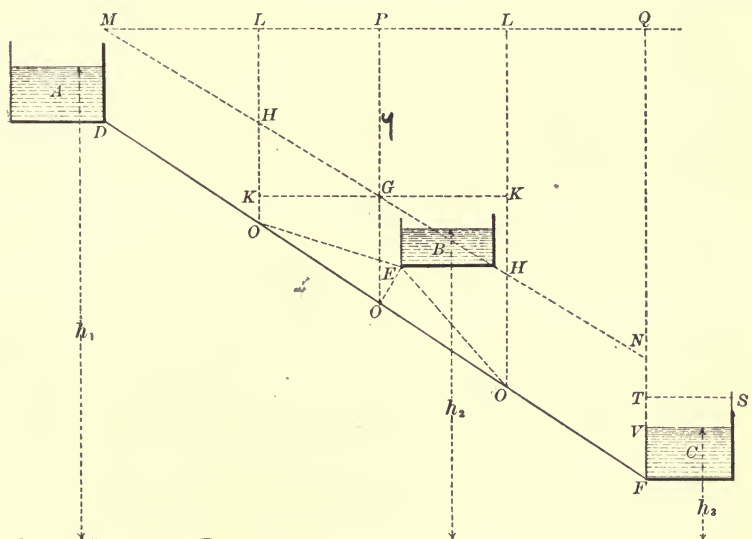


FIG. 113.

available for producing flow either from E towards O or from O towards E , and hydrostatic equilibrium is established.

If the junction O is on the left of G , and a vertical line $OKHL$ is drawn intersecting KK , MN , and MQ in the points K , H , and L , there is the head HK available for producing flow from O towards E .

If the junction O is on the right of G , and the vertical line $OHKL$ is drawn, the head HK is now available for producing flow from E towards O .

Let the vertical through G meet MQ in P , and take $PG = Y$. Then, approximately,

$$\frac{l_1}{l_1 + l_3} = \frac{MG}{MN} = \frac{PG}{QN} = \frac{Y}{h_1 - h_3},$$

and therefore

$$Y = \frac{h_1 - h_3}{l_1 + l_3} \cdot l_1.$$

If $HL < Y$, the flow is from O towards E .

If $HL > Y$, " " " " E " O .

Again,

$$\left(h_1 + \frac{p_0}{w}\right) - \left(h_3 + \frac{p_0}{w} + \frac{V^2}{2g}\right) = \alpha \frac{V^2}{r} (l_1 + l_3),$$

and therefore, approximately,

$$h_1 - h_3 = \alpha \frac{V^2}{r} (l_1 + l_3). \quad \dots \dots (1)$$

Next assume the junction O to be on the left of G , and open the valve at E . Then

$$\frac{h_1 - z}{l_1} = \alpha \frac{v_1^2}{r}; \quad \dots \dots (2)$$

$$\frac{z - h_2}{l_2} = \alpha \frac{v_2^2}{r}; \quad \dots \dots (3)$$

$$\frac{z - h_3}{l_3} = \alpha \frac{v_3^2}{r}; \quad \dots \dots (4)$$

$$\text{and} \quad Q_1 = Q_2 + Q_3,$$

$$\text{or} \quad v_1 = v_2 + v_3.$$

Thus

$$\alpha \frac{V^2}{r} (l_1 + l_3) = h_1 - h_3 = \frac{\alpha}{r} (l_1 v_1^2 + l_3 v_3^2) = \frac{\alpha}{r} \left\{ l_1 (v_2 + v_3)^2 + l_3 v_3^2 \right\};$$

and therefore

$$v_3^2 (l_1 + l_3) + 2l_1 v_2 v_3 + l_1 v_2^2 - (l_1 + l_3) V^2 = 0.$$

Hence, assuming v_2 to be very small as compared with V ,

$$v_3 = V - \frac{l_1 v_2}{l_1 + l_3},$$

or

$$Q_3 = Q - \frac{l_1 Q_2}{l_1 + l_3},$$

where $Q = \pi r^2 V$.

Thus it appears that if a quantity Q_2 of water is drawn off by means of a branch from a main capable of giving a total end-service Q , this end-service will be diminished by $\frac{1}{2}Q_2$, $\frac{1}{3}Q_2$, $\frac{1}{4}Q_2$, etc., according as the junction O divides the pipe DF into two portions in the ratio of 1 to 1, 1 to 2, 1 to 3, etc.

NOTE.—The more correct value of v_3 is

$$v_3 = -\frac{l_1 v_2}{l_1 + l_3} + \sqrt{V^2 - \frac{l_1 l_3 v_2^2}{(l_1 + l_3)^2}},$$

and the maximum value of $\frac{l_1 l_3}{(l_1 + l_3)^2}$ does not exceed $\frac{1}{4}$.

Orifice Fed by Two Reservoirs.—Neglect all losses of head except the losses due to frictional resistance.

When the valve at O is closed the flow is wholly from A to C , and the delivery is

$$Q = \sqrt{\frac{\pi^2 r^5}{\alpha} \frac{h_1 - h_2}{l_1 + l_3}}.$$

The line of charge (hydraulic gradient) is MN , where

$$MR = \frac{p_0}{w} = NV.$$

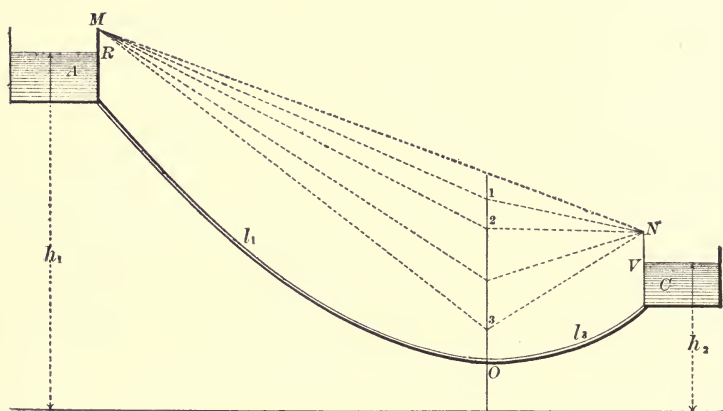


FIG. 114.

Open the valve a little: a volume Q_2 will now flow through O , and a volume Q_3 into C , where

$$Q_3 = Q - \frac{l_1 Q_2}{l_1 + l_3}.$$

The "line of charge" becomes the broken line $M1N$.

As the opening of the valve continues, the pressure-head at O diminishes, and when it is equal to $h_3 + \frac{p_0}{w}$ the line of charge is $M2N$, $2N$ being horizontal. Hydrostatic equilibrium is now established between O and C , and the whole of the water from A passes through O , the delivery being given by

$$Q_2 = \sqrt{\frac{\pi^2 r^5}{\alpha} \frac{h_1 - h_2}{l_1}} = Q \sqrt{\frac{l_1 + l_3}{l_1}}.$$

Opening O still further, both reservoirs will serve the orifice, and the line of charge will continue to fall.

If a piezometer is inserted at O , the water will rise in it to a height z above datum. Then

(a) Fig. 115:

Between A and O

$$\frac{250 - z}{2000} = \alpha \frac{v_1^3}{1} = \alpha v_1^3.$$

Between O and B

$$\frac{\pm 150 \mp z}{3000} = \alpha \frac{v_2^3}{\frac{1}{2}} = 2\alpha v_2^3. \quad \dots \dots \dots (I)$$

Between O and C

$$\frac{z}{4000} = \alpha \frac{v_3^3}{1} = \alpha v_3^3.$$

To find the *direction* of the flow in OC , let $z = 150$, then $v_2 = 0$, $\alpha v_1^3 = \frac{1}{20}$, $\alpha v_3^3 = \frac{3}{80}$, and therefore $v_1 > v_3$. Thus more water flows from A to O than is required for the lowest reservoir, and a portion must flow to the intermediate reservoir. Hence $z > 150$ ft., and

$$Q_1 = Q_2 + Q_3,$$

or

$$v_1 = \frac{v_2}{4} + v_3.$$

Therefore

$$\sqrt{\frac{250 - z}{2000\alpha}} = \frac{1}{4} \sqrt{\frac{z - 150}{6000\alpha}} + \sqrt{\frac{z}{4000\alpha}},$$

or

$$\sqrt{1500 - 6z} = \frac{1}{4} \sqrt{2z - 300} + \sqrt{3z}.$$

By trial this gives $z = 161$ ft., very nearly, and then, substituting in eqs. (I),

$$v_1^3 = 222.5, \text{ or } v_1 = 14.916 \text{ ft. per sec.,}$$

$$v_2^3 = 9\frac{1}{8}, \text{ or } v_2 = 3.027 \text{ " "}$$

$$v_3^3 = 201.25, \text{ or } v_3 = 14.186 \text{ " "}$$

Hence, also,

$$Q_1 = \frac{22}{7} \cdot \frac{2^3}{4} \times 14.916 = 46.879 \text{ cu. ft. per sec.,}$$

$$Q_2 = \frac{22}{7} \cdot \frac{1^3}{4} \times 3.027 = 2.378 \text{ " "}$$

$$Q_3 = \frac{22}{7} \cdot \frac{2^3}{4} \times 14.186 = 44.584 \text{ " "}$$

and $Q_2 + Q_3 = 46.962 = Q_1$, very nearly.

(b) Fig. 116:

Between A and O

$$\left. \begin{array}{l} \text{Between } O \text{ and } B \\ \text{Between } O \text{ and } C \end{array} \right\} \begin{array}{l} \frac{250 - z}{4000} = \alpha v_1^2 \\ \frac{\pm 150 \mp z}{3000} = 2\alpha v_2^2 \\ \frac{z}{2000} = \alpha v_3^2 \end{array} \dots \dots \dots (II)$$

If $z = 150$, $v_2 = 0$, $\alpha v_1^2 = \frac{1}{40}$, and $\alpha v_3^2 = \frac{3}{40}$. Thus $v_3 > v_1$ and therefore $Q_3 > Q_1$, so that more water flows to the lowest reservoir than is supplied by the highest reservoir. Hence the balance must come from the intermediate reservoir and $z < 150$ ft.

$$\text{Also,} \quad Q_1 + Q_2 = Q_3,$$

$$\text{or} \quad v_1 + \frac{v_2^2}{4} = v_3.$$

Therefore

$$\sqrt{\frac{250 - z}{4000\alpha}} + \frac{1}{4}\sqrt{\frac{150 - z}{6000}} = \sqrt{\frac{z}{2000\alpha}},$$

$$\text{or} \quad \sqrt{1500 - 6z} + \frac{1}{4}\sqrt{300 - 2z} = \sqrt{6z}.$$

By trial this gives $z = 96$, very nearly, and then, substituting in eqs. (II),

$$v_1^2 = 192.5, \quad \text{or} \quad v_1 = 13.874 \text{ ft. per sec.,}$$

$$v_2^2 = 45, \quad \text{or} \quad v_2 = 6.709 \text{ " "}$$

$$v_3^2 = 240, \quad \text{or} \quad v_3 = 15.492 \text{ " "}$$

Hence, also,

$$Q_1 = \frac{22}{7} \cdot \frac{2^2}{4} \times 13.874 = 43.604 \text{ cu. ft. per. sec.,}$$

$$Q_2 = \frac{22}{7} \cdot \frac{1}{4} \cdot 1^2 \times 6.709 = 5.271 \text{ " "}$$

$$Q_3 = \frac{22}{7} \cdot \frac{1}{4} \cdot 2^2 \times 15.492 = 48.689 \text{ " " } = Q_1 + Q_2 \text{ very nearly.}$$

(c) Let $AO = x$. Then, since $v_2 = 0$, $z = 150$ ft., and therefore

$$\frac{250 - 150}{x} = \alpha v_1^2 = \alpha v_3^2 = \frac{150}{6000 - x}.$$

Hence

$$\frac{6000 - x}{x} = \frac{150}{100} = \frac{3}{2}, \quad \text{and} \quad x = 2400 \text{ ft.}$$

PROBLEM II. Given $h_1, h_2, h_3; Q_2, Q_3$, and therefore $Q_1 (= \pm Q_2 + Q_3)$; to find $r_1, r_2, r_3, v_1, v_2, v_3, z$.

As before, let z be the pressure-head at O . Then

$$\frac{h_1 - z}{l_1} = \alpha \frac{v_1^2}{r_1} \quad . \quad . \quad . \quad (1) \quad \text{and} \quad Q_1 = \pi r_1^2 v_1; \quad . \quad . \quad (2)$$

$$\frac{\pm z \mp h_2}{l_2} = \alpha \frac{v_2^2}{r_2} \quad . \quad . \quad . \quad (3) \quad \text{"} \quad Q_2 = \pi r_2^2 v_2; \quad . \quad . \quad (4)$$

$$\frac{z - h_3}{l_3} = \alpha \frac{v_3^2}{r_3} \quad . \quad . \quad . \quad (5) \quad \text{"} \quad Q_3 = \pi r_3^2 v_3. \quad . \quad . \quad (6)$$

These six equations contain the seven required quantities, viz., $r_1, r_2, r_3, v_1, v_2, v_3$, and z . Thus a seventh equation must be obtained before their values can be found. This equation is given by the condition "that the cost of the piping laid in place should be a minimum," it being assumed that the cost of a pipe laid in place is proportional to its diameter.

Hence

$$l_1 r_1 + l_2 r_2 + l_3 r_3 = \text{a minimum.} \quad . \quad . \quad . \quad (7)$$

From equations (1) and (2), $\frac{h_1 - z}{l_1} = \frac{\alpha Q_1^2}{\pi^2 r_1^5};$

" " (3) " (4), $\frac{\pm z \mp h_2}{l_2} = \frac{\alpha Q_2^2}{\pi^2 r_2^5};$

" " (5) " (6), $\frac{z - h_3}{l_3} = \frac{\alpha Q_3^2}{\pi^2 r_3^5}.$

Differentiating these three equations,

$$\frac{dz}{l_1} = \frac{5\alpha Q_1^2}{\pi^2 r_1^6} \cdot dr_1;$$

$$\frac{dz}{l_2} = \mp \frac{5\alpha Q_2^2}{\pi^2 r_2^6} \cdot dr_2;$$

$$\frac{dz}{l_3} = - \frac{5\alpha Q_3^2}{\pi^2 r_3^6} \cdot dr_3.$$

But, by equation (7),

$$l_1 dr_1 + l_2 dr_2 + l_3 dr_3 = 0.$$

Hence

$$\frac{r_1^6}{Q_1^2} = \pm \frac{r_2^6}{Q_2^2} + \frac{r_3^6}{Q_3^2}, \quad \dots \dots \dots (8)$$

which is the seventh equation required.

This last equation may be written in the forms

$$\frac{r_1^2}{v_1^2} = \pm \frac{r_2^2}{v_2^2} + \frac{r_3^2}{v_3^2}$$

and

$$\frac{Q_1}{v_1^3} = \pm \frac{Q_2}{v_2^3} + \frac{Q_3}{v_3^3}.$$

25. Mains with any Required Number of Branches.

Let there be n junctions and m pipes.

Let h_1, h_2, \dots, h_m be the m pressure-heads at the end of each successive length of pipe.

Let z_1, z_2, \dots, z_n be the n pressure-heads at the 1st, 2d, 3d, \dots n th junctions.

Let l_1, l_2, \dots, l_m be the lengths of the m pipes.

PROBLEM I. Given $h_1, h_2, \dots, h_m, r_1, r_2, \dots, r_m$; to find $v_1, v_2, \dots, v_m, z_1, z_2, \dots, z_n$.

There are m equations of the type $\frac{\pm h \mp z}{l} = \alpha \frac{v^2}{r}$.

Also, the quantity flowing through the first portion of the main is equal to the sum of the quantities flowing through all the branches at the first junction, and an analogous equation will hold for each of the remaining $n - 1$ junctions. Thus n additional equations are obtained.

From these $m + n$ equations $v_1, v_2, \dots, v_m, z_1, z_2, \dots, z_n$ may be found analytically or by the method of repeated approximation.

PROBLEM II. Given $h_1, h_2, \dots, h_m, Q_1, Q_2, \dots, Q_m$; to find $r_1, r_2, \dots, r_m, z_1, z_2, \dots, z_n$.

There are now only m equations of the type

$$\frac{\pm h \mp z}{l} = \alpha \frac{v^2}{r},$$

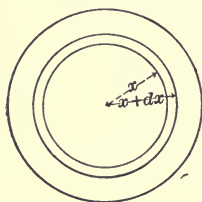
involving $m + n$ unknown quantities, and the problem admits of an infinite number of solutions.

It is therefore assumed that the cost of the piping laid in place is to be a *minimum*. Thus n new equations are obtained, and the $m + n$ equations may be solved analytically or by repeated trial.

NOTE.—The maximum velocity of flow in town mains is from 2 to 7 ft. per second.

26. Variation of Velocity in a Transverse Section.—

Assumption.—That the water in any portion of a pipe is made up of an infinite number of hollow concentric cylinders of fluid, each moving parallel to the axis with a certain definite velocity.



Let u be the velocity of one of these cylinders of radius x and thickness dx . Then the flow across a transverse section

FIG. 117.

is given by the equation

$$dq = 2\pi x \, dx \cdot u,$$

and the total flow

$$Q = 2\pi \int_0^r ux \, dx, \quad \dots \dots \dots (1)$$

r being the radius of the pipe.

If v_m be the mean velocity for the whole transverse section of the pipe,

$$v_m = \frac{Q}{\pi r^2} = \frac{2 \int_0^r ux \, dx}{r^2} \dots \dots \dots (2)$$

Again, assuming with Navier that the surface resistance between two concentric cylinders is of the nature of a viscous resistance and may be represented by $k \frac{du}{dx}$ per unit of area at the radius x , k being a coefficient called the coefficient of viscosity, then the total resistance at the radius x for a length ds of the cylinder

$$= -2\pi x \cdot ds \cdot k \frac{du}{dx} = -2\pi k \cdot ds \cdot x \frac{du}{dx}.$$

The total resistance at the radius $x + dx$

$$= +2\pi k \cdot ds \left[x \frac{du}{dx} + \frac{d}{dx} \left(x \frac{du}{dx} \right) dx \right].$$

Hence the total resultant resistance for the length ds of the cylinder under consideration

$$= 2\pi k ds \frac{d}{dx} \left(x \frac{du}{dx} \right) dx.$$

The component of the weight of the slice of the cylinder in the direction of the axis

$$= w \cdot 2\pi x \cdot dx \cdot ds \cdot \sin \theta,$$

θ being the inclination of the axis to the horizon.

Let $-dz$ be the fall of level in the distance ds . Then

$$-dz = ds \cdot \sin \theta.$$

Therefore component of weight in direction of axis

$$= -w \cdot 2\pi x \cdot dx \cdot dz.$$

The resultant pressure on the slice in the direction of motion

$$= \{p - (p + dp)\} 2\pi x \cdot dx = -2\pi x \cdot dx \cdot dp.$$

Then, since the motion is uniform,

$$w \cdot 2\pi k \cdot ds \cdot \frac{d}{dx} \left(x \frac{du}{dx} \right) dx - w \cdot 2\pi x \cdot dx \cdot dz - 2\pi x \cdot dx \cdot dp = 0,$$

and therefore

$$\frac{k \cdot ds}{x} \frac{d}{dx} \left(x \frac{du}{dx} \right) - dz - \frac{dp}{w} = 0.$$

Integrating only for the cylinder under consideration,

$$\frac{ks}{x} \frac{d}{dx} \left(x \frac{du}{dx} \right) - \left(z + \frac{p}{w} \right) = \text{a constant.}$$

But $z + \frac{p}{w}$ is evidently independent of x and is a linear function of s (Art. 5, Chap. II). Hence

$$\frac{1}{x} \frac{d}{dx} \left(x \frac{du}{dx} \right) = \text{a constant} = A, \text{ suppose.}$$

Therefore

$$\frac{d}{dx} \left(x \frac{du}{dx} \right) = Ax. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Integrating,

$$x \frac{du}{dx} = A \frac{x^2}{2} + B.$$

Assuming that the central fluid filament is the filament of maximum velocity, then when $x = 0$, $\frac{du}{dx}$ is also nil. Therefore

$$B = 0, \quad \text{and} \quad x \frac{du}{dx} = \frac{Ax^2}{2},$$

and therefore

$$\frac{du}{dx} = A \frac{x}{2} \quad \dots \quad (4)$$

Integrating, Eq. 4,

$$u = A \frac{x^2}{4} + C,$$

C being a constant of integration.

Since dp is the difference of intensity of pressure on the ends of the cylindrical slice,

$$-2\pi x \cdot ds \cdot k \frac{du}{dx} = \pi x^2 \cdot dp = \pi x^2 w \cdot dh.$$

Therefore

$$\frac{du}{dx} = -\frac{wx}{2k} \frac{dh}{ds} = -\frac{wx}{2k},$$

and, by equation (4),

$$A = -\frac{wi}{k}.$$

Let $u_{\max.}$ be the velocity of the central filament, i.e., the value of u when $x = 0$. Then

$$u_{\max.} = c,$$

and

$$u_{\max.} - u = -\frac{A}{4}x^2 = Dx^2, \quad \dots \quad (5)$$

where $D = -\frac{A}{4}$.

Again, by equation (1),

$$Q = 2\pi \int_0^r (u_{\max.} - Dx^2)x \cdot dx = \pi r^2 \left(u_{\max.} - \frac{Dr^2}{2} \right);$$

and by equation (2),

$$v_m = u_{\max.} - \frac{Dr^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If u_s = velocity at pipe wall, then, by equation (5),

$$u_s = u_{\max.} - Dr^2. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence, by equations (6) and (7),

$$u_s + u_{\max.} = 2v_m. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If $u = 0$ when $x = r$, then $C = -A\frac{r^2}{4}$, and

$$u = -\frac{A}{4}(r^2 - x^2).$$

Therefore

$$\begin{aligned} Q &= -\frac{A\pi}{2} \int_0^r x(r^2 - x^2)dx. \\ &= -\frac{A\pi r^4}{8} = \frac{w\pi r^4}{8k}. \end{aligned}$$

NOTE.—In a paper by Gardner S. Williams and others, in the Proceedings of the Am. Soc. of C. E. for May, 1901, giving the results of experiments on the flow of water in pipes, the inferences are made: that at ordinary velocities of flow, and under normal conditions, the ratio of the mean velocity to the maximum is .84; that in a straight pipe there will be, under some conditions, a difference of pressure at different points in the circumference of the same cross-section; that the normal curve of velocities is an ellipse; that the effect of a flow disturbance is felt many diameters beyond the point at which it occurs; that for a maximum flow careful alignment is as necessary as a smooth interior.

27. Gauging of Pipe-flow.—A variety of meters have been designed to register the quantity of water delivered by a pipe. The principal requisites of such a meter are:

1. That it should register with accuracy the quantity of water delivered under different pressures.
2. That it should not appreciably diminish the effective pressure of the water.
3. That it should be compact and adaptable to every situation.
4. That it should be simple and durable.

The Venturi Meter (Fig. 118) is so called from Venturi, who first pointed out the relation between the pressures and velocities of flow in converging and diverging tubes.

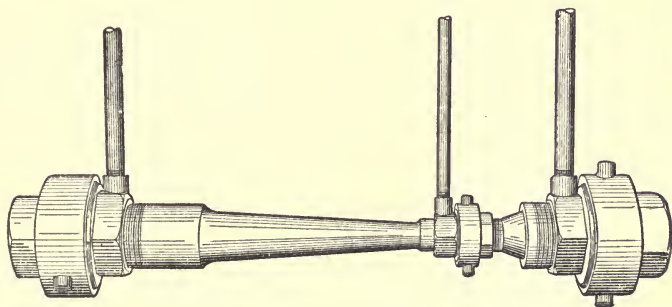


FIG. 118.

As shown by the longitudinal section, Fig. 119, this meter consists of two truncated cones joined at the smallest sections by a short throat-piece. At *A* and *B* there are air-chambers with holes for the insertion of piezometers, by which the fluid pressure may be measured. By Art. 5, Chap. I, the theoretical quantity *Q* of flow through the throat at *A* is

$$Q = \frac{a_1 a_2}{\sqrt{a_2^2 - a_1^2}} \sqrt{2g(H_2 - H_1)},$$

a_1 , a_2 being the sectional areas at A and B , respectively, and $H_2 - H_1$ the difference of head in the piezometers, or the "head on Venturi," as it is called.

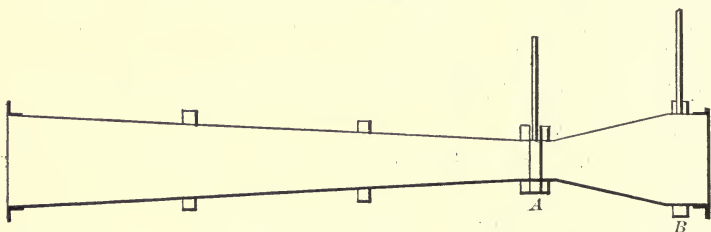


FIG. 119.

Introducing a coefficient of discharge C , the actual delivery through A is

$$Q = C \frac{a_1 a_2}{\sqrt{a_2^2 - a_1^2}} \sqrt{2g(H_2 - H_1)}.$$

An elaborate series of experiments by Herschel gave C values varying between .94 and 1.04, but the great majority of the values lay between .96 and .99.

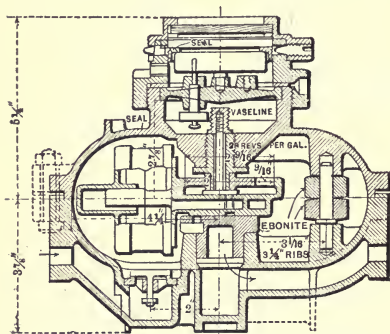


FIG. 120.—Schonheyder's Positive Meter.

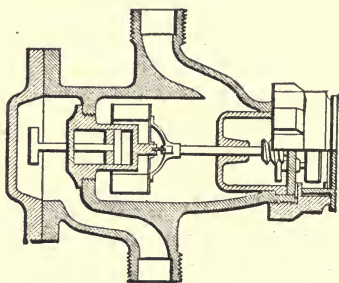


FIG. 121.—The Universal Meter.

The piezometers may be connected with a recorder, and thus a continuous register of the quantity of water passing through the meter may be obtained at any convenient position within a radius of 1000 ft. This distance may be extended to several miles by means of an electric device.

Other meters may be generally classified as Piston or Reciprocating Meters and Inferential or Rotary Meters. They are all provided with recorders which register the delivery with a greater or less degree of accuracy.

The piston meter (Fig. 120) is the most accurate and gives a positive measurement of the actual delivery of water as recorded by the strokes of the piston in a cylinder which is filled from each end alternately. Thus an additional advantage

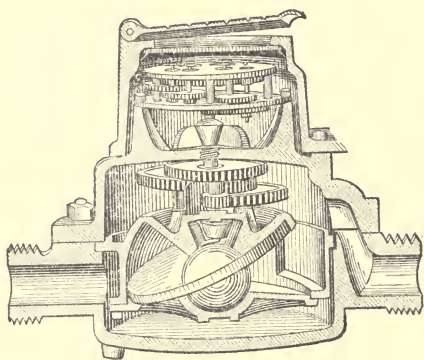


FIG. 122.—The Buffalo Meter.

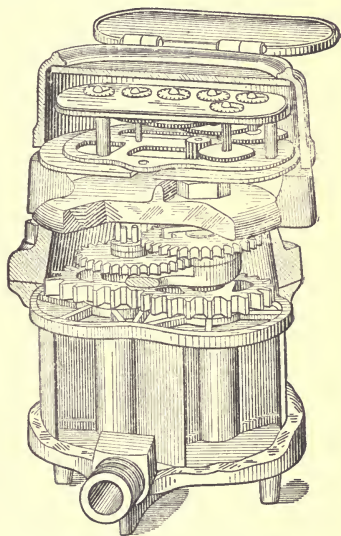


FIG. 123.—The Union Rotary Piston Meter.

possessed by a water-engine is that the working cylinder will also serve as a meter.

In inferential meters a drum or turbine is actuated by the force of the current passing through the pipe, but it often happens that when the flow is small the force is insufficient to cause the turbine to revolve, and there is consequently no register of the corresponding quantity of water passing through the meter.

EXAMPLES.

(N.B. Take $g = 32$ and $6\frac{1}{2}$ gallons = 1 cu. ft. unless otherwise specified.)

1. A water-main is to be laid with a virtual slope of 1 in 850, and is to give a maximum discharge of 55 cubic feet per second. Determine the requisite diameter of pipe and the maximum velocity, taking $f = .0064$.

Ans. 3.679 ft.; 3.2888 ft. per sec.

2. Find the loss of head due to friction in a pipe; diameter of pipe = 12 in., length of pipe = 5280 ft., velocity of flow = 3 ft. per second; $f = .0064$. Also find the discharge.

Ans. 19.008 ft.; 2.3562 cu. ft. per sec.

3. A pipe has a fall of 10 ft. per mile; it is 10 miles long and 4 ft. in diameter. Find the discharge, assuming $f = .0064$.

Ans. 54.7 cu. ft. per sec.

4. A pipe discharges 250 gallons per minute, and the head lost in friction is 3 ft. Find approximately the head lost when the discharge is 300 gallons per minute; also find the work consumed by friction in both cases.

Ans. 4.32 ft.; 7500 ft.-lbs.; 12,960 ft.-lbs.

5. What is the mean hydraulic depth in a circular pipe when the water rises to the height $\frac{\text{diameter}}{2\sqrt{2}}$ above the centre?

Ans. $\frac{10}{33} \times \text{diameter}$.

6. A 12-inch pipe has a slope of 12 feet per mile; find the discharge. ($f = .005$.)

Ans. 2.118 cu. ft. per sec.

7. The mean velocity of flow in a 24-in. pipe is 5 ft. per second; find its virtual slope, f being .0064.

Ans. 1 in 200.

8. Calculate the discharge per minute from a 24-in. pipe of 4000 ft. length under a head of 80 ft., using a coefficient suitable for a clean iron pipe.

Ans. 34.909 cu. ft. per sec.

9. How long does it take to empty a dock whose depth is 31 ft. 6 ins. and which has a horizontal sectional area of 550,000 sq. ft., through two 7-ft. circular pipes 50 ft. long, taking into account resistance at entrance?

Ans. 214 min. 6 sec.

10. The virtual slope of a pipe is 1 in 700; the delivery is 180 cubic feet per minute. Find the diameter and velocity of flow.

Ans. 1.26 ft.; 2.401 ft. per sec.

11. Determine the diameter of a clean iron pipe, 100 feet in length, which is to deliver .5 cu. ft. of water per second under a head of 5 feet. Assume $f = .006$.

Ans. .328 ft.

12. A reservoir of 10,000 sq. ft. area and 100 ft. deep discharges

through a pipe 24 ins. in diam. and 2000 ft. in length. Find the velocity of flow. What should the diam. be in order that the reservoir may be emptied in two hours? ($f = .0064$.) *Ans.* 15.37 ft. per sec.; 4.0923 ft.

13. The pressure from an accumulator at the entrance of a $\frac{1}{2}$ -in. pipe L ft. long is 1000 lbs. per sq. in. If N is the total H.P. available at the inlet, show that the H.P. absorbed in frictional resistance is $\left(\frac{N}{5760}\right)^{\frac{3}{2}} \frac{L}{\bar{d}^{\frac{5}{2}}}$,

f being $\frac{2}{245} = .0081$.

✓ 14. The delivery at the end of a 3-inch pipe is 11.06 H.P. The total effective head at the entrance to pipe is 896 feet. The loss in frictional resistance is 21 per cent. Find the distance to which the energy is transmitted. *Ans.* 15,000 ft., f being .0064.

✓ 15. A reservoir has a superficial area of 12,000 ft. and a depth of 60 ft.; it is emptied in 60 minutes through four horizontal circular pipes, equal in diameter and 50 ft. long. Find the diameter. ($f = .0064$.) *Ans.* 1.786 ft.

Explain how the *total* head is made up, and draw the plane of charge.

16. A 3-inch pipe is very gradually reduced to $\frac{1}{2}$ inch. If the pressure-head in the pipe is 40 ft., find the greatest velocity with which the water can flow through. *Ans.* 1.4 ft. per sec.

17. Water flows through a 24-inch pipe 5000 yards in length. At 1000 yards it yields up 300 cubic feet per minute to a branch. At 2800 yards it yields up 400 cubic feet per minute to a second branch. At 4000 yards it yields up 600 cubic feet per minute to a third branch. The delivery at the end is 500 cubic feet per minute. Find the head absorbed by friction. ($f = .0075$.) *Ans.* 177.801 ft.

✓ 18. Find the H.P. required to raise 550 gallons per minute to a height of 60 feet, through a pipe 100 feet in length and 6 in. in diameter, the coefficient of friction being .0064. *Ans.* 10.74.

19. What head of water is required for a 5-in. pipe, 150 ft. in length, to carry off 25 cu. ft. of water per minute? *Ans.* 1.56223 ft.

What head will be required if the pipe contains two rectangular knees? *Ans.* 1.84918 ft.

20. Determine the delivery of a 2-in. pipe, 48 ft. long, under a 5-ft. head, f being .005. *Ans.* .1449 cu. ft. per sec.

What will be the delivery if the pipe has 5 small curves of 90° curvature, the ratio of the radius of the pipe to that of the curves being 1 : 2? *Ans.* .1381 cu. ft. per sec.

21. The curved buckets of a turbine form channels 12 in. long, 2 in. wide, and 2 in. deep; the mean radius of curvature of the axis is 8 in.; the water flows along the channel with a velocity of 50 ft. per minute. What is the head lost through curvature? *Ans.* .00138 ft.

22. Find the power transmitted by water flowing at 80 ft. per sec. in a 36-inch pipe, the metal being $1\frac{1}{8}$ inches thick and the allowable stress

2800 lbs. per square inch. If the pipe is $1\frac{1}{4}$ miles in length, find the loss of power. *Ans.* 576 H.P.; 720.2 ft.-lbs.

23. Find the diameter of a pipe $\frac{1}{2}$ mile long to deliver 1500 gallons of water per minute with a loss of 20 feet of head. ($f = .005$)

Ans. 1.0135 ft.

24. Water is to be raised 20 ft. through a 30-ft. pipe of 6 in. diameter. Find the velocity of flow, assuming that 10 per cent of additional power is required to overcome friction, and that $f = .0075$.

Ans. 8.44 ft. per sec.

25. In a pipe 3280 ft. in length and delivering 6750 gallons per min., the loss of head in friction is 83 ft. Taking $f = .0064$, find the diameter.

Ans. 1.527 ft.

26. Calculate, by Thrupp's formula, the flow through a 4-in. rough wrought-iron pipe having a fall of 33 feet per mile.

Ans. .1426 cu. ft. per sec.

27. A clean 6-in. pipe has a virtual slope of 1 per 400. Taking $f = .005$, find the velocity of steady flow, the discharge, and the energy absorbed in frictional resistance in 1000 feet.

Ans. 2 ft. per sec.; $\frac{1}{8}$ cu. ft. per sec.; $61\frac{43}{112}$ ft.-lbs.

28. A 6-in. pipe, 500 ft. long, discharges into a 3-in. pipe, also 500 ft. long. The effective head between the inlet and outlet is 10 feet. Find the discharge, taking $f = .0064$, and making allowance for the resistance at the inlet.

Ans. .1703 cu. ft. per sec.

29. How far can 100 H.P. be transmitted by a $3\frac{1}{2}$ -in. pipe with a loss of head not exceeding 25 per cent under an effective head of 750 lbs. per square inch?

Ans. 5426.3 ft.

30. A pipe 2000 ft. long and 2 ft. in diameter discharges at the rate of 16 ft. per second. Find the increase in the discharge if for the last 1000 ft. a second pipe of same size be laid by the side of the first and connected with it so that the water may flow equally well along either pipe.

Ans. 7.24 cu. ft. per sec.

31. A pipe of length l and radius r gives a discharge Q . How will the discharge be affected (1) by doubling the radius for the whole length; (2) by doubling the radius for half the length; (3) by dividing it into three sections of equal length, of which the radii are r , $\frac{r}{2}$, and $\frac{r}{4}$, respectively? (f = coefficient of friction.)

$$\text{Ans. 1. New discharge} = 4Q \left(\frac{3r + 4fl}{3r + 2fl} \right)^{\frac{1}{2}};$$

$$2. \quad " \quad " \quad = Q \left(\frac{48r + 64fl}{66r + 33fl} \right)^{\frac{1}{2}};$$

$$3. \quad " \quad " \quad = Q \left(\frac{9r + 12fl}{524.712r + 4228fl} \right)^{\frac{1}{2}}.$$

32. A 24-in. pipe 2000 ft. long gives a discharge of Q cubic feet of water per minute. Determine the change in Q by the substitution for the foregoing of either of the following systems: (1) two lengths, each

of 1000 ft., whose diameters are 24 ins. and 48 ins. respectively ; (2) four lengths, each of 500 ft., whose diameters are 24 ins., 18 ins., 16 ins., and 24 ins.

Draw the "plane of charge" in each case.

Ans. (1) Discharge is increased 33.2 per cent taking loss at change of section into account ;

Discharge is increased 35.7 per cent disregarding loss at change of section.

(2) Discharge is diminished 45 per cent disregarding losses at change of section.

33. Q is the discharge from a pipe of length l and radius r ; examine the effect upon Q of increasing r to nr for a length ml of the pipe.

$$\text{Ans. New discharge} = Q \left\{ \frac{\frac{3}{2} + \frac{2fl}{r}}{\frac{3}{2} + \frac{2fl}{r} \left(1 - m + \frac{m}{n^5} \right) + \frac{(n^2 - 1)^{\frac{1}{2}}}{n^4}} \right\}^{\frac{1}{2}}$$

34. A 5-in. pipe, 300 ft. long, discharges into a 3-in. pipe, 200 ft. long, the total fall being 5 feet. Find the quantity of flow in gallons per hour.

Ans. 4080.

35. A main, 1000 ft. long and with a fall of 5 ft., discharges into two branches, the one 750 ft. long with a fall of 3 ft., the other 250 ft. long with a fall of 1 ft. The longer branch passes twice as much water as the other and the total delivery is $47\frac{1}{2}$ cu. ft. per minute. The velocity of flow in the main is $2\frac{1}{2}$ ft. per second. Find the diameters of the main and branches. ($f = .0064$.)

Ans. 63245 ft. ; .51 ft. ; .36 ft.

36. The water in a 12-in. main, 800 ft. long, flows at the rate of 1 ft. per second and *one third* of the water is discharged into a branch 200 ft. long with a fall of 1 in 40, while the remainder passes into a 600-ft. branch with a fall of 1 in 60. The effective head between the inlet and outlet of the main is $2\frac{1}{2}$ ft. Find the total discharge and the diameters of the branches, taking $f = .0064$, and making allowance for loss at inlet but disregarding losses at the junction.

Ans. $94\frac{2}{3}$ cu. ft. per sec., .27 ft. ; .39 ft.

37. If a pipe whose diameter is 8 ins. suddenly enlarges to one whose diameter is 12 ins., find the power required to force 1000 gallons per minute through the enlargement, and draw to scale the plane of charge.

Ans. Energy expended = .1377 H.P.

38. 1000 gallons per minute are forced through a system of pipes AB , BC , CD , of which the lengths are 100 ft., 50 ft., and 120 ft., and the radii 6 ins., 3 ins., and 4 ins., respectively. Draw to scale the plane of charge.

Ans. Loss in friction from A to B = 14.744 ft.; loss at B = 14.56 ft.

" " " " B to C = 235.9 " ; " " C = 8.819 "

" " " " C to D = 134.36 "

39. A pipe 4 ins. in diameter suddenly contracts to one 3 ins. in diameter; find the power necessary to force 250 gallons per minute through the sudden contraction.

Ans. 1.23997 H.P.

40. Water flows from a 3-in. pipe through a $1\frac{1}{2}$ -in. orifice in a diaphragm into a 2-in. pipe. What head is required if the delivery is to be 8 cu. ft. of water per minute?

Ans. 2.826 ft.

41. 500 gallons of water per minute are forced through a continuous line of pipes AB , BC , CD , of which the radii are 3 ins., 4 ins., 2 ins., and the lengths 100 ft., 150 ft., and 80 ft., respectively. Find the *total* loss of head (*a*) due to the sudden changes of form at B and C , (*b*) due to friction. Find (*c*) the diameter of an equivalent uniform pipe of the same total length.

Ans. (*a*) .1378 ft.; 1.152 ft.

(*b*) 3.688 ft. in AB ; 1.313 ft. in BC ; 22.393 ft. in CD .

(*c*) .4212 ft.

42. AB , BC , CD is a system of three pipes of which the lengths are 1000 ft., 50 ft., and 800 ft., and the diameters 24 ins., 12 ins., and 24 ins., respectively; the water flows from CD through a 1-in. orifice in a thin diaphragm, and the velocity of flow in AB is 2 ft. per second. Draw the plane of charge and find the mechanical effect of the efflux, f being .0064.

Ans. Loss at $C = \frac{9}{16}$ ft.; at $B = \frac{81}{256}$ ft.; in friction from A to $B = .8$ ft.; from B to $C = 1.28$ ft.; from C to $D = .64$ ft.; energy of jet = 14,811 $\frac{3}{4}$ H.P.

43. 1000 gallons per minute flows through a sudden contraction from 12 ins. to 8 ins. at A , then through a sudden enlargement from 8 ins. to 12 ins. at B , the intermediate pipe AB being 100 ft. long. Draw the plane of charge, f being .0064.

Ans. Loss at $A = .288$ ft.; at $B = .281$ ft.; in friction from A to $B = 3.499$ ft.

44. Water flows from one tube into another of *twice* the diameter; the velocity in the latter is 10 ft. Find the head corresponding to the resistance.

Ans. 14.0625 ft.

45. A 2-in. pipe A suddenly enlarges to a 3-in. pipe B , the quantity of water flowing through being 100 gallons per minute. Find the loss of head and the difference of pressure in the pipes (1) when the flow is from A to B ; (2) when the flow is from B to A , C being .66.

Ans. (1) Loss of head = 8.639 in.

Gain of pressure-head = 13.83 "

(2) Loss of head = 7.428 "

Diminution of pressure-head = 29.88 "

46. A 3-in. horizontal pipe rapidly contracts to a 1-in. mouthpiece, whence the water emerges into the air, the discharge being 660 lbs. per minute. Find the pressure in the 3-in. main.

If the 3-in. pipe is 200 ft. in length and receives water from an open tank, find the height of the tank, f being .005.

Ans. 1003.5 lbs. per sq. ft.; 19.92 ft.

47. A horizontal pipe 4 ins. in diameter suddenly enlarges to a diameter of 6 ins.; find the force required to cause a flow of 300 gallons of water per minute through the sudden enlargement.

Ans. .06 H.P.

48. 1000 gallons per minute is to be forced through a system of pipes AB , BC , CD , of which the lengths are 100 ft., 50 ft., 120 ft., and the radii 4 ins., 6 ins., and 3 ins., respectively. What must be diameter of equivalent uniform pipe? Draw the plane of charge, f being .0064.

Ans. Diameter = 3.4 ins.;

loss in friction from A to B = 111.96 ft.; loss at B = 4.499 ft.;

" " " " B to C = 7.372 " " " C = 14.56 "

" " " " C to D = 566.17 "

49. Find the H.P. required to pump 1,000,000 gallons of water per day of 24 hours to a height of 300 ft. through a line of straight piping 3000 ft. long, the diameter of the pipe being 8 ins. for the first 1000 ft., 6 ins. for the second, and 4 ins. for the third, allowance being made for the loss at inlet and the losses at abrupt changes of section; also 4 is to be taken as the coefficient of resistance for pump-valves. (At changes of section $c_c = .64$.) What is the diameter of an equivalent uniform pipe? ($f = .0064$.)

Ans. 196; diam. = .403 ft., or say 5 ins.

50. In a given length l of a circular pipe whose inner radius is r and thickness e , a column of water flowing with a velocity v is suddenly checked by the shutting off of cocks, etc. Show that

$$gh = \frac{Ee\lambda^2}{r} \left\{ 1 + \frac{e}{2r} \left(1 + \frac{E}{E_1} \right) + \frac{e^2}{r^2} \right\},$$

in which h = head due to the velocity v , E = coefficient of elasticity, E_1 = coefficient of compressibility of water, λ = extension of pipe circumference due to E .

51. The water surface in one reservoir is 500 ft. above datum, and is 100 ft. above the surface of the water in a second reservoir 20,000 ft. away, and connected with the first by an 18-in. main. Find the delivery per second, taking into account the loss of head at the entrance.

Ans. 7.64 cu. ft. per sec., f being .0064.

52. Determine the discharge from a pipe of 12 in. radius and 3280 ft. in length which connects two reservoirs having a difference of level of 128 ft. Take into account resistance at entrance. Draw the plane of charge. ($f = .005$.)

Ans. 48,571 cu. ft. per sec.

53. Determine the diameter of a clean iron pipe 5000 ft. in length which connects two reservoirs having a total head of 40 ft. and discharges into the lower at the rate of 20 cu. ft. per second. Draw to scale the line of charge. ($f = .005$.)

Ans. 1.9219 ft.

54. The difference of level between the two reservoirs is 100 ft., and they are connected by a pipe 10,000 ft. long. Find the diameter of the pipe so as to give a discharge of 2000 cubic feet per minute (*a*) by Darcy's formula, (*b*) assuming $f = .0064$. (Allow for loss of head at entrance.)

Ans. (*a*) 2.256 ft. if $\alpha = .0001622$; (*b*) 2.360 ft.

55. Two reservoirs are connected by a 12-inch pipe $1\frac{1}{4}$ miles long. For the first 500 yards it has a slope of 1 in 30, for the next half mile a slope of 1 in 100, and for the remainder of its length it is level. The head of water over the inlet is 55 ft. and that over the outlet is 15 ft. Determine the discharge in gallons per minute. (Take $f = .0064$.)

Ans. 1950.66.

56. Two reservoirs are connected by a 6-in. pipe in three sections, each section being three quarters of a mile in length. The head over the inlet is 20 ft., that over the outlet 9 ft. The virtual slope of the first section is 1 in 50, of the second 1 in 100, and the third section is level. Find the velocity of flow, and the delivery, f being .005.

Ans. 4.5 ft. per sec.; 332 gallons per minute.

57. A pipe 5 miles long, of uniform diameter equal to 12 in., conveys water from a reservoir in which the water stands at a height of 300 ft. above Trinity high-water mark, to a reservoir in which the water stands at a height of 150 ft. above the same datum. To what height will water rise in a supply-pipe taken one mile from the lower end? For what pressure would you design the main at this point, if it lies 20 ft. above the level of the lower reservoir? ($f = .0064$.)

Ans. 179.755 ft.; 19.13 lbs. per sq. in.

58. A clean 6-in. pipe, 1000 ft. long, has four sharp knees, viz., one of 60° , two of 90° , and one of 120° . Find the head wasted at the knees and in the straight pipe, the flow being at the rate of 150 gallons per minute.

Ans. .2734 ft.; 3.0237 ft.

59. A 6-in. pipe, 4000 feet in length, leads from a reservoir *A* to a point *O*, at which it divides into two 6-inch branches, each 4000 feet in length, the one leading to a reservoir *B*, the other to a reservoir *C*. The surface of the water in *A* is 100 feet above that in *B* and 200 feet above that in *C*. Find the velocities of flow in the three branches, f being .0064.

Ans. $v_1 = 7.89$ ft. per second = v_3 ; $v_2 = 0$.

60. A pipe 24 in. in diameter and 2000 ft. long leads from a reservoir in which the level of the water is 400 ft. above datum to a point *B*, at which it divides into two branches, viz., a 12-in. pipe *BC*, 1000 ft. long, leading to a reservoir in which the surface of the water is 250 feet above datum, and a branch *BD*, 1500 ft. long, leading to a reservoir in which the surface of the water is 50 ft. above datum. Determine the diameter of *BD* when the free surface-level at *B* is (*a*) 300 ft.; (*b*) 250 ft., and (*c*) 200 above datum.

Ans. (*a*) 1.454 ft.; (*b*) 1.783 ft.; (*c*) 2.096 ft.

61. Two reservoirs *A* and *B* are connected by a line of piping *MON*, 2000 ft. in length. From the middle point *O* of this pipe a branch *OP*, 1000 ft. in length, leads to a reservoir *C*. The reservoirs *A* and *B* are

200 feet and 100 feet, respectively, above the level of C . The deliveries in MO , OP , ON , in cubic feet per second, are $\frac{2}{3}\pi$, $\frac{1}{3}\pi$, and π respectively. Find (a) the velocities of flow in MO , OP , ON ; (b) the radii of these lengths; (c) the height of the free surface-level at O above C , f being .0064. *Ans.* (a) 11.121 ft. per sec. in MO ; 10.158 ft. per sec. in OP ; 14.145 ft. per sec. in ON .

(b) .5 ft.; .41831 ft.; .26588 ft. (c) 150.5 ft., very nearly.

62. Find the amount of water in gallons per day which will be delivered by a 24-inch cast-iron pipe, 15,000 ft. in total length, when the water surface at the outlet is $87\frac{1}{2}$ ft. below the water surface at the inlet, taking $f = .001$ and allowing for resistance at inlet.

If the water, instead of flowing into a reservoir, is made to drive a reaction turbine, what must be the velocity of flow in the pipe to give a max. speed? What will be the H.P. of the turbine if its efficiency is .84?

A third reservoir is connected with the system by means of a 24-in. cast-iron pipe, 7500 ft. long, joined to the main at the middle point. The water surface of this intermediate reservoir is 50 ft. above that of the lowest reservoir. Discuss the distribution.

Ans. 22,628,571 $\frac{3}{4}$; 7.7 ft. per sec.; 12.63 H.P.; $z = 73.68$ or 51.32 ft.; $v_1 = 7.76$ or 12.42 ft. per sec.; $v_2 = 10.05$ or 2.373 ft. per sec.; $v_3 = 17.73$ or 14.8 ft. per sec.

63. The water-levels in two reservoirs A and B are, respectively, 300 ft. and 200 ft. above that in C . The reservoir A supplies 3 cu. ft. of water, of which 2 cu. ft. go to B and 1 cu. ft. goes to C . A pipe 2500 ft. long leads from A to a junction at O , from which two branches, each 2500 ft. in length, lead, the one to B and the other to C . Assuming that the cost of laying a pipe in place is proportional to the diam. and that this cost is to be a minimum, find the pressure head at O and the diams. of the pipes.

Ans. 164 ft.; diam. of $AO = .66$ ft., of $OB = 63$ ft., of $OC = .4$ ft.

64. An engine pumps a volume of Q cubic feet of water per second through a hose 1 ft. in length, and d feet in diameter, having at the end a nozzle D feet in diameter. Find the pumping H.P. and apply your result to the determination of the H.P. of an engine which is to pump 30 cu. ft. of water per minute through a 1-in. nozzle at the end of a 3-in. hose 400 ft. in length ($f = .00625$). Also find the force required to hold the nozzle.

Ans. 11 $\frac{3}{8}$ H.P.; 89 $\frac{3}{8}$ lbs.

65. A fire-engine pumps water through a 400-ft. length of $2\frac{1}{2}$ -in. bore at the rate of 12 ft. per second, and discharges through a 1-in. nozzle. Find the pressure in the hose, and the pumping H.P. Also find the force required to hold the nozzle. ($f = .00125$.)

Ans. .6702 lbs. per ft.; 5.0916; 59.95 lbs.

66. The conduit-pipe for a fountain is 250 ft. long and 2 in. in diameter; the coefficient of resistance for the mouthpiece is .32; the entrance orifice is sufficiently rounded, and the bends have sufficiently long radii

of curvature to allow of the corresponding coefficient of resistance being disregarded. How high will a $\frac{1}{2}$ -in. jet rise under a head of 30 ft. ?

Ans. 20.4 ft.

67. Water surface of a reservoir is 300 ft. above datum, and a 4-in. pipe 600 ft. long leads from reservoir to a point 200 ft. above datum. Find the height to which the water would rise (*a*) if end of pipe is open to atmosphere, (*b*) if it terminates in a 1-in. nozzle. In latter case find longitudinal force on nozzle. *Ans.* (*a*) $2\frac{3}{8}$ ft.; (*b*) 87.52 ft.; 59.693 lbs.

68. The surface of the water in a tank is 388 ft. above datum and is connected by a 4-in. pipe 200 ft. long with a turbine 146 ft. above datum. Determine the velocity of the water in the pipe at which the power obtained from the turbine will be a maximum. Assuming the efficiency of the turbine to be 85 per cent, determine the power, *f* being .005. *Ans.* 19.928 ft. per sec.; 27.11075 H.P.

69. A pipe 12 ins. in diameter and 900 ft. long is used as an inverted siphon to cross a valley. Water is lead to it and away from it by an aqueduct of rectangular section 3 ft. broad and running full to a depth of 2 ft. with an inclination of 1 in 1000. What should be the difference of level between the end of one aqueduct and the beginning of the other, *f* being .0064 for the pipe, and .008 for the aqueduct ?

Ans. 14.39.

70. Water flows through a pipe 20 ft. long with a velocity of 10 ft. per second. If the flow is stopped in $\frac{1}{10}$ second and if retardation during the stoppage is uniform, find the increase in the pressure produced. ($g = 32$ and the density of the water = 62.5 lbs. per cu. ft.)

Ans. $62\frac{1}{2}$ cu. ft. of water.

71. An hydraulic motor is driven by means of an accumulator giving 750 lbs. per square inch. The supply-pipe is 900 ft. long and 4 ins. in diameter. Find the maximum power attainable, and velocity in pipe. ($f = .0075$.) *Ans.* 242.4 H.P.; 21.203 ft. per sec.

72. A 2-in. hose conveys 2 gallons of water per second. Find the longitudinal tension in the hose. *Ans.* 9.18 lbs.

73. Find the pumping H.P. to deliver 1 cu. ft. of water per second through a 1-in. nozzle at end of a 3-in. hose 200 ft. long, *f* being .016.

Ans. 97.335 H.P.

74. The surface of the water in a tank is 286 ft. above datum. The tank is connected by a 4-in. pipe 500 ft. long with a 36-in. cylinder 170 ft. above datum. Find (*a*) the velocity of flow in the pipe for which the available power will be a maximum; (*b*) the power. If the piston moves at the rate of 1 ft. per minute, find (*c*) the pressure on the piston. Also find the height to which the water would rise if (*d*) the cylinder end of the pipe were open to the atmosphere and if (*e*) the pipe terminated in a nozzle 1 in. in diameter, neglecting the frictional resistance of the nozzle. Finally, find (*f*) the power required to hold the nozzle. (Coeff. of friction = .005.) *Ans.* (*a*) 8.93 ft. per sec.; (*b*) 6.85 H.P.; (*c*) 22.8 tons per sq. ft.; (*d*) 3.74 ft.; (*e*) 103.8 ft.; (*f*) 70.8 lbs.

75. A 3-in. hose, 400 ft. in length, terminates in a $\frac{3}{8}$ -in. nozzle; Water enters the hose under a head of $297\frac{1}{2}$ ft. Find the velocity of efflux, the height to which the issuing jet will rise, the pressure-head at the nozzle inlet, and the force required to hold the hose, f being .00625. *Ans.* 128 ft. per sec.; 256 ft.; 18,437 $\frac{1}{2}$ lbs. per sq. ft.; 98 $\frac{3}{4}$ lbs.

76. A reducer, 10 ft. long, conveys 400 gallons of water per minute, and its diameter diminishes from 12 ins. to 6 ins.; find the total loss of head due to friction. *Ans.* .05529.

77. A reservoir is to be supplied with water at the rate of 11,000 gallons per minute, through a vertical pipe 30 ft. high; find the minimum diameter of pipe consistent with economy. Cost of pipe per foot = $\$d$, d being the diameter; cost of pumping = 1 cent per H.P. per hour; original cost of engine per H.P. = \$100.00; add 10 per cent for depreciation. Engine works 12 hours per day for 300 days in the year, f being .0064. *Ans.* 4.375 ft.

78. A city is supplied with water by means of an aqueduct of rectangular section, 24 ft. wide, running 4 ft. deep, and sloping 1 in 2400. One-fourth of the supply is pumped into a reservoir through a pipe 3000 ft. long, rising 25 ft. in the first 1500 ft., and 75 ft. in the second 1500 ft. The pumping is effected by an engine burning $2\frac{1}{2}$ lbs. of coal per H.P. per hour, and working constantly through the year. A percentage is to be allowed for repairs and maintenance; the cost of the coal per ton of 2000 lbs. is \$4; the prime cost of the engine is \$100 per H.P.; the efficiency of the engine is $\frac{2}{3}$; the coefficient of pipe friction is .0064, the cost of the piping is \$30 per ton. Determine the most economical diameter of pipe, and the H.P. of the engine, f being .0064 for the pipe and .08 for the channel. *Ans.* 4.84 ft.; 456.455 H.P.

79. A vessel with 500 sq. ft. of surface experiences a resistance of 150 lbs. per sq. ft. when steaming at 5 knots. How much H.P. will be absorbed in frictional resistance by a vessel with 10,000 sq. ft. of surface steaming at 18 knots? *Ans.* 2140 $\frac{1}{2}$.

80. The performances of two similarly designed ships are to be compared. The one, with a length of 300 ft. and a displacement of 8000 tons, is to steam at 20 knots. What should be the length and displacement of the other, which is to steam at 21 knots? Compare also the I.H.P.s. *Ans.* 330 $\frac{3}{4}$ ft.; 10,720 tons; 1.34.

81. From a central junction four mains, each 10,000 ft. long, lead to four reservoirs, A, B, C, D, the water-levels in A, B, C being 600, 400, and 200 ft., respectively, above that in D. If the diameter of each main is 12 ins., find (a) the effective head at the junction and the velocities of flow. If the velocity in each main is 5 ft. per sec., find (b) the effective head at the junction and the diameters of the mains.

Ans. (a) 300 ft.; 8.66 f/s in highest and lowest mains; 5 f/s in intermediate mains.

(b) 300 ft.; 4 ins. for highest and lowest mains; 12 ins. for intermediate mains.

CHAPTER III.

FLOW OF WATER IN OPEN CHANNELS.

1. Channel-flow Assumptions.—A transverse section of the water flowing in an open channel may be supposed to consist of an infinite number of elementary areas representing the sectional areas of fluid filaments or stream-lines. The velocities of these stream-lines are very different at different points of the same transverse section, and the distribution of the pressure is also of a complicated character. Generally speaking, the side and bed of a channel exert the greatest retarding influence on the flow, and therefore along these surfaces are to be found the stream-lines of minimum velocity. The stream-lines of maximum velocity are those farthest removed from retarding influences. If the stream-line velocities for any given section are plotted, a series of equal velocity-curves may be obtained. In a channel of symmetrical section



FIG. 124.

the depth of the stream-line of maximum velocity below the water-surface is less than one fourth of the depth of the water, while the mean velocity-curve cuts the central vertical line at

a point below the surface about three fourths of the depth of the water.

In the ordinary theory of flow in open channels the variation of velocity from point to point in a transverse section is disregarded, and it is assumed that all the stream-lines are sensibly parallel and move normally to the section with a common velocity equal to the mean velocity of the stream. With this assumption, it also necessarily follows that the distribution of pressure over the section is in accordance with the hydrostatic law.

Again, it is assumed that the laws of fluid friction already enunciated are applicable to the flow of water in open channels. Thus the resistance to flow is proportional to some function of the velocity ($F(v)$), to the area (S) of the wetted surface, is independent of the pressure, and may be expressed by the term $S \cdot F(v)$. An obvious error in this assumption is that v is the *mean* velocity of the stream and not the velocity of the stream-lines along the bed and sides of the channel. In practice, however, the errors in the formulæ based upon these imperfect hypotheses are largely neutralized by giving suitable values to the coefficient of friction (f).

When a constant volume (Q) of water feeds a channel of given form, the water assumes a definite depth, a permanent régime is said to be established and the flow is *steady*. If the transverse sectional area (A) is also constant, then, since $Q = vA$, the velocity v is constant from section to section and the flow is said to be uniform. Usually the sectional area A is variable and therefore the velocity v also varies, so that the motion is steady with a varying velocity. Any convenient short stretch of a channel, free from obstructions, may be selected and treated, without error of practical importance, as being of a uniform sectional area equal to that of the mean section for the whole length under consideration.

2. Steady Flow in Channels of Constant Section (A).—The flow is evidently uniform; and since A is constant, the

depth of the water is also constant, so that the water-surface is parallel to the channel-bed.

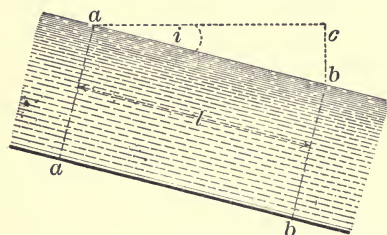


FIG. 125.

Consider a portion of the stream, of length l , between the two transverse sections aa , bb .

Let i be the inclination of the bed (or water-surface) to the horizon.

Let P be the length of the wetted perimeter of a cross-section.

Then, since the motion is uniform, the external forces acting upon the mass between aa and bb in the direction of motion must be in equilibrium.

These forces are:

(1) The component of the weight of the mass, viz.,

$$wAl \sin i = wAli = wAl \frac{h}{l} = wAh,$$

h being the fall of level in the length l .

NOTE.—When i is small, as is usually the case in streams,

$$\frac{h}{l} = \tan i = \sin i = i, \text{ approximately.}$$

(2) The pressures upon the areas aa and bb , which evidently neutralize each other.

(3) The frictional resistance developed by the sides and bed, viz.,

$$P \cdot l \cdot F(v).$$

Hence

$$wAh - PlF(v) = 0,$$

or

$$\frac{F(v)}{w} = \frac{Ah}{Pl} = mi,$$

m being the hydraulic mean depth.

It now remains to determine the form of the function $F(v)$.

In ordinary English practice it is usual to take

$$\frac{F(v)}{w} = f \frac{v^2}{2g},$$

f being the coefficient of friction. Then

$$f \frac{v^2}{2g} = mi,$$

or

$$v = \sqrt{\frac{2g}{f}} \sqrt{mi} = c \sqrt{mi},$$

c being a coefficient whose value depends upon the roughness of the channel surface and upon the form of its transverse section.

The total head H in a stream is made up of two parts, the one being utilized in producing the velocity of flow and the other being absorbed in frictional resistance. Thus

$$H = \frac{v^2}{2g} + \frac{l}{m} \frac{F(v)}{w}.$$

In long channels and in rivers in which the slope of the bed does not exceed 3 ft. per mile the term $\frac{v^2}{2g}$ is very small as compared with $\frac{l}{m} \frac{F(v)}{w}$ and may be disregarded without sensible error. In this case

$$H = \frac{l}{m} \frac{F(v)}{w}.$$

EX. 1. A channel of regular trapezoidal section, with banks sloping at 30° to the vertical, has a bottom width of 8 ft., and a width of 16 ft. at the free surface. It conveys 288 cu. ft. of water per sec., and the fall is 1 in 2000. Find the mean depth, the mean velocity of flow, and the coefficients f and c .

Depth of waterway $= 4 \tan 60^\circ = 4\sqrt{3}$ ft.

$$A = \frac{1}{2}(8 + 16)4\sqrt{3} = 48\sqrt{3} \text{ sq. ft.}$$

$$P = 8 + 2 \times 4\sqrt{3} \sec 30^\circ = 24 \text{ ft.}$$

Therefore the mean depth $= \frac{A}{P} = 2\sqrt{3} = 3.464$ ft.

The mean velocity of flow $= \frac{288}{48\sqrt{3}} = 2\sqrt{3} = 3.464$ ft. per sec.

Hence

$$2\sqrt{3} = \sqrt{\frac{64}{f}} \times \sqrt{2\sqrt{3} \times \frac{1}{2000}} = c \sqrt{2\sqrt{3} \times \frac{1}{2000}}.$$

Therefore

$$f = .009237 \quad \text{and} \quad c = 82.63.$$

Ex. 2. How much water is conveyed away by a horizontal trench 10 ft. wide, the depth of the water at entrance being 5 ft., and the surface falling 1 ft. in 2400 feet? (Take $f = .008$.)

Area at upper end = 50 sq. ft.; at lower end = 40 sq. ft.;

$$\text{the mean area} = \frac{1}{2}(40 + 50) = 45 \text{ sq. ft.}$$

Therefore, if Q cu. ft. are conveyed,

$$\frac{Q}{50} = \text{velocity at upper end}; \quad \frac{Q}{40} = \text{that at lower end};$$

and

$$\text{mean velocity} = \frac{Q}{45}.$$

The wetted perimeter = 20 ft. at upper end; = 18 ft. at lower end; and

$$\text{mean wetted perimeter} = \frac{1}{2}(20 + 18) = 19 \text{ ft.}$$

Thus the hydraulic mean depth $m = \frac{45}{19}$.

Hence

$$1 + \left(\frac{Q}{50}\right)^2 \frac{1}{64} = \left(\frac{Q}{40}\right)^2 \frac{1}{64} + \frac{.008 \times 2400}{\frac{45}{19}} \left(\frac{Q}{45}\right)^2 \frac{1}{64},$$

and

$$Q = 389 \text{ cu. ft. per sec.}$$

3. Retarding Effect of Air, etc.—The retarding effect of the air upon the free surface of a river or of the water in a canal or in any channel has not yet been accurately determined. It may be assumed that the resistance per unit of free surface

due to the air is about *one tenth* of the resistance due to similar units at the bottom and sides of smooth channels. Thus if X is the width of the free surface in a smooth channel, the wetted perimeter becomes $P + \frac{X}{10}$.

In general, the wetted perimeter may be expressed in the form $P + \frac{X}{\beta}$, β being 10 for smooth channels and greater than 10 for rough channels. The value of β is evidently diminished by opposing winds and increased by following winds.

Again, in the formula

$$mi = \frac{F(v)}{w},$$

$m \left(= \frac{A}{P} \right)$ and $i \left(= \frac{h}{l} \right)$ are similarly related in the determination of v , the mean velocity of flow. If v is constant, the product mi must also be constant, so that if m increases i must diminish, and *vice versa*. Thus in a very flat country the flow may be maintained by making m sufficiently large, while, again, if the channel-bed is steep m is small.

The erosion caused by a watercourse increases with the rapidity of flow. At the same time the sectional area (A) of the waterway also increases, so that the velocity of flow v diminishes. Thus there is a tendency to approximate to a "permanent régime" when the resistance to erosion balances the tendency to scour.

Hence, throughout any long stretch of a river passing through a specific soil, the mean velocity of flow will be very nearly constant if the amount of flow (Q) does not vary. Generally speaking, the volume conveyed by a river increases from source to mouth on account of the additions received from tributaries, etc. Since Q increases, A must also increase; and if mi or v is to remain constant, i must diminish. It is to be observed that the surface slopes of large rivers diminish gradually from source to mouth.

For a given discharge (Q) the mean depth (m) diminishes as i increases, and, as the cost of constructing a canal is approximately proportional to the mean depth, it is advisable to give the bed as large a slope as possible. But the velocity of flow (v) also increases with i , and the slope must therefore not exceed that for which v would be so great as to cause the erosion of the banks. On the other hand, v must not be so small as to allow of the growth of aquatic plants or of the deposition of sand, gravel, and other detritus, which would soon obstruct the waterway and add a considerable item to the cost of maintenance. Between these extreme limits the slope may be varied in any required manner, the controlling influences being the configuration of the ground and the nature of the soil through which the canal passes. In every case a careful determination should be made of the best combination of the three elements v , i , and A which would give a specified discharge. In France the canal beds have slopes varying from $1\frac{1}{2}$ to 20 in 10,000, and the magnitudes of both v and i may be considerable when the canal passes through rock or through a well-compacted material capable of resisting erosion. According to Belgrand the value of v for water carrying fine particles of loam should exceed 1 ft. (.25 m.) per second, and should not be less than 2 ft. (.5 m.) per second if the waters are laden with coarse particles of loam or sand. In clear water, the growth of weeds, etc., which would seriously interfere with the flow, is prevented if the velocity of flow is from 2 to 3 ft. (.5 m. to .8 m.) per second.

The slope of an *aqueduct*, in which no trouble is to be anticipated from plant-growth, may be as small as 3 in 10,000, and may even fall to 1 in 10,000 when the waters are exceptionally clear, as in the case of the aqueducts on the Dhuis and Vanne. On the other hand, the slope should rarely, if ever, exceed 12 in 10,000, and as a general rule the slope should be less than 10 in 10,000. The ordinary channel formula, viz., $v = c \sqrt{mi}$, is applicable to the flow in a *conduit*, so long as the

conduit does not run full, and since v is proportional to \sqrt{m} it is a maximum for some definite depth of water. When the water fills the conduit, the formula for channel-flow ought to change suddenly so as to agree with that for pipe-flow, and in this respect the theory is therefore imperfect. The mean velocity of flow in a conduit should not be less than about 2 ft. (0.5 m.) per second, and may be as great as $5\frac{3}{4}$ ft. (1.5 m.) per second. High velocities enable the waters to carry off floating débris and sand particles. There should be no sudden changes of slope or of section, as they favor the formation of eddies and the deposition of detritus.

The following table of slopes and mean velocities is taken from the article by Daries in the *Encycl. Sc. des Aide-Mémoire*:

	Slope in 10,000.	Mean Velocity per Second.		Nature of Canal Sides.
		Feet	Metres.	
Craponne Canal	10	Alluvial soil
Marseille "	3 to 7	3.3	1	Earth and rock
Carpentras "	2 to 4	2.53 to 6.56	.77 to 2	Vegetable soil, fissured limestone
Saint-Martory Canal .	5.4	1.64	.5	Clay
Verdon Canal	1.5 to 2	2.5	.76	Calcareous rocks
Neste "	2	1.64	.5	Clay, pudding-stone, rock
Beaucaire Canal	2 to 1	1.21	.37	Alluvial soil
Laroche "	2	1.67	.51	Calcareous rocks
I'Ourcq "	1.2	1.31	.4	Earth
Dhuis Aqueduct	1	1.18	.36	Millstone-grit masonry with a $\frac{2}{3}$ -inch (= .02 m.) facing in cement
Avre Aqueduct	3	3.3	1	Millstone-grit masonry with a $\frac{2}{3}$ -inch (= .02 m.) facing in cement
Naples Aqueduct	5	2.95	.9	Rubble masonry with a $\frac{1}{8}$ -in. (= .015 m.) facing in cement
Montpellier Aqueduct	2.5	2.62	.8	Rubble masonry with a $\frac{1}{8}$ -in. (= .015 m.) facing in cement
Croton Aqueduct	2.1	2.36	.72	

4. **On the Form of the Section of a Channel.**—The fundamental formulæ governing the form of the transverse section of a channel are

$$Q = Av$$

and

$$v^2 = c^2 m i = c^2 \frac{A}{P} i.$$

Therefore, also,

$$Pv^3 = c^3 Qi.$$

For channels of the *same slope*

$$v^3 \propto m.$$

Take $v^3 = am$, a being some constant.

Then, if dv is a small change in the velocity corresponding to a small change dm in the hydraulic mean depth,

$$2v \cdot dv = a \cdot dm,$$

and therefore

$$\frac{dv}{v} = \frac{dm}{2m}.$$

Thus the hydraulic mean depth must be changed 20 per cent to produce a change of 10 per cent in the velocity.

Again,

$$Q \propto Pv^3.$$

But P increases with Q , and therefore Q increases more rapidly than v^3 . For example, an increase in the velocity of less than $3\frac{1}{3}$ per cent will cause an increase of 10 per cent in the discharge.

For channels giving the *same discharge*

$$Pv^3 \propto i.$$

For a given volume of water there must be a sensible change in the slope to produce an appreciable change in the

velocity of flow, although, generally speaking, the wetted perimeter (P) diminishes or increases as i increases or diminishes, and thus v and therefore $\frac{1}{A^3}$, increases or diminishes more rapidly than i . An increase of 10 per cent in the velocity causes a diminution of about 4 per cent in the sectional area of the waterway.

For channels of the *same slope* and giving the *same discharge* Pv^3 and also $\frac{A^3}{P}$ are constant. A further condition is required before the sectional area can be determined.

PROBLEM I. A canal of rectangular section and of width x is to convey water of depth y with the condition that *either* the sectional area (A) of the waterway is to be a constant quantity *or* the wetted perimeter (P) is to be a minimum. It is proposed to find the relation between x and y so that (a) the velocity of flow may be a maximum, (b) the quantity of flow may be a maximum.

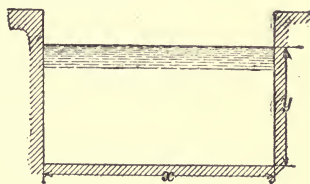


FIG. 126.

$$v \propto \sqrt{m} \propto \sqrt{\frac{A}{P}},$$

and

$$Q = Av \propto \sqrt{\frac{A^3}{P}}.$$

If v is a maximum,

$$d\left(\frac{A}{P}\right) = 0 = \frac{P \cdot dA - A \cdot dP}{P^2}.$$

If Q is a maximum,

$$d\left(\frac{A^3}{P}\right) = 0 = \frac{3A^2 \cdot PdA - A^3 \cdot dP}{P^2}.$$

In each case, if $dA = 0$, i.e., if the area is constant, then

$$dP = 0;$$

and if $dP = 0$, i.e., if the wetted perimeter is a minimum, then

$$dA = 0.$$

Thus the same results are obtained for the problem in its different conditions.

Now

$$A = xy \quad \text{and} \quad P = x + 2y.$$

Therefore

$$dA = y \cdot dx + x \cdot dy = 0,$$

and

$$dP = dx + 2 \cdot dy = 0.$$

Hence

$$y = \frac{x}{2}.$$

Therefore, also,

$$A = \frac{x^2}{2}, \quad P = 2x,$$

$$m = \frac{A}{P} = \frac{x}{2},$$

$$v = c \sqrt{\frac{xi}{2}},$$

and

$$Q = \frac{x^2}{2} v = \frac{c \sqrt{i}}{2 \sqrt{2}} x^{\frac{5}{2}}.$$

A suitable value for c corresponding to the slope i or to the value of $m \left(= \frac{x}{2} \right)$ can be obtained from the Tables of Bazin, Kutter, or Manning at the end of the chapter.

PROBLEM II. The section is usually in the form of a quadrilateral, the non-parallel sides sloping at an angle, θ ,

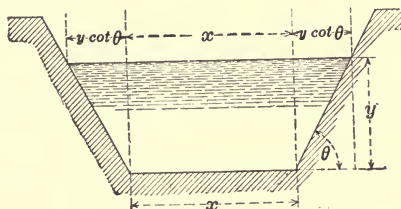


FIG. 127.

depending upon the nature of the soil through which the channel passes.

For example, in a canal

- with retaining walls $\theta = 63^\circ 36'$,
- with stiff earthen sides, faced, $\theta = 45^\circ$,
- with stiff earthen sides, unfaced, $\theta = 33^\circ 41'$,
- with sides in light or sandy soils $\theta = 26^\circ 34'$.

In such a channel let x be the bottom width and y the depth of the water. Then, the remaining conditions being the same as those in Problem I, it again follows that

$$dA = 0 \quad \text{and} \quad dP = 0.$$

But

$$A = y(x + y \cot \theta) \quad \text{and} \quad P = x + 2y \operatorname{cosec} \theta.$$

First. If θ is given,

$$dA = 0 = y \cdot dx + (x + 2y \cot \theta) dy,$$

and

$$dP = 0 = dx + 2 \operatorname{cosec} \theta \cdot dy.$$

Therefore

$$2y \operatorname{cosec} \theta = x + 2y \cot \theta,$$

or

$$x \sin \theta = 2y(1 - \cos \theta),$$

or

$$\tan \frac{\theta}{2} = \frac{x}{2y}.$$

The section may be easily sketched, as in Figs. 128 and 129.

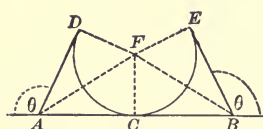


FIG. 128.

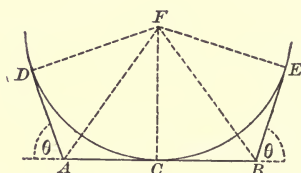


FIG. 129.

From the middle point C of AB , the bottom width, draw CF at right angles to AB and equal in length to the depth of the water. Then

$$\frac{AB}{CF} = 2 \tan \frac{\theta}{2},$$

θ being the given slope of the sides.

With F as centre and FC as radius describe a circle. From the points A and B draw tangents to touch this circle at D and E . FA evidently bisects the angle CAD . Therefore

$$\tan \frac{CAD}{2} = \tan CAF = \frac{CF}{AC} = \frac{CF}{\frac{1}{2}AB} = \cot \frac{\theta}{2}.$$

Hence $\pi - CAD = \theta$, and AD , BE have the slope required.

Again,

$$\begin{aligned} A &= y \left(2y \frac{1 - \cos \theta}{\sin \theta} + y \cot \theta \right) \\ &= y^2 \frac{2 - \cos \theta}{\sin \theta}, \end{aligned}$$

or

$$y = \sqrt{\frac{A \sin \theta}{2 - \cos \theta}}$$

and

$$\begin{aligned} P &= 2y \frac{1 - \cos \theta}{\sin \theta} + 2y \operatorname{cosec} \theta \\ &= 2y \frac{2 - \cos \theta}{\sin \theta} = \frac{2A}{y}. \end{aligned}$$

Therefore

$$m = \frac{A}{P} = \frac{y}{2},$$

$$v = \frac{c}{\sqrt{2}} \sqrt{yi},$$

and

$$Q = \frac{cA}{\sqrt{2}} \sqrt{yi},$$

the coefficient c being obtained from the tables.

The following Table gives the best relative values, *per unit of area*, of x , y , m , and P , corresponding to specified values of θ , and the actual values may be obtained by multiplying those of the Table by \sqrt{A} :

θ	x	y	m	P
90°	1.414	.707	.3535	2.828
60°	.877	.760	.380	2.633
45°	.613	.740	.370	2.706
40°	.525	.722	.361	2.772
36° 52'	.471	.707	.3535	2.828
35°	.439	.697	.3485	2.870
30°	.336	.664	.332	3.012
26° 34'	.300	.636	.318	3.144

The above values cannot always be exactly adopted in actual practice. The character of the soil, the importance of preventing excessive filtration, and the difficulties of construc-

2 to 1 slope.

tion and maintenance, often render it necessary to insure that the depth of the water shall not exceed a certain limit, say 8 to 12 ft. (2 m. to 3 m.). In France the depth of irrigation-canals is between 4 and $6\frac{1}{2}$ ft. (1.2 m. and 2 m.).

Second. If the bottom width x is fixed, then

$$dA = 0 = (x + 2y \cot \theta) dy - y^2 \operatorname{cosec}^2 \theta \cdot d\theta$$

and

$$dP = 0 = 2 \operatorname{cosec} \theta \cdot dy - 2y \frac{\cos \theta}{\sin^2 \theta} \cdot d\theta.$$

Hence

$$\frac{x + 2y \cot \theta}{2 \operatorname{cosec} \theta} = \frac{y}{2 \cos \theta},$$

or

$$x \sin \theta \cos \theta = -y(2 \cos^2 \theta - 1),$$

or

$$x \frac{\sin 2\theta}{2} = -y \cos 2\theta,$$

and therefore

$$\tan (\pi - 2\theta) = -\tan 2\theta = \frac{2y}{x}.$$

It may be observed that as the width (x) of the bottom increases, θ also increases.

If the width is *nil*, then $\tan 2\theta = \infty$ and $\theta = 45^\circ$, so that the triangular section of minimum perimeter is a semi-square.

Third. If the depth y is fixed, then

$$dA = 0 = y dx - y^2 \operatorname{cosec}^2 \theta \cdot d\theta$$

and

$$dP = 0 = dx - 2y \frac{\cos \theta}{\sin^2 \theta} \cdot d\theta.$$

Therefore

$$y = \frac{y}{2 \cos \theta},$$

or

$$\cos \theta = \frac{1}{2} \quad \text{and} \quad \theta = 60^\circ.$$

PROBLEM III. To find the proper sectional form of a channel of bottom width $2a$ so that the mean velocity of flow may be constant for all depths of water.

Let x, y , Fig. 130, be the co-ordinates of any point P in the profile referred to the middle point O of AB , the bottom width, as origin, and let s be the length of AP .

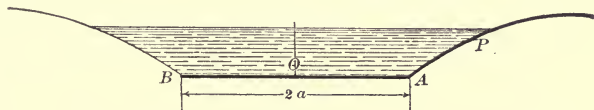


FIG. 130.

Since v is to be constant, m must also be constant, and therefore

$$\frac{A}{P} = \frac{\int y \cdot dx}{s + a} = \text{a const.} = m,$$

which may be written

$$\int y \cdot dx = m(s + a).$$

Differentiating,

$$y \cdot dx = m \cdot ds = m(dx^2 + dy^2)^{\frac{1}{2}},$$

and therefore

$$\frac{dx}{m} = \frac{dy}{(y^2 - m^2)^{\frac{1}{2}}}.$$

Integrating,

$$\frac{x}{m} = \log_e (y + \sqrt{y^2 - m^2}) + c,$$

c being a constant of integration.

But $y = a$ when $x = 0$, and $\therefore 0 = \log_e (a + \sqrt{a^2 - m^2}) + c$.

$$\text{Hence } \frac{x}{m} = \log_e \frac{y + \sqrt{y^2 - m^2}}{a + \sqrt{a^2 - m^2}} = \log_e \frac{y + \sqrt{y^2 - m^2}}{b},$$

where

$$b = a + \sqrt{a^2 - m^2}.$$

$$\text{Therefore } y + \sqrt{y^2 - m^2} = be^{\frac{x}{m}}.$$

Hence, too,

$$y - \sqrt{y^2 - m^2} = \frac{m^2}{b} e^{-\frac{x}{m}}.$$

Adding together the last two equations,

$$2y = b e^{\frac{x}{m}} + \frac{m^2}{b} e^{-\frac{x}{m}},$$

or

$$y = \frac{1}{2b} \left(b^2 e^{\frac{x}{m}} + m^2 e^{-\frac{x}{m}} \right),$$

which is the equation to the required profile, and is a curve which belongs to the class of catenaries and which evidently flattens out very rapidly.

If the bottom width is such that

$$a = m = b,$$

the equation becomes

$$y = \frac{m}{2} \left(e^{\frac{x}{m}} + e^{-\frac{x}{m}} \right)$$

and the profile is a true catenary of parameter m , with its axis coincident with the bottom and its directrix coincident with the vertical at the middle of the section.

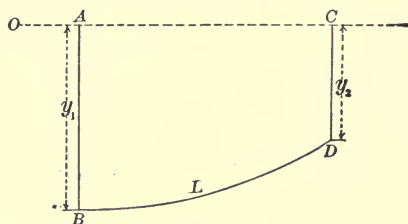


FIG. 131.

PROBLEM IV. A channel of given slope has a given surface width AC , vertical sides AB ($= y_1$) and CD ($= y_2$) of given depths, and a curved bed BD ($= L$) of given length.

The amount and velocity of flow in the channel will be a maximum when the form of the bed BD is a circular arc. This can be easily proved as follows:

Since the slope is constant, $v \propto \sqrt{m} \propto \sqrt{\frac{A}{P}}$.

But $P (= L + y_1 + y_2)$ is a constant quantity, and therefore v and also Q will be a maximum when A is a maximum.

Hence, too, the area between the chord BD and the curve must be a maximum, and therefore the curve must be a circular arc. The proof of this by the Calculus of Variations is as follows:

Take O in CA produced as the origin, OC as the axis of x , and the vertical through O as the axis of y . Then

$A = \int_{x_1}^{x_2} y dx$ is to be a maximum.

$$L = \int_{x_1}^{x_2} \frac{ds}{dx} dx = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_{x_1}^{x_2} \sqrt{1 + p^2} dx$$

is a given quantity, OA being $= x_1$, $OC = x_2$, and $\frac{dy}{dx} = p$.

Let $V = y + a \sqrt{1 + p^2}$, a being some constant.

Then

$$\int_{x_1}^{x_2} V \cdot dx \text{ is to be a maximum,}$$

and therefore

$$V = p \frac{dV}{dp} + c_1;$$

that is,

$$y + a \sqrt{1 + p^2} = \frac{ap^2}{\sqrt{1 + p^2}} + c,$$

and thus

$$y + \frac{a}{\sqrt{1 + p^2}} = c_1.$$

Therefore

$$\frac{dx}{dy} = \frac{1}{p} = \frac{c_1 - y}{\sqrt{a^2 - (c_1 - y)^2}}.$$

Integrating,

$$x + c_2 = \sqrt{a^2 - (c_1 - y)^2},$$

the equation to a circle of radius a .

Hence the profile BD is a circular arc.

The maximum depth of the channel is $c_1 - a$.

The constants c_1 , c_2 , a can be found from the three conditions that the arc is of given length and has to pass through the two fixed points B and D .

PROBLEM V. *The Semicircular Channel*.—Theoretically, the best form of channel for a given waterway is one in which the bed is a circular arc (Prob. IV), as the wetted perimeter is then a minimum and the mean depth (or radius) a maximum.

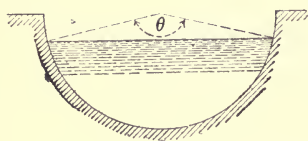


FIG. 132.

In the semicircular channel, Fig. 132, let the free surface subtend an angle θ at the centre.

Then

$$A = \frac{r^2}{2}(\theta - \sin \theta) = \frac{r^2}{2}\theta\left(1 - \frac{\sin \theta}{\theta}\right)$$

and

$$P = r\theta,$$

r being the radius.

Therefore

$$m = \frac{A}{P} = \frac{r}{2}\left(1 - \frac{\sin \theta}{\theta}\right).$$

Hence, since $mi = bv^2 = b\frac{Q^2}{A^2}$,

$$r^5\theta^2\left(1 - \frac{\sin \theta}{\theta}\right)^3 i = 8bQ^2.$$

If the channel runs full, $\theta = \pi$, and then

$$r^5\pi^2 i = 8bQ^2.$$

As a first approximation it may be assumed that

for small channel sections with cement faces..... $b = .00022$

“ channels of mean dimensions with smooth faces $b = .00017$

“ channels of large dimensions..... $b = .00011$

In metric measure these coefficients become .0004, .0003, and .0002, respectively.

Miscellaneous Problems. — The bed of the aqueduct at Naples is semi-elliptic, but beds in the form of a semi-ellipse, a cycloid, a parabola, or an hyperbola, would only be adopted under very exceptional conditions, as when a curved profile is required with a limited depth. The waterway and the wetted perimeter can, of course, be approximately calculated from the known properties of these curves.

For the *semi-elliptic section*, if a and b are the semi-major and minor axes,

$$A = \pi\frac{ab}{2},$$

and

$$P = \frac{\pi}{2} \frac{a}{b}(a + b)v,$$

$$\left\{ \begin{array}{l} \text{where } v = 1, \quad 1.0025, \quad 1.01, \quad 1.0226, \quad 1.0404, \quad 1.0635, \quad 1.0922, \quad 1.1267, \quad 1.1677, \quad 1.2155; \\ \text{when } \frac{a-b}{a+b} = 0, \quad .1, \quad .2, \quad .3, \quad .4, \quad .5, \quad .6, \quad .7, \quad .8, \quad .9. \end{array} \right\}$$

For the *cycloidal section*, if r is the radius of the generating circle,

$$A = 3\pi r^2 \quad \text{and} \quad P = 8r.$$

Therefore

$$m = \frac{8}{3}\pi r,$$

and the flow equation becomes

$$8bQ^2 = 27r^5i\pi^3.$$

If the water-line is at AA , defined by the angle θ which the radius OA of the generating circle makes with the vertical, then

$$\begin{aligned} &= 3\pi r^2 - r^2 \left(3\theta - 4 \sin \theta + \frac{\sin 2\theta}{2} \right) \\ &\quad - 2r^2(1 - \cos \theta)(\pi - \theta + \sin \theta) \\ &= r \left(\pi - \theta + 2 \sin \theta + 2\pi \cos \theta - 2\theta \cos \theta + \frac{\sin 2\theta}{2} \right) \end{aligned}$$

and

$$P = 8r \cos \frac{\theta}{2}.$$

5. Aqueducts.—The aqueduct of the ancients was of rectangular section and was sometimes of very large dimensions as compared with the volume of water to be conveyed. Although in modern times there are examples of rectangular sections, it is now more usual to make them circular, egg-shaped, square with a diagonal vertical, or trapezoidal. Aqueducts are also constructed of forms which are combinations of the circle and egg-shaped, or of the trapezoid and circle. When a mean volume of water is to be conveyed and when provision has to be made for a definite height, as, for example, for a man standing upright, preference is given to the egg-shaped aqueduct.

In the sections shown by Figs. 133 to 137 it will be observed that a rise of the water-line near the top causes an

appreciable increase in the wetted perimeter, while there is no proportional increase in the waterway. Thus the mean depth (m) and therefore also the mean velocity (v) of flow continually diminish. The *à priori* conclusion may be drawn that the discharge (Q) is not a maximum when the pipe runs full, but when the water-line is some distance below the top. The

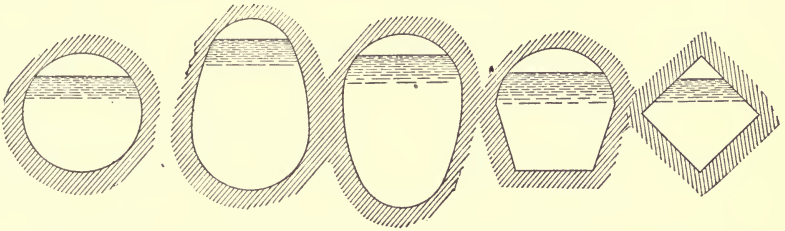


FIG. 133.

FIG. 134.

FIG. 135.

FIG. 136.

FIG. 137.

differential equation defining this position may be easily found as follows (Prob. 1, p. 229):

$$b \frac{Q^2}{A^2} = bv^2 = mi = \frac{A}{P}i.$$

Therefore

$$Q^2 = \frac{A^3}{P} \frac{i}{b}.$$

Since Q is to be a maximum,

$$dQ = 0.$$

Therefore

$$d\left(\frac{A^3}{P}\right) = 0 = \frac{3PA^2 \cdot dA - A^3 \cdot dP}{P^2},$$

or

$$3P \cdot dA - A \cdot dP = 0$$

is the equation required.

If the velocity of flow is to be a maximum,

$$dv = 0,$$

and therefore

$$dm = 0 = d\left(\frac{A}{P}\right) = \frac{P \cdot dA - A \cdot dP}{P^2},$$

or

$$P \cdot dA - A \cdot dP = 0.$$

EX. I. *Circular Section*.—Let the wetted perimeter subtend an angle θ at the centre. Then

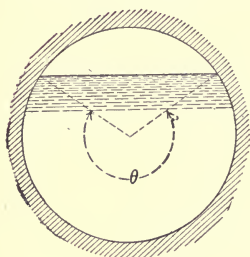


FIG. 138.

$$A = \frac{r^2}{2}(\theta - \sin \theta) \text{ and } dA = \frac{r^2 \cdot d\theta}{2}(1 - \cos \theta);$$

$$P = r\theta \text{ and } dP = r \cdot d\theta.$$

Hence for a *maximum discharge*

$$\frac{3r^3}{2} \cdot d\theta \cdot \theta(1 - \cos \theta) - \frac{r^3}{2}(\theta - \sin \theta) \cdot d\theta = 0,$$

or

$$2\theta - 3\theta \cos \theta + \sin \theta = 0.$$

$\theta = 308^\circ$ is the value of θ satisfying this equation.

For a *maximum velocity*

$$\frac{r^3}{2} d\theta \cdot \theta(1 - \cos \theta) - \frac{r^3}{2}(\theta - \sin \theta) = 0,$$

or

$$\theta = \tan \theta,$$

and $\theta = 257^\circ 27'$ is the value of θ which satisfies this equation.

In circular aqueducts the angle θ is usually about 240° , which insures a certain clear space above the water-line.

Then, also,

$$P = 4.189r; \quad A = 2.528r^2; \quad m = .6r.$$

EXAMPLE 2. *A Square Section with Vertical Diagonal.*

Let a side of the square = a ,
and let x be the length of the
portion of the side which is not
wetted. Then

$$A = a^2 - \frac{x^2}{2}$$

and

$$dA = -x \cdot dx;$$

$$P = 4a - 2x$$

and

$$dP = -2 \cdot dx.$$

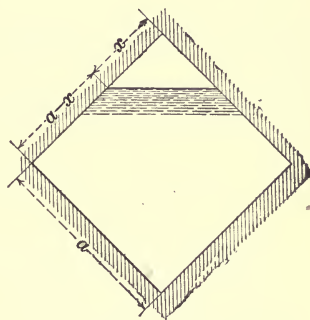


FIG. 139.

Hence for a *maximum discharge*

$$-3(4a - 2x)x \cdot dx + 2\left(a^2 - \frac{x^2}{2}\right)dx = 0,$$

or

$$5x^2 - 12ax + 2a^2 = 0.$$

Therefore

$$x = \frac{a}{5}(6 - \sqrt{26}) = .18a,$$

and the depth below the apex of the water-line

$$= \frac{x}{\sqrt{2}} = .1274a.$$

For a *maximum velocity of flow*

$$-x(4a - 2x)dx + 2\left(a^2 - \frac{x^2}{2}\right)dx = 0,$$

or

$$x^2 - 4ax + 2a^2 = 0,$$

and therefore

$$x = a(2 - \sqrt{2}) = .5858a,$$

and the depth of the water-line below the apex

$$= \frac{x}{\sqrt{2}} = .4142a.$$

EXAMPLE 3. *Egg-shaped Section*.—This form of aqueduct consists essentially of three parts, a lower portion bounded by a semicircle of radius r_1 , an upper portion bounded by a cir-

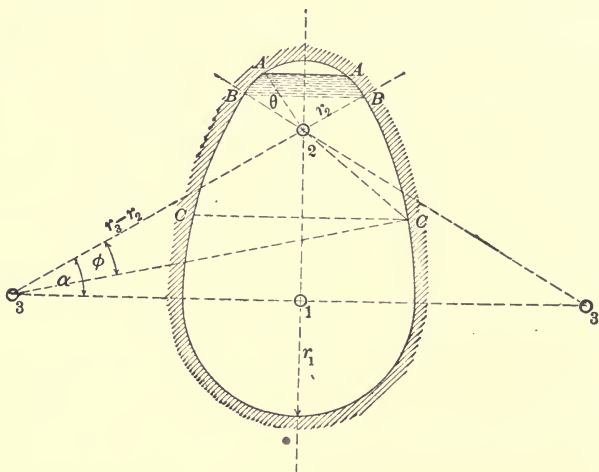


FIG. 140.

cular arc of lesser radius r_2 , and an intermediate portion bounded by circular arcs of radius r_3 , which meet the lower and upper arcs tangentially.

The depth of the intermediate portion is defined by the angle α which the radius O_3O_2 makes with the horizontal, and the position of the water-line AA is defined by the angle θ which O_2A makes with O_3O_2 produced. Then

If the water-line is *above* BB ,

$$A = \frac{\pi r_1^2}{2} + r_3^2 \alpha - (r_3 - r_1)(r_3 - r_2) \sin \alpha \\ + r_2^2 \theta + \frac{r_2^2}{2} \sin 2(\alpha + \theta),$$

and

$$P = \pi r_1 + 2r_3 \alpha + 2r_2 \theta.$$

If the water-line coincides with BB , $\theta = 0$, and then

$$A = \frac{\pi r_1^2}{2} + r_3^2 \alpha - (r_3 - r_1)(r_3 - r_2) \sin \alpha + \frac{r_2^2}{2} \sin 2\alpha$$

and

$$P = \pi r_1 + 2r_3 \alpha.$$

If z is the vertical distance between O_1 and the highest point,

$$z = r_2 + (r_3 - r_1) \sin \alpha.$$

Also,

$$r_3 - r_1 = (r_3 - r_2) \cos \alpha.$$

If the water-line CC is below BB , let ϕ be the angle subtended at O_3 by the arc BC , and let $O_2C = x$. Then, since B_2OC is now θ ,

$$x \sin (\theta - \phi) = (r_3 - r_2) \sin \phi$$

and

$$x \cos (\theta - \alpha) = r_3 \cos (\alpha - \phi) - (r_3 - r_1),$$

two equations giving x and ϕ in terms of θ and the radii.

The area of the waterway is now the area up to BB diminished by the area of the slice between BB and CC , and this area

$$= \frac{r_2^2}{2} \sin 2\alpha + r_3^2 \phi - r_3(r_3 - r_2) \sin \phi + \frac{x^2}{2} \sin 2(\theta - \alpha).$$

Hence

$$\begin{aligned}
 A &= \frac{\pi r_1^2}{2} + r_3^2 \alpha - (r_3 - r_1)(r_3 - r_2) \sin \alpha + \frac{r_2^2}{2} \sin 2\alpha \\
 &\quad - \left\{ \frac{r_2^2}{2} \sin 2\alpha + r_3^2 \phi - r_3(r_3 - r_2) \sin \phi + \frac{x^2}{2} \sin 2(\theta - \alpha) \right\} \\
 &= \frac{\pi r_1^2}{2} + r_3^2(\alpha - \phi) - (r_3 - r_2) \{ (r_3 - r_1) \sin \alpha - r_3 \sin \phi \} \\
 &\quad - \frac{x^2}{2} \sin 2(\theta - n).
 \end{aligned}$$

and

$$P = \pi r_1 + 2r_3(\alpha - \phi).$$

The larger diameter is usually at the bottom for aqueducts, but almost invariably at the top for sewers.

The discharge for sewers may be calculated by Bazin's formula, but an allowance of 20 per cent should be made in order to make provision for deposits and, where they occur, for water-pipes, electric conduits, etc. Care should also be taken that the section is sufficient to carry away the water from the heaviest rains and from the branch drains in such manner that the water in the sewer does not rise above a certain level.

Assuming that the time of flow in the sewer is three times that of the rainfall and that the maximum downfall is 27.5 gallons (= 125 litres) per second, Belgrand has proposed for the discharge of the Paris sewers the formula

$$S \times .188 = A \sqrt{mi},$$

S being the drainage area in acres.

In metric measure, S being the drainage area in hectares,

$$S \times .0239 = A \sqrt{mi}.$$

In branch drains and in smaller systems the influx of water is much more rapid and the time of flow should not be estimated at more than twice the duration of the rainfall.

NOTE.—In designing sections for open channels or aqueducts, complicated preliminary calculations may be generally avoided by employing a graphical method. Selecting a provisional section, the water areas and wetted perimeters may be obtained for different depths of water and the corresponding mean depths plotted to any convenient scale. Repeating these operations for different sections, the mean-depth curves will quickly indicate the best section to be adopted.

6. Formulæ of Prony, Eytelwein, Beardmore and Tadini.

—A careful study of Chezy's experiment on the Courpalet cut (Orleans canal) and of twenty-three experiments made by Dubuat on wooden channels of small section, led Prony, in 1804, to adopt the equation

$$\frac{F(v)}{w} = av + bv^2 = mi,$$

in which $\frac{1}{a} = 22472.5$ and $\frac{1}{b} = 10607.02$.

About the year 1815, Eytelwein, taking into account sixty additional experiments on the Rhine and Weser by Woltmann, Funk and Brunings, proposed slightly different values for a and b , viz.,

$$\frac{1}{a} = 41211.11 \quad \text{and} \quad \frac{1}{b} = 8975.43.$$

The expression mi has the same value with Prony's as with Eytelwein's coefficients when the velocity is about 72 ft. per minute, and for a small change in this velocity the variation in the value of mi is also small and of little practical importance. For other velocities the value of mi with Prony's coefficients will be greater or less than the value with Eytelwein's coefficients according as the velocity of flow is greater or less than 72 ft. per minute.

The formula with Eytelwein's coefficients was for a long

time used by engineers, and was preferred as giving the most reliable results.

For values of v exceeding 20 ft. per minute the term av is small as compared with bv^2 , and may be disregarded without much error. This formula then becomes

$$bv^2 = mi,$$

and therefore, according to Prony,

$$v = \frac{1}{\sqrt{b}} \sqrt{mi} = 103 \sqrt{mi},$$

and according to Eytelwein,

$$v = \frac{1}{\sqrt{b}} \sqrt{mi} = 95 \sqrt{mi}.$$

Intermediate between these is Beardmore's formula, viz.,

$$v = 100 \sqrt{mi}.$$

Barré de St. Venant has suggested the relation

$$mi = .000136v^{\frac{8}{3}}$$

(or $mi = .0004v^{\frac{8}{3}}$, if a metre is the unit).

The above formulæ, now obsolete, involve a grave error, as it is assumed that the resistance due to the roughness of the wetted surface is a constant quantity. Bazin's experiments have clearly shown that the resistance may vary between very wide limits depending upon the nature of the materials and soil which form the bed and sides of the channel. For a deep and wide channel, in which the slope of the bed is small, approximately accurate results are given by Tadini's formula,

$$v = 91 \sqrt{mi}$$

(or $v = 50 \sqrt{mi}$, if a metre is the unit).

7. Bazin's Formulæ.—Between 1855 and 1859, Darcy and Bazin carried out a number of experiments in a cut leading from the Bourgogne canal. The channel sections were of different forms and dimensions, the sides were faced with wood, cement, hewn ashlar, bricks, rubble masonry, and earth, and the slope of the beds varied from .001 to .10.

The results, for the rectangular and trapezoidal sections, sensibly agreed with the calculations obtained from the formula

$$mi = \left(\alpha + \frac{\beta}{m} \right) v^2,$$

but with circular and egg-shaped sections the calculated are about 10 per cent less than the actual results.

In practice it is most convenient to take

$$v = \frac{1}{\sqrt{b}} \sqrt{mi} = c \sqrt{mi},$$

where $b = \frac{1}{c^2} = \alpha + \frac{\beta}{m}$.

α and β are not constant, but have values depending upon the character of the channel faces and bed. Bazin gives the following table:

Character of the Wetted Surface.	Value of α , the Unit being		Value of β .
	A Foot.	A Metre.	
Smooth cement, planed wood, etc.....	.000046	.00015	.0000045
Cut masonry, bricks, planks.....	.000058	.00019	.0000133
Rubble masonry.....	.000073	.00024	.00006
Earth.....	.000085	.00028	.00035
Boulders (Kutter).....	.00012	.00040	.0007

Tables at the end of the chapter give the values of the coefficients b and c , a *metre* being the unit.

Reviewing the results of more than 700 experiments carried out in France, Europe, the United States, and British India,

etc., upon canals and rectangular, trapezoidal, semicircular, and circular aqueducts, of different dimensions, Bazin, in 1897, (*Ann. des Ponts et Chaussées*,) deduced the formula

$$v = \frac{157.6}{1 + \frac{\gamma}{\sqrt{m}}} \sqrt{mi}$$

(or $v = \frac{87}{1 + \frac{\gamma}{\sqrt{m}}} \sqrt{mi}$, if a metre is the unit).

This equation, again, may be most conveniently written in the form

$$v = c \sqrt{mi},$$

and Tables at the end of the chapter give the values of c for the six different classes into which Bazin has divided all channels, the corresponding values of the coefficient γ being given by the following table:

Class.	Character of the Wetted Surface.	γ , the Unit being	
		A Foot.	A Metre.
I.	Smooth cement, planed wood.....	.109	.06
II.	Planks, bricks, cut masonry, etc.....	.290	.16
III.	Rubble masonry.....	.833	.46
IV.	Earth, dry rubble, etc.....	1.540	.85
V.	Earthen channels in ordinary condition.....	2.355	1.30
VI.	Earthen channels or rivers, presenting exceptional resistance; the beds covered with boulders and the sides with grass, etc.....	3.170	1.7

8. Ganguillet and Kutter's Formula.—Bazin's is the only formula used in France, but in England, Germany, and the United States engineers prefer the formula of Ganguillet and Kutter, viz.,

$$v = c \sqrt{mi},$$

the value of c being given in a Table at the end of the chapter.

Also the coefficient

$$c = \frac{a + \frac{1}{n} + \frac{p}{i}}{1 + \left(a + \frac{p}{i}\right) \frac{n}{\sqrt{m}}}$$

a , l , and p being certain constants and n a coefficient depending only on the roughness of the channel sides and bed.

If the unit is a *foot*, $a = 41.6$; $l = 1.8112$; $p = .00281$.

If the unit is a *metre*, $a = 23$; $l = 1$; $p = .00155$.

The unit being a foot, n varies from .008 to .05 and the following table gives the values of n which will be found of most use in practice:

Character of Sides.	n	Authority.
Planed timber.....	.009	Ganguillet and Kutter
Smooth cement.....	.01	
A mixture of 2 of cement to 1 of sand.....	.011	
Rough planks.....	.012	
Ashlar or brickwork013	
Canvas on frames.....	.015	
Rubble masonry.....	.017	
Rivers and channels in very firm gravel.....	.02	
“ “ “ “ perfect order, free from detritus (stones, weeds, etc.).	.025	
“ “ “ “ moderately good order, not quite free from detritus or weeds03	
“ “ “ “ bad order, with weeds and detritus035	Jackson
Torrential streams encumbered with detritus....	.05	
Canals in earth above the average order.....	.0225	
“ “ “ in fair order.....	.025	
“ “ “ below the average order.....	.0275	
“ “ “ in rather bad order, overgrown with weeds and covered with detritus..	.03	

The difficulty of properly selecting the value of n is due to the fact that there is no absolute measure of the roughness of channel beds.

In obtaining the above results Ganguillet and Kutter made a careful study of:

(a) *The Experiments of Darcy and Bazin.*—These show that c depends both upon the roughness and the sectional dimensions. The values of α and β in Bazin's formula vary with the character of the channel sides and bed; but while in small channels the influence upon the flow of differences of roughness must be very great, it is certain that this influence diminishes as the sectional area increases, and that it will be nil when the area is infinitely great.

(b) *The Measurements of Humphreys and Abbot* on the Mississippi, a stream of very large sectional area with a bed of very small slope.

(c) *Their own Gaugings* in the regulated channels of certain Swiss torrents with exceptionally steep slopes and running through extremely rough channels.

(d) *The Effect of the Slope.*—The coefficient c diminishes as the slope, i , increases. The value of c does not vary much with the slope of the bed in small rivers, but in large rivers with small slopes the variation is considerable.

9. Formulæ of Manning, Tutton, Humphreys and Abbot, and Gauckler.—In 1890, Manning proposed the formula

$$v = c_1 m^{\frac{2}{3}} i^{\frac{1}{2}} = \frac{1.486}{n} m^{\frac{2}{3}} i^{\frac{1}{2}}, \text{ if the unit is a foot,}$$

or

$$v = c_1 m^{\frac{2}{3}} i^{\frac{1}{2}} = \frac{1}{n} m^{\frac{2}{3}} i^{\frac{1}{2}}, \text{ if the unit is a metre.}$$

In this formula, which gives good results, the coefficient n has the same value as the n in Kutter's formula.

Bazin's and Manning's formulæ are identical if

$$c \sqrt{mi} = \frac{1}{\sqrt{b}} \sqrt{mi} = c_1 m^{\frac{2}{3}} i^{\frac{1}{2}},$$

i.e., if

$$c = \frac{1}{\sqrt{b}} = c_1 m^{\frac{1}{6}}.$$

By an independent method, Tutton, in 1893, deduced the corresponding formula,

$$v = \frac{1.54}{n} m^{\frac{2}{3}} i^{\frac{1}{2}},$$

n being again the same as in Kutter's formula.

As a result of observations on the Mississippi in 1865, Humphreys and Abbot deduced the rather complicated formula

$$v = \{3.873(m'i^{\frac{1}{2}})^{\frac{1}{4}} - .0388\}^2, \text{ the unit being a foot,}$$

or

$$v = \{(69m'i^{\frac{1}{2}})^{\frac{1}{4}} - .0214\}^2, \text{ the unit being a metre.}$$

In this expression, which is of especial value for large watercourses, m' is the ratio of the sectional area to the *total* perimeter.

Gauckler's formulæ for canals,

$$v = cm^{\frac{4}{3}}i, \text{ if the slope is } < 7 \text{ per } 10000,$$

and

$$v = c'm^{\frac{2}{3}}i^{\frac{1}{2}}, \quad \text{“} \quad \text{“} \quad > 7 \text{ per } 10000,$$

and Hagen's formula,

$$v = 2.43m^{\frac{1}{2}}i^{\frac{1}{6}},$$

the unit in each case being a metre, have not been used in practice.

Ex. 1. A channel with a fall of 1 in 10,000 has brickwork faces, is of rectangular section, 20 ft. wide, and is to convey 200 cu. ft. of water per second. What must be the depth of the water?

Let x be the required depth. Then

$$A = 20x; \quad P = 20 + 2x,$$

and

$$m = \frac{10x}{10 + x}$$

Also,

$$\frac{200}{20x} = \frac{10}{x} = v = c \sqrt{\frac{10x}{10+x} \cdot \frac{1}{10000}} = \frac{c}{100} \sqrt{\frac{10x}{10+x}},$$

This equation can be best solved by trial.

Let $x = 5$ ft. Then

$$m = \frac{50}{15} = 3.33 \text{ ft.},$$

and the Tables give 127.2, as the corresponding value of c .

Therefore

$$v = \frac{127.2}{100} \sqrt{3.33} = 2.3223 \text{ ft. per sec.},$$

and $Q = 100v = 232.23 \text{ cu. ft. per sec.},$

which is too great.

Let $x = 4$ ft. Then

$$m = \frac{40}{14} = 2.85,$$

and the Tables give 126.4 as the corresponding value of c .

Therefore

$$v = \frac{126.4}{100} \sqrt{\frac{40}{14}} = 2.1365 \text{ ft. per sec.},$$

and $Q = 80v = 170.92 \text{ cu. ft. per sec.},$

which is too small.

Thus x must lie between 4 and 5 ft. Try $x = 4.5$ ft. Then

$$m = \frac{45}{14.5} = 3.1,$$

and the corresponding value of c is 126.8.

Therefore

$$v = \frac{126.8}{100} \sqrt{\frac{90}{29}} = 2.2338 \text{ ft. per sec.},$$

and $Q = 90v = 201.042 \text{ cu. ft.},$

which is very nearly correct. By further trials the depth can be obtained within a fraction of an inch.

Ex. 2. A canal in earth with sides sloping at 40° is to convey 100 cu. ft. of water per sec., at a velocity of 1 ft. per second. What is the fall of the canal, and what are its most suitable dimensions?

$A = 100$. Then (see Table, p. 233),

$$\text{bottom width} = .525 \sqrt[4]{100} = 5.25 \text{ ft.},$$

$$\text{depth of water} = .722 \sqrt[4]{100} = 7.22 \text{ ft.},$$

$$\text{mean depth, } m = .361 \sqrt[4]{100} = 3.61 \text{ ft.}$$

By the Tables the corresponding value of c is 93.3. Therefore

$$1 = 93.3 \sqrt[4]{3.61 \times i},$$

and

$$i = \frac{1}{31424}.$$

Ex. 3. A length of the La Roche cut is in compact rock. Its bottom width is 0.70 m., the depth of the water is 0.50 m., one bank is vertical and the other slopes at $26^\circ 34'$ to the vertical. If the fall is 1 in 500, find the mean velocity and quantity of flow.

The width of section at the surface = $.70 + .50 \tan 26^\circ 34' = 0^m.95$.

$$A = \frac{1}{2}(.95 + .70) \cdot 50 = 0.4125 \text{ sq. m.}$$

$$P = .50 + .70 + .50 \sec. 26^\circ 34' = 1.759 \text{ m.}$$

Therefore

$$m = \frac{.4125}{1.759} = .2345,$$

and the corresponding value of b in the Tables is .0423. Hence

$$.0423 \times v = \sqrt[4]{.2345 \times \frac{1}{500}} = .02165,$$

and

$$v = .512 \text{ m. per sec.}$$

Therefore, also,

$$Q = .512 \times .4125 = .2112 \text{ c.m. per sec.}$$

Again, using Bazin's formula for the filament of max. vel.,

$$v_{\max.} = v + 14 \sqrt{mi} = .512 + 14 \times .02165 = 0^m.815 \text{ per sec.},$$

and

$$v_b = \text{bottom velocity} = \frac{3}{5}(v_{\max.}) = 0^m.489 \text{ per sec.}$$

Ex. 4. In another length of La Roche cut, in earth, the banks slope at 45° , the bottom width is 0.3 m., and the depth of the water is 0.5 m. Find the coefficients b and c , the discharge being .2112 cu. ft. per second, and the fall 1 in 500.

$$A = \frac{1}{2}(1.3 + .5) \cdot 5 = 0.4 \text{ sq. m.}$$

$$P = .3 + 2\sqrt{.5} = 1.714.$$

Therefore $m = \frac{.4}{1.714} = .23337.$

Also, $v = \frac{.2112}{.4} = 0.528 \text{ per sec.}$

Hence

$$\begin{aligned} .528 &= c\sqrt{.2337\frac{1}{500}} = \frac{1}{\sqrt{b}}\sqrt{.2337\frac{1}{500}} \\ &= c \times .02162 = \frac{1}{\sqrt{b}} \times .02162 \end{aligned}$$

and $c = 24.4, \quad b = .00168,$

which closely agree with the results given by the Tables.

Ex. 5. Find the quantity of water conveyed by a channel of trapezoidal section lined with brickwork and having a fall of 6 in 1000. The water-surface width is 7.185 ft., the bottom width is 6.56 ft., and the depth of the water is 4.92 feet.

$$A = \frac{1}{2}(7.185 + 6.56) \times 4.92 = 33.813 \text{ sq. ft.,}$$

$$P = 6.56 + 2 \times 4.954 = 16.468 \text{ ft.}$$

Therefore $m = \frac{33.813}{16.468} = 2.053.$

Hence

$$Q = Av = c \times 33.813\sqrt{2.053 \times \frac{6}{1000}} = c \times 3.7528.$$

For $m = 2.053$

Bazin's Tables give $c = 124.6$, and then $Q = 467.6$ cu. ft. per sec.;

Manning's " " $c = 128.6$, and then $Q = 482.6$ " "

Kutter's " " $c = 130.4$, and then $Q = 489.36$ " "

and the differences in the three cases are not considerable.

10. **Variation of Velocity in the Transverse Section of a Watercourse.**—The discharge (Q) across any transverse section of a watercourse is the product of the area (A) of the section and the mean velocity (v) of flow. Thus

$$Q = Av.$$

The value of v for channels of small section can easily be found by discharging into a suitable reservoir for a definite interval of time, when Q can be estimated; and since A is known, v can be at once calculated. This method is impracticable with watercourses of large dimensions. The profile of the section must then be carefully plotted, when its area can be obtained with a planimeter or by the method of mean heights. The velocity of flow varies from point to point throughout the section in a most irregular manner, and its value has not been fixed by any single law. By using a meter or gauge the velocity may be measured at a large number of points, and in this manner the mean velocity (v) and the maximum velocity ($v_{\max.}$) can be very approximately determined. The velocity, however, varies so much and depends so largely upon the conditions under which the flow takes place, that it seems hopeless to expect that the complicated law of velocity distribution can be expressed in a general formula. The numerous experiments of Bazin on the Bourgogne canal and on the Seine and Saone, of Cunningham on the Ganges canal, and of Humphreys and Abbot on the Mississippi, all go to prove this and at the same time throw much light upon the whole subject. It has been shown that the ratio $\frac{v_{\max.}}{v}$ diminishes as the resistance of the sides and bed,

which is measured by the expression $\frac{mi}{v^2}$, increases. The ratio, for example, is about .85 in a channel with a very smooth surface and falls to about .50 when the channel is cut through

earth. As the surface resistance diminishes the value of $\frac{mi}{v^2}$ tends to become very small and ultimately zero, while the ratio $\frac{v_{\max.}}{v}$ tends to become *unity*. Bazin therefore expressed the relation between $v_{\max.}$ and v in the form

$$= 1 + F\left(\frac{mi}{v^2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which the function $F\left(\frac{mi}{v^2}\right)$ vanishes with $\frac{mi}{v^2}$.

A special case is Bazin's empirical formula,

$$\frac{v_{\max.}}{v} = 1 + K\sqrt{\frac{mi}{v^2}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= 1 + K\sqrt{b} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$= 1 + \frac{K}{c}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the values of b and c being given by the Tables, and K being a coefficient depending upon the form of the section and the conditions of flow. For example, if $v'_{\max.}$ is the *maximum surface velocity* for a given section,

$$\frac{v'_{\max.}}{v} = 1 + 36.3\sqrt{\frac{mi}{v^2}}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

for a watercourse of great width as compared with the depth and

$$\frac{v'_{\max.}}{v} = 1 + 25.4\sqrt{\frac{mi}{v^2}}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

for a channel of restricted dimensions, as in ordinary practice.

Again, if $v_{\max.}$ is the *maximum* velocity for the whole section of such a channel, and if v_m is the *mean* velocity along

the vertical in which the maximum velocity lies, then, approximately,

$$v_{\max.} = v_m + 10.9 \sqrt{hi}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which h is the depth of the water on the vertical in question. (If a metre is the unit, the values of K in the three last formulæ are 20, 14, and 6, respectively.)

For channels of mean dimensions Prony has suggested the formula

$$\frac{v}{v'_{\max.}} = \frac{7.78 + v'_{\max.}}{10.34 + v'_{\max.}}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

(If the unit is a metre, substitute 2.37 for 7.78, and 3.15 for 10.34.)

In the same case Dubuat gives

$$v = \frac{v'_{\max.} + v_b}{2}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which v_b is the velocity at the bottom of the channel.

For values of $v'_{\max.}$ up to about 11 or 12 ft. (3.5 m.) per second the calculated values of the ratio $\frac{v}{v'}$ vary but little from the average value .8, a result which has been verified in certain special experiments. It is therefore considered sufficient to take

$$\frac{v}{v'_{\max.}} = .8, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and then, by eq. (9),

$$\frac{v_b}{v'_{\max.}} = .6. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

When the water is of great depth the ratio $\frac{v}{v'_{\max.}}$ falls to .75, and to .60 if the bottom is covered with reeds.

Sonnet has theoretically deduced for watercourses of great width the relation

$$v = \frac{2v'_{\max.} + v_b}{3}, \quad . \quad . \quad . \quad . \quad . \quad (12)$$

so that if $v = \frac{4}{5}v'_{\max.}$, then $v_b = \frac{2}{5}v'_{\max.}$

For a long time it was supposed that the maximum velocity ($v_{\max.}$) was in the free surface, and its value was determined by observing the time in which floats passed between two transverse sections at a specified distance apart. Experiments have now demonstrated that this maximum velocity is at some point below, although in general near the free surface, and the floats will not give the proper value of the maximum velocity unless they are suitably submerged. It has also been found that the depth of this point of maximum velocity increases as the ratio of the width to the depth of the waterway diminishes, and may be as great as *one third* of the depth of the water.

On any horizontal line at right angles to the axis of the channel the velocity diminishes with the depth of the water, is greatest towards the centre, and diminishes at an increasing rate on approaching the sides.

The experiments of Darcy and Bazin have shown that the air-resistance is not the most important factor in causing the variation in the velocity throughout the section. With a gauge they determined the velocities at a number of points in the cross-section, and plotted the corresponding equal-velocity curves:

(a) For a *closed* wooden pipe, of rectangular section, running full (Fig. 141);

(b) For an *open* wooden channel running *half* full and formed by removing the upper side of the pipe in (a) (Fig. 142).

The curves for the pipe are approximately rectangular and parallel to the sides of the pipe. The discharge in the open channel is slightly greater than one half of the pipe's discharge,

but there is no similarity between the equal-velocity curves in the two cases. In the open channel they become more elliptical, tend to close at the centre, and cut the free surface obliquely, the angle of incidence becoming more and more acute towards the centre. The curves are also at a greater distance from the centre than the corresponding curves in the pipe. This very marked modification in the form of the

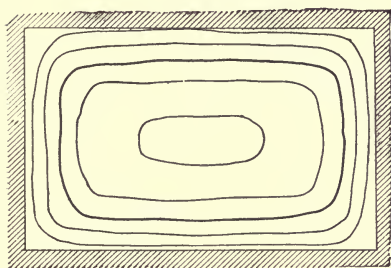


FIG. 141.

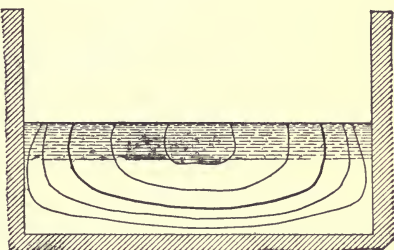


FIG. 142.

velocity curves is due especially, in Bazin's opinion, to the absence of the upper boundary and to the consequent practical impossibility of an absolutely constant cross-section. Eddies and other irregular movements are produced in the surface and give rise to corresponding losses of energy and velocity. Actual experiment, too, has shown that, even with a strong wind blowing down-stream, tending, as might be supposed, to cause an excessive surface velocity, the maximum velocity is still at some point *below* the free surface.

For any given vertical in the section it appears to be approximately true that the velocity at about *three fifths* of the total depth is sensibly the *mean* velocity for the whole depth, and that the difference between the maximum and bottom velocities, viz., $v_{\max.} - v_b$, increases with the roughness and lies between $\frac{1}{4}v_{\max.}$ and $\frac{1}{2}v_{\max.}$

In a semicircular channel of radius r the equal-velocity curves are circular, Fig. 143, and concentric with the bed, the

velocity v at the distance y from the centre being given by

$$v = v_c - \frac{38 \sqrt{ri}}{r^3} y^3,$$

v_c being the velocity at the centre.

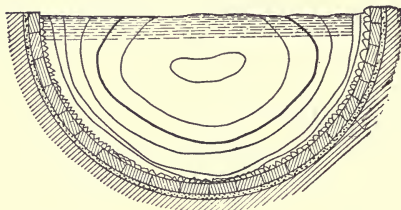


FIG. 143.

Generally speaking, the equal-velocity curves are approximately of the same form as the profile of the section (Figs. 143 to 146), and this is especially the case near the sides and

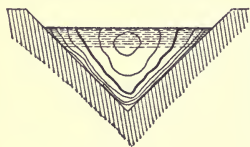


FIG. 144.

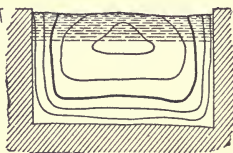


FIG. 145.

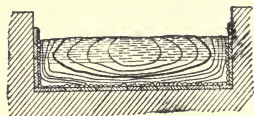


FIG. 146.

bed. The curves at the bottom do not always reach the surface, but sometimes cut the sides.

Again, experiments indicate that the law of velocity distribution along any vertical in the section may be represented by a parabola of the 2d degree, with its axis horizontal and at the same depth as the point of maximum velocity. Defontaine in an experiment on an arm of the Rhine deduced for the vertical at the centre of the current the analogous law

$$u = 4.8222 - .066y^2, \quad . \quad . \quad . \quad (13)$$

u being the velocity at the depth y .

(If the unit is a metre, $u = 1.266 - .252y^2$.)

The following theoretical investigation of the velocity curve is based on the assumptions that:

(a) The watercourse is of very great width as compared with the depth;

(b) The watercourse is of sensibly uniform depth;

(c) The fluid particles flow across a transverse section in sensibly parallel lines;

(d) A permanent régime has been established so that the pressure is distributed over the section in sensibly parallel lines;

(e) The resistance to the relative flow of consecutive fluid filaments is of the nature of a viscous resistance.

Let Fig. 147 represent a portion of a vertical longitudinal

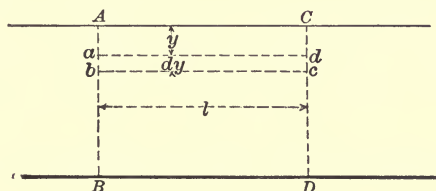


FIG. 147.

section of the stream intersected by two transverse sections AB , CD , l being the distance between them.

Consider a thin layer $abcd$ of thickness dy and width b , bounded by the sections AB , CD , and by the planes ad , bc , at depths y and $y + dy$, respectively, below the free surface.

The forces acting upon the layer in the direction of motion are:

(1) The pressures on the ends ab , cd , which evidently neutralize each other.

(2) The component of the weight $= wbl \cdot dy \cdot \sin i = wbl i \cdot dy$; i being the slope of the bed.

(3) The viscous resistances on the lateral faces of the layer under consideration. These are nil, since in a stream of indefinite width there will be no relative sliding between $abcd$ and the vertical faces on each side.

(4) The viscous resistances along the planes ad and bc .

The frictional resistance to distortion, i.e., to shearing, along such planes, is found to be proportional to the shear per unit of time, and is measured by the shear per unit of area at the actual rate of shearing. The coefficient of viscosity, or simply the viscosity, is the quotient $\frac{\text{shear per unit of area}}{\text{shear per unit of time}}$, and defines that quality of the fluid in virtue of which it resists a change of shape.

Adopting Navier's hypothesis,

$$\text{the viscous resistance along } ad = -kbl \frac{du}{dy},$$

k being the coefficient of viscosity, and u the velocity at the depth y . The sign is negative as, since u increases with y , $\frac{du}{dy}$ is positive, and, at the same time, the action of the layers above ad is of the character of a retardation.

$$\begin{aligned} \text{The viscous resistance along } bc &= kbl \frac{du}{dy} + kbl \cdot d\left(\frac{du}{dy}\right) \\ &= kbl \frac{du}{dy} + kbl \frac{d^2u}{dy^2} dy. \end{aligned}$$

Then, as the motion is uniform,

$$wbli \cdot dy - kbl \frac{du}{dy} + kbl \frac{du}{dy} + kbl \frac{d^2u}{dy^2} dy = 0.$$

Hence

$$\frac{d^2u}{dy^2} + \frac{wi}{k} = 0.$$

Integrating twice,

$$u = -\frac{wi}{2k} y^2 + ay + v, \quad . \quad . \quad . \quad (14)$$

a and v , being constants of integration.

It is evident that v_s is the surface velocity, i.e., the value of u when $y = 0$.

The equation may be written in the form

$$u - v_s - \frac{ka^2}{2wi} = -\frac{wi}{2k} \left(y - \frac{ka}{wi} \right)^2 \quad \dots \quad (15)$$

Thus the velocity curve is a parabola having a horizontal axis at a depth $Y = \frac{ka}{wi}$ below the free surface. This is also the depth of the filament of maximum velocity

$\left(\frac{du}{dy} = 0 \right)$ and

$$v_{\max.} = v_s + \frac{ka^2}{2wi} = v_s + \frac{wi}{2k} Y^2 \quad (16)$$

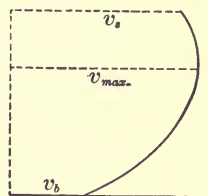


FIG. 148.

Hence, by equations (14) and (16),

$$u = v_{\max.} - \frac{wi}{2k} (y - Y)^2 \quad \dots \quad (17)$$

Let v_m be the "mean" velocity for the whole depth h . Let $v_{\frac{1}{2}}$ be the mid-depth velocity. Then

$$\begin{aligned} v_m &= \frac{\int_0^h \left\{ v_{\max.} - \frac{wi}{2k} (y - Y)^2 \right\} dy}{h} \\ &= v_{\max.} - \frac{wi}{6k} (h^2 - 3hY + 3Y^2), \quad \dots \quad (18) \end{aligned}$$

and

$$v_{\frac{1}{2}} = v_{\max.} - \frac{wi}{2k} \left(\frac{h}{2} - Y \right)^2 \quad \dots \quad (19)$$

$\frac{Y}{h} = \frac{1}{3}$ in some artificial channels and in others ranged from 0 to .2.

These last results are not in accord with the Mississippi measurements.

In the case of a rectangular channel of such width that the influence of the sides on the flow may be disregarded, the mean radius, m , may be substituted for h and the mean velocity, v_m , is sensibly the same as the mean velocity, v , for the whole section. Hence equation (22) may be written

$$\frac{u}{v} = \frac{v_{\max.}}{v} - 36.3 \sqrt{\frac{mi}{v^2} \left(\frac{y - Y}{h - Y} \right)^2},$$

or

$$\frac{u}{v} = \frac{v_{\max.}}{v} - 36.3 \sqrt{b} \left(\frac{y - Y}{h - Y} \right)^2, \quad . \quad . \quad . \quad (23)$$

the value of b being given by the Tables.

Filament of Maximum Velocity in the Surface.—In this case $Y = 0$, and equation (21) becomes

$$v'_{\max.} - v_b = \frac{wi}{2k} h^2, \quad . \quad . \quad . \quad . \quad (24)$$

$v'_{\max.}$ being the value of $v_{\max.}$ when the maximum velocity is in the surface.

Equation (22) also becomes

$$u = v'_{\max.} - 36.3 \sqrt{hi} \frac{y^2}{h^2}. \quad . \quad . \quad . \quad (25)$$

Again, by equations (18) and (24),

$$v'_{\max.} - v_b = \frac{wi}{2k} h^2 = 3(v'_{\max.} - v_m),$$

II. Tables of Erosion and Viscosity.

* TABLE INDICATING THE VELOCITIES ABOVE WHICH
EROSION COMMENCES.

Nature of the Channel Bed.	v_s		v_m		v_b	
	Met.	Feet.	Met.	Feet.	Met.	Feet.
Soft and clayey soils.....	.15	.49	.11	.36	.08	.26
Rich clay.....	.30	.98	.23	.75	.16	.52
Firm sand.....	.60	1.97	.46	1.51	.31	1.02
Gravel.....	1.22	4.00	.96	3.15	.70	2.30
Broken stone.....	1.52	5.00	1.23	4.03	.94	3.08
Soft schist.....	2.22	7.28	1.86	6.10	1.49	4.90
Stratified rocks.....	2.75	9.02	2.27	7.45	1.82	6.00
Hard rocks.....	4.27	14.00	3.69	12.10	3.14	10.3

* TABLE OF VISCOSITIES (*Everett's System of Units*).

WATER.				MERCURY.			
Temp. (Cent.)	Viscosity.	Temp. (Cent.)	Viscosity.	Temp. (Cent.)	Viscosity.	Temp. (Cent.)	Viscosity.
0°	.0181	35°	.0073	0°	.0169	315°	.00918
5	.0154	40	.0067	10	.0162	340	.00897
10	.0133	45	.0061	18	.0156		
15	.0116	50	.0056	99	.0123		
20	.0102	60	.0047	154	.0109		
25	.0091	80	.0036	197	.0102		
30	.0081	90	.0032	249	.00964		

The viscosity is given by

$$\frac{.0183}{1 + .0369t}, \text{ according to Meyer,}$$

and by

$$\frac{.5212}{26 + t} - .00131, \text{ according to Slotte;}$$

t being the temperature centigrade.

12. River-bends.—The following explanation is due to Professor James Thomson (Inst. Mechl. Engs., 1879; Proc. Royal Soc. 1877). In rivers flowing in alluvial plains, the

* N. B. The viscosities are in C. G. S. units. To reduce to F. P. S. units and centigrade degrees multiply by 2.0481.

curvature of the windings which already exist tends to increase owing to the scouring away of material from the outer bank and to the deposition of detritus along the inner bank. The sinuosities often increase until a loop is formed, with only a narrow isthmus of land between two encroaching banks of a river. Finally a cut-off occurs, a short passage for the water is opened through the isthmus, and the loop is separated from the river-course, taking the form of a horseshoe shaped lagoon or swamp. The ordinary supposition, that the water always tends to move forward in a straight line, rushing against the outer bank and wearing it away, and at the same time causing deposits at the inner bank, is correct, but it is very far from being a complete explanation of what takes place.

When water flows round a circular curve under the action of gravity only, it takes a motion like that in a free vortex. Its velocity parallel to the axis of the stream is greater at the inner than at the outer side of the curve.

Thus, too, the water in a river-bank flows more quickly along courses adjacent to the inner bank of the bend than

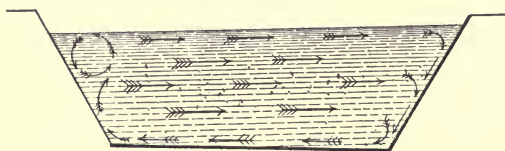


FIG. 150.

along courses adjacent to the outer. The water, in virtue of centrifugal force, presses outwards so that the water-surface of a transverse section (Fig. 150) has a slope rising upwards from the inner to the outer bank. Hence the free level, for any particle of the water near the outer bank, is higher than the free level for any particle in the same transverse section near the inner bank, but the tendency to flow from the higher to the lower level is counteracted by centrifugal action. Now the water immediately in contact with the bottom and sides of the course is retarded, and its centrifugal force is not sufficient

to balance the pressure due to the greater depth at the outside of the bend. This water therefore tends to flow from the outer bank towards the inner (Fig. 151), carrying with it detritus

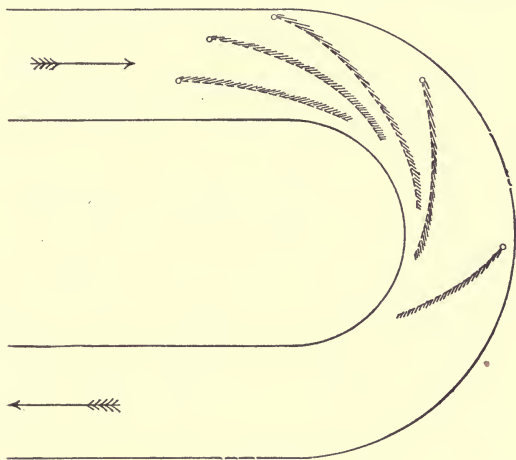


FIG. 151.

which is deposited at the inner bank. Simultaneously with the flow of water inwards, the mass of the water must necessarily flow outwards to take its place.

13. Flow of Water in Open Channels of Varying Cross-section and Slope.

Assumptions.—(a) That the motion is steady.

Thus the mean velocity is constant for any given cross-section, but varies *gradually* from section to section.

(b) That the change of cross-section is also gradual.

(c) That, as in cases of *uniform* motion, the work absorbed by the frictional resistance of the channel bed and sides is the only *internal* work which need be taken into consideration.

Let Fig. 152 represent a longitudinal section of the stream. The fluid molecules which are found in any plane section *ab* at the commencement of an interval will be found in a curved surface *dc* at the end of the interval, on account of the different velocities of the fluid filaments.

Suppose that the mass of water bounded by the two transverse sections ab , ef comes into the position $cdhg$ in a unit of time. Then the change of kinetic energy in this mass is equal to the algebraic sum of the work done by gravity, of the work

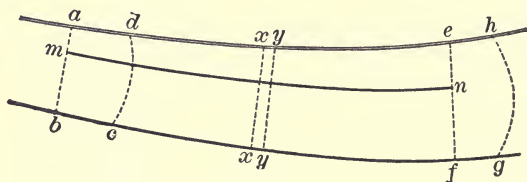


FIG. 152.

done by pressure, and of the work done against the frictional resistance.

Change of Kinetic Energy.—This is evidently the difference between the kinetic energies of the masses $efgh$ and $abcd$, since, as the motion is steady, the kinetic energy of the mass between cd and ef remains constant.

Let A_1 be the area of the cross-section ab .

“ u_1 “ “ mean velocity across this section.

“ v “ “ velocity at this section of any given fluid filament of sectional area a .

Let $v = u_1 \pm V$.

Then

$$A_1 u_1 = \Sigma(av) \quad \text{and} \quad \Sigma(aV) = 0.$$

The kinetic energy of the mass $abcd$

$$\begin{aligned} &= \frac{w}{2g} \Sigma(av^3) = \frac{w}{2g} \Sigma\{a(u_1 \pm V)^3\} \\ &= \frac{w}{2g} \Sigma\{a(u_1^3 \pm 3u_1^2 V + 3u_1 V^2 \pm V^3)\} \\ &= \frac{w}{2g} \Sigma a\{u_1^3 + V^2(2u_1 + v)\}, \end{aligned}$$

since $\Sigma(aV) = 0$ and $3u_1 \pm V = 2u_1 + v$.

Now $2u_1 + v$ is evidently positive. Hence the kinetic energy of the mass $abcd$

$$\begin{aligned} &= \frac{w}{2g} \Sigma av^3 > \frac{w}{2g} \Sigma au_1^3 \\ &> \frac{w}{2g} A_1 u_1^3 \\ &= \alpha \frac{w}{2g} A_1 u_1^3, \end{aligned}$$

α being a coefficient of correction whose value depends upon the law of the distribution of the velocity throughout the section ab . It is positive and greater than unity. Assume that α has the same value for the sections ab and ef . Then if A_2 , u_2 are the area and mean velocity at the transverse section ef , the kinetic energy of the mass $efgh$

$$= \alpha \frac{w}{2g} A_2 u_2^3.$$

Hence the change of kinetic energy in the mass under consideration

$$\begin{aligned} &= \alpha \frac{w}{2g} (A_2 u_2^3 - A_1 u_1^3) \\ &= \alpha \frac{wQ}{g} \frac{u_2^3 - u_1^3}{3}, \end{aligned}$$

since

$$A_2 u_2 = Q = A_1 u_1.$$

Work done by Gravity.—Consider any fluid filament mn , the depth of m below the surface being y_1 , and of n , y_2 .

Let z be the fall in the surface-level from a to e .

Then the fall from m to n

$$= z + y_2 - y_1,$$

and the work done by gravity on the elementary volume dQ in a unit of time

$$= +w \cdot dQ(z + y_2 - y_1).$$

Work done by Pressure.

The pressure per unit of area at $m = wy_1 + p_0$;

“ “ “ “ “ “ “ “ $n = wy_2 + p_0$;

p_0 being the atmospheric pressure.

Hence the work due to these pressures per unit of time

$$\begin{aligned} &= dQ(wy_1 + p_0) - dQ(wy_2 + p_0) \\ &= w \cdot dQ(y_1 - y_2). \end{aligned}$$

Thus the *total* work done by gravity and by pressure

$$\begin{aligned} &= \Sigma \{w \cdot dQ(z + y_2 - y_1) + w \cdot dQ(y_1 - y_2)\} \\ &= \Sigma (w \cdot dQ \cdot z) = wQz \end{aligned}$$

for the mass under consideration.

Work absorbed by Friction.—Consider a thin lamina of water of thickness ds , bounded by the transverse planes xx , yy , the distance of xx from ab being s .

Since the change of velocity is gradual, the mean velocity from xx to yy may be assumed to be constant.

Let u be this mean velocity.

“ P be the wetted perimeter at the section xx .

“ A be the area of the waterway at the section xx .

Then the work absorbed by friction per second from xx to yy

$$= P \cdot ds \cdot u \cdot F(u),$$

and the total work absorbed between ab and ef

$$= Q \int_0^l \frac{P}{A} F(u) ds,$$

l being the distance between ab and ef . Hence

$$\alpha \frac{wQ}{g} \frac{u_2^2 - u_1^2}{2} = wQz - Q \int_0^l \frac{P}{A} F(u) ds,$$

and therefore $z = \alpha \frac{u_2^2 - u_1^2}{2g} + \int_0^l \frac{P}{A} \frac{F(u)}{w} ds.$

Take $\frac{F(u)}{w} = f \frac{u^2}{2g}$ and $\frac{A}{P} = m.$ Then

$$z = \alpha \frac{u_2^2 - u_1^2}{2g} + \int_0^l \frac{f}{m} \frac{u^2}{2g} ds. \quad \dots \quad (1)$$

If the two planes ab and cf are indefinitely near one another (Fig. 153), the last equation evidently gives

$$dz = \frac{\alpha}{g} u \cdot du + \frac{f}{m} \frac{u^2}{2g} ds, \quad \dots \quad (2)$$

which is the fundamental differential equation of steady varied motion, dz being the fall of surface level in the distance ds .

In the figure aa' is drawn parallel to the bed and aa'' is horizontal. The distance $a'e$ may, without sensible error, be assumed equal to dz .

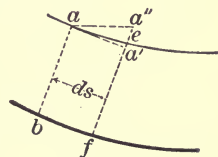


FIG. 153.

Also, $a'a' = i \cdot aa' = i \cdot ds$, very nearly.

Hence

$$ids = a'a' = a'e + a'e = dh + dz. \quad \dots \quad (3)$$

Substituting the value of dz from this equation in equation (2),

$$i \cdot ds - dh = \frac{\alpha}{g} u \cdot du + \frac{f}{m} \frac{u^2}{2g} \cdot ds. \quad \dots \quad (4)$$

Also, since $Au = Q$, a constant,

$$A \cdot du + u \cdot dA = 0,$$

and $dA = x \cdot dh$, very nearly, if x is the width of the stream.

Therefore

$$Adu + ux \cdot dh = 0,$$

and hence, by equation (4),

$$i \cdot ds - dh = -\alpha \frac{u^2 x}{g A} \cdot dh + \frac{f}{m} \frac{u^2}{2g} ds.$$

Therefore

$$\frac{dh}{ds} = \frac{i - \frac{f}{m} \frac{u^2}{2g}}{1 - \alpha \frac{u^2 x}{g A}} = i \frac{1 - \frac{f}{m} \frac{u^2}{2gi}}{1 - \alpha \frac{u^2 x}{g A}}. \quad (5)$$

Let the position of any point a in the surface be defined by its perpendicular distance h from the bed and by the distance s of the transverse section at a from an origin in the bed.

Then $\frac{dh}{ds}$ is the tangent of the angle which the tangent to the surface at a makes with the bed. It is positive or negative according as the depth increases or diminishes in the direction of flow, thus defining two states of steady varied motion.

Between these there is an intermediate state defined by

$$\frac{dh}{ds} = 0 = i - \frac{f}{m} \frac{u^2}{2g},$$

and $i = \frac{f}{m} \frac{u^2}{2g}$ is the equation for steady flow with *uniform* motion.

Let U, M, H be the corresponding values of u, m, h in the case of uniform motion. Then

$$i = \frac{f}{M} \frac{U^2}{2g} = b \frac{U^2}{M}, \quad (6)$$

and eq. (5) becomes

$$\frac{dh}{ds} = i \frac{1 - \frac{M}{m} \frac{u^2}{U^2}}{1 - \alpha \frac{u^2 x}{g A}} = \frac{i - b \frac{u^2}{m}}{1 - \alpha \frac{u^2 x}{g A}}. \quad (7)$$

If the section of the channel is a rectangle,

$$A = xh, \quad xhu = xHU, \quad m = \frac{xh}{x + 2h}, \quad \text{and} \quad M = \frac{xH}{x + 2H}.$$

Substituting these values in eq. (7),

$$\frac{dh}{ds} = i \frac{1 - \frac{x + 2h}{x + 2H} \left(\frac{H}{h}\right)^3}{1 - \alpha \frac{u^2}{gh}}. \quad (8)$$

Three cases will be considered and, in each case, a line PQ , drawn parallel to the bed, represents the surface of *uniform motion*, H being the distance between PQ and the bed.

CASE I. $\alpha u^2 < gh$ and $H < h$, Fig. 154.

$\frac{dh}{ds}$ is positive, and therefore h increases in the direction of flow. Thus the actual surface MN of the stream is wholly above the line PQ .

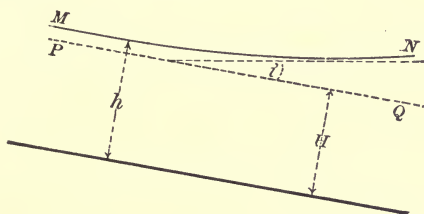


FIG. 154.

Proceeding up-stream, h becomes more and more nearly equal to H , so that the numerator of eq. (8), and therefore also $\frac{dh}{ds}$, approximates more and more closely to zero.

Again, proceeding down-stream, h increases and u diminishes, so that both the numerator and denominator in eq. (8) approximate more and more closely to the value *unity*, and therefore $\frac{dh}{ds}$ becomes more and more nearly equal to i , the slope corresponding to uniform motion.

Hence up-stream MN is asymptotic to PQ , and down-stream MN is asymptotic to a horizontal line. This form of surface is produced when a weir is built across a channel in which the water had previously flowed with a uniform motion.

CASE II. $\alpha u^2 < gh$ and $H > h$, Fig. 155.

$\frac{dh}{ds}$ is now negative, and the depth diminishes in the direction of flow.

Up-stream h increases and approaches H in value, so that MN is asymptotic to PQ .

Down-stream h diminishes, u increases, and therefore the value of $\frac{\alpha u^2}{gh}$ is more and more nearly equal to *unity*.

Thus, in the limit, the denominator in eq. (8) becomes zero, and therefore $\frac{dh}{ds} = \infty$. Hence theory indicates that at a certain point down-stream the surface line MN takes a direction which is at right angles to the general direction of flow. This is contrary to the fundamental hypothesis that the fluid filaments flow in sensibly parallel lines. In fact, before the

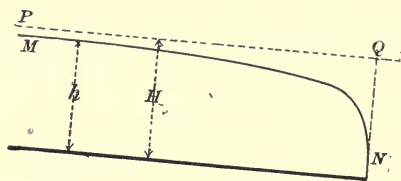


FIG. 155.

limit could be reached this hypothesis would cease to be even approximately true, and the general equation would cease to be applicable. This form of water-surface is produced when there is an abrupt depression in the bed of the stream.

Fig. 156 shows one of the abrupt falls in the Ganges canal as at first constructed. The surface of the water flowing freely over the crest of the fall took a form similar to MN below the line PQ of uniform motion. The diminution of depth in the

approach to the fall caused an increase in the velocity of flow, with the result that for several miles above the fall a serious erosion of the bed and sides took place. In order to remedy this, temporary weirs were constructed so as to raise the level

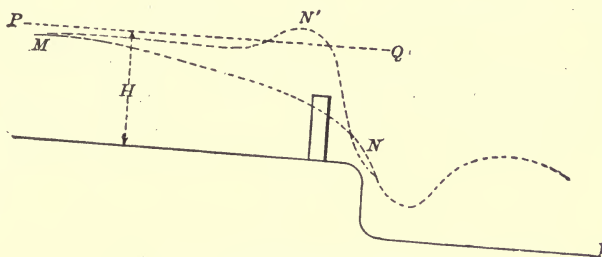


FIG. 156.

of the water until the surface line assumed a form MN' corresponding approximately to PQ . In some cases the water was raised above its normal height and a backwater produced.

CASE III. $au^2 > gh$ and $H < h$, Fig. 157.

$\frac{dh}{ds}$ is negative and the surface line of the stream is wholly above PQ .

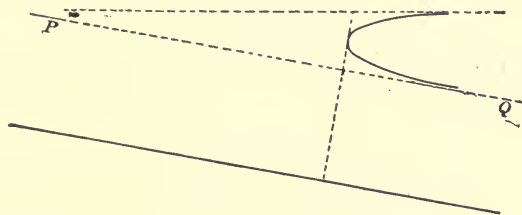


FIG. 157.

If h gradually increases, u diminishes and $\frac{dh}{ds}$ approximates to $-i$ in value.

If h gradually diminishes, it approximates to H in value, and in the limit $\frac{dh}{ds} = 0$.

Between these two extremes there is a value of h for which the denominator of eq. (8) becomes nil, viz.,

$$h = \alpha \frac{u^2}{g},$$

and the corresponding value of $\frac{dh}{ds}$ is infinity.

Thus one part of the surface line is asymptotic to PQ , the line of uniform motion, another part is asymptotic to a horizontal line, while at a certain point at which the depth is

$$h = \alpha \frac{u^2}{g},$$

the surface of the stream is normal to the bed.

This is contrary to the fundamental hypothesis that the fluid filaments flow in sensibly parallel lines, and the general equation no longer represents the true condition of flow.

In cases such as this there has been an abrupt rise of the surface of the stream, and what is called a "standing wave" has been produced.

In a stream of depth H flowing with a uniform velocity U , which is $> \sqrt{\frac{gH}{\alpha}}$, construct a weir so as to increase the depth to h_1 , which is $> \frac{\alpha U^2}{g}$.

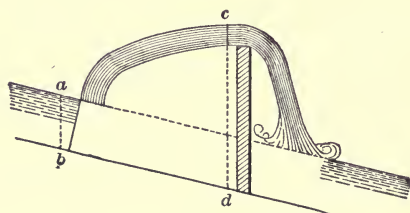


FIG. 158.

Then in one portion of the stream near the weir the depth is $> \frac{\alpha U^2}{g}$, while further up the stream the depth is $< \frac{\alpha U^2}{g}$.

Thus at some intermediate point the depth $= \alpha \frac{U^2}{g}$, the corresponding value of $\frac{dh}{ds}$ being ∞ , and at this point a *standing wave is produced*.

Now

$$\frac{fU^2}{2g} = Mi = Hi,$$

and since $H < \alpha \frac{U^2}{g}$,

$$\frac{fU^2}{2g} < \alpha \frac{U^2}{g} i,$$

and therefore

$$i > \frac{f}{2\alpha},$$

which condition must be fulfilled for a standing wave.

Bazin gives the following table of values of f :

Nature of Bed.	Slope $\left(\frac{h}{l} = i\right)$ below which standing wave is impossible. In Metres per Metre.	Standing Wave Produced.	
		Slope in Metres per Metre (or Feet per Foot).	Least Depth in Metres.
Very smooth cemented surface....	.00147	{ .002 .003 .004	.08 .03 .02
Ashlar or brickwork.00186	{ .003 .004 .006	.12 .06 .03
Rubble masonry.....	.00235	{ .004 .006 .010	.36 .16 .08
Earth.....	.00275	{ .006 .010 .015	1.06 .47 .28

A standing wave rarely occurs in channels with earthen beds, as their slope is almost always less than the limit, .00275.

The formation of a standing wave was first observed, by Bidone in a small masonry canal of rectangular section.

The width of the canal $= 0^m.325 = x$;

“ slope $\left(= \frac{h}{l}\right)$ of the canal $= .023$;

“ uniform velocity of flow $= 1^m.69 = U$;

“ depth corresponding to $U = 0^m.064 = H$.

A weir built across the canal increased the depth of the water near the weir to $0^m.287 = h_1$.

It was found that the “uniform régime” was maintained up to a point within $4^m.5$ of the weir. At this point the depth suddenly increased from $0^m.064$ to about $0^m.170$, and between the point and the weir the surface of the stream was slightly convex in form (Fig. 158).

With the preceding data and taking $\alpha = 1.1$, $\frac{\alpha U^2}{gH} = 5$ and is therefore > 1 at a section ab , Fig. 159.

At the section cd ,

$$u = \frac{H}{h}U = \frac{.064}{.287} \times 1.69 = 0^m.377,$$

and $\frac{\alpha U^2}{gh_1} = .055$ and is therefore < 1 .

Thus the expression $1 - \frac{\alpha u^2}{gh}$ is negative for a section ab and positive for a section cd , so that i must change sign between these sections, and $\frac{dh}{ds}$ will then become infinite.

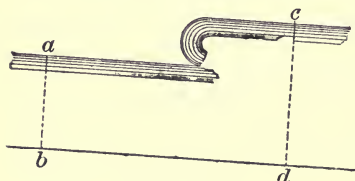


FIG. 159.

Consider a portion of a stream bounded by two transverse sections, ab , cd , in which a standing wave occurs, Fig. 159.

Assume that the fluid filaments flow across the sections in sensibly parallel lines.

Let the velocities and area at section ab be distinguished by the suffix 1, and those at cd by the suffix 2. Then

Change of momentum in direction of flow } = impulse in same direction.

Hence

$$\frac{w}{g} \{ \Sigma(av_2)v_2 - \Sigma(av_1)v_1 \} = w(A_1y_1 - A_2y_2),$$

and therefore

$$\frac{1}{g}(\Sigma av_2^2 - \Sigma av_1^2) = A_1y_1 - A_2y_2, \quad . \quad . \quad (9)$$

y_1, y_2 being the depths below the surface of the centres of gravity of the sections ab, cd , respectively.

Now, $v_1 = u_1 + V_1$. Therefore

$$\begin{aligned} \Sigma av_1^2 &= \Sigma a(u_1^2 + 2u_1V_1 + V_1^2) \\ &= u_1^2A_1 + \Sigma aV_1^2. \end{aligned}$$

Also, as already shown,

$$\alpha A_1 u_1^3 = \Sigma av_1^3 = A_1 u_1^3 + \Sigma a V_1^2 (3u_1 + V_1),$$

and, neglecting V_1 as compared with $3u_1$,

$$\alpha A_1 u_1^3 = A_1 u_1^3 + 3u_1 \Sigma a V_1^2.$$

Thus

$$\Sigma a V_1^2 = \frac{A_1 u_1^2}{3} (\alpha - 1),$$

and hence

$$\begin{aligned} \Sigma av_1^2 &= u_1^2 A_1 + \frac{A_1 u_1^2}{3} (\alpha - 1) \\ &= \frac{u_1^2 A_1}{3} (\alpha + 2) = \alpha' A_1 u_1^2, \end{aligned}$$

where $\alpha' = \frac{\alpha + 2}{3}$, and is 1.033 if $\alpha = 1.1$.

Similarly it may be shown that

$$\Sigma av_2^2 = \alpha' A_2 u_2^2.$$

Thus equation (9) becomes

$$\frac{\alpha'}{g}(A_2 u_2^2 - A_1 u_1^2) = A_1 y_1 - A_2 y_2. \quad \dots \quad (10)$$

Let the section of the canal be a rectangle of depth H_1 at ab and H_2 at cd . Then

$$u_1 H_1 = u_2 H_2; \quad \frac{H_1}{2} = y_1; \quad \frac{H_2}{2} = y_2.$$

Therefore, by equation (10),

$$\frac{\alpha'}{g} A_1 u_1^2 \left(\frac{H_1}{H_2} - 1 \right) = A_1 \left(\frac{H_1}{2} - \frac{H_2^2}{2H_1} \right),$$

which reduces to

$$\frac{\alpha' u_1^2}{g H_2} (H_2 - H_1) = \frac{1}{2 H_1} (H_2^2 - H_1^2).$$

$H_2 = H_1$ satisfies the equation and corresponds to a condition of uniform motion.

Also;

$$\frac{\alpha' u_1^2}{g} = \frac{H_2}{H_1} \frac{H_2 + H_1}{2}. \quad \dots \quad (11)$$

In Bidone's canal, $u_1 = 1^m.69$, $H_1 = 0^m.064$. Substituting these values in equation (11), the value of H_2 is found to be $0^m.16$, which agrees somewhat closely with the actual measurements.

N.B.—The coefficients α and α' have not been very accurately determined, but their exact values are not of great importance. They are often taken equal to *unity*.

14. Longitudinal Profile and Rühlmann's Law.—In the preceding article, put $F\left(i - b\frac{u^2}{m}\right) = 1 - \alpha\frac{u^2}{g} \frac{x}{A}$ in eq. (7), then

$$ds = F \cdot dh.$$

If the transverse profile has been determined, the value of F corresponding to the depth h at any point O can be at once found and, by means of the last equation, the surface profile between the depths h and H can be easily plotted.

Let F_1, F_2, F_3, \dots be the values of F at a series of points at which the depths, differing successively by a small quantity dh , are h_1, h_2, h_3, \dots respectively. Then

$$ds_1 = F_1 \cdot dh; \quad ds_2 = F_2 \cdot dh; \quad ds_3 = F_3 \cdot dh; \dots;$$

and the corresponding distances s_1, s_2, s_3, \dots of these points from O are

$$s_1 = \frac{ds_1 + ds_2}{2}; \quad s_2 = s_1 + \frac{ds_2 + ds_3}{2}; \quad s_3 = s_2 + \frac{ds_3 + ds_4}{2}; \dots$$

EXAMPLE. A cut of rectangular section, with a fall of 1 in 10,000, is 10 ft. wide and delivers 40 cu. ft. of water per second. At a certain point the depth is increased to 4 ft. by a dam. Assuming that the faces of the cut are not very smooth and that, consequently, .0001 may be taken as an approximate value of b , then the depth, H , for uniform motion is given by

$$\left(\frac{40}{10H}\right)^2 = U^2 = M \frac{i}{b} = \frac{10H}{10 + 2H},$$

or

$$80 + 16H = 5H^2,$$

and an approximate solution of this equation is $H = 2.9$ ft.

The following Table can now be easily prepared for a series of depths, commencing at the dam and diminishing successively by 3 ins., α being unity:



h	A	P	m	u	$1 - a \frac{u^2}{g^2 h}$	$i - b \frac{u^2}{m}$	F	ds	s
4.00	40	18	2.2	1	.991406	.000045	22031.2	5508	
3.75	37.5	17.5	2.1	1.1	.991943	.000041	24193.7	6048	5,778
3.50	35	17	2.1	1.2	.992480	.0000372	26679.6	6670	12,137
3.25	32.5	16.5	1.9	1.3	.993017	.0000335	29642.3	7410	19,177
3.00	30	16	1.8	1.4	.993554	.00003	33184.7	8546	27,155

The tenth column gives the distances from the dam of the sections in which the depths are 3.75, 3.50, 3.25, and 3 ft.

Rühlmann's formula for the distance between two sections between which the depth of the water gradually increases from $y + H$ to $Y + H$ is

$$s = \frac{H}{i} \left\{ f\left(\frac{Y}{H}\right) - f\left(\frac{y}{H}\right) \right\},$$

the function $f\left(\frac{y}{H}\right)$ being given by the following table:

$\frac{y}{H}$	$f\left(\frac{y}{H}\right)$	$\frac{y}{H}$	$f\left(\frac{y}{H}\right)$	$\frac{y}{H}$	$f\left(\frac{y}{H}\right)$
0.01	0.0067	0.3	1.3428	1.4	2.7264
0.02	0.2444	0.4	1.5119	1.5	2.8337
0.03	0.3863	0.5	1.6611	1.6	2.9401
0.04	0.4889	0.6	1.7980	1.7	3.0458
0.05	0.5701	0.7	1.9266	1.8	3.1508
0.06	0.6376	0.8	2.0495	1.9	3.2553
0.07	0.6958	0.9	2.1683	2.0	3.3594
0.08	0.7472	1.0	2.2839	2.5	3.8745
0.09	0.7933	1.1	2.3971	3.0	4.3843
0.10	0.8353	1.2	2.5083	3.5	4.8910
0.20	1.1361	1.3	2.6179	4.0	5.3958

Applying this formula to the preceding example, in order to determine the distance between the 3- and 4-ft. depths, at the dam

$$\frac{Y}{H} = \frac{1.1}{2.9} = .3793,$$

and, by interpolation,

$$f\left(\frac{Y}{H}\right) = 1.4769.$$

At the 3-ft. depth

$$\frac{y}{H} = \frac{.1}{2.9} = .03448,$$

and

$$f\left(\frac{y}{H}\right) = .4323.$$

Hence

$$s = \frac{2.9}{.0001} (1.4769 - .4323) = 30,293 \text{ feet.}$$

15. Channel of Rectangular Section with a nearly Horizontal Bed.—In this case i is very small and may be disregarded in eq. (4), Art. 13, which may therefore be written in the form

$$ds = -\frac{2g}{f} \frac{m}{u^2} dh - \alpha \frac{2g}{f} \frac{m}{g} \frac{du}{u}.$$

But $xhu = Q = \text{a constant}$, and therefore $h \cdot du + u \cdot dh = 0$.
Also,

$$m = \frac{xh}{x + 2h}.$$

Hence

$$ds = -\frac{x^3}{bQ^2} \frac{h^3 \cdot dh}{(x + 2h)} + \frac{\alpha x}{bg} \frac{dh}{x + 2h}.$$

Integrating,

$$s = -\frac{x^3}{8bQ^2} \left\{ \frac{4}{3} h^3 - xh^2 + x^2h - \frac{x^3}{2} \log_e (x + 2h) \right\} \\ + \frac{2\alpha x}{bg} \log_e (x + 2h) + c,$$

c being a constant of integration.

Hence the distance $s_1 - s_2$ between two points at which the depths are h_1 and h_2 ($< h_1$) is given by

$$s_1 - s_2 = \frac{x^3}{bQ^2} \int_{h_1}^{h_2} \frac{h^3 dh}{x + 2h} + \frac{\alpha x}{bg} \int_{h_1}^{h_2} \frac{dh}{x + 2h}.$$

The last term is usually very small and may be disregarded without appreciable error, and therefore

$$Q^2 = \frac{x^3}{b(s_1 - s_2)} \int_{h_1}^{h_2} \frac{h^3 dh}{x + 2h},$$

a formula by means of which the discharge may be found.

16. Channel of Great Width as compared with the Depth.—In this case

$$A = xh \quad \text{and} \quad P = x, \text{ approximately.}$$

Therefore

$$m = h \quad \text{and} \quad M = H.$$

Also,

$$u_2 = \frac{H^2}{h^2} U^2 = \frac{H^3 i}{h^2 b}.$$

Hence, eq. (7), Art. 13, may be written in the form

$$i \frac{ds}{dh} = \frac{\left(\frac{h}{H}\right)^3 - \frac{\alpha i}{gb}}{\left(\frac{h}{H}\right)^3 - 1} = 1 + \frac{1 - \frac{\alpha i}{gb}}{\left(\frac{h}{H}\right)^3 - 1}.$$

Take $z = \frac{H+y}{H} = \frac{h}{H}$, y being the rise or fall above or below the surface of uniform motion. Then $dh = H \cdot dz$, and

$$\frac{i}{H} \frac{ds}{dz} = 1 + \frac{1 - \frac{\alpha i}{gb}}{z^3 - 1}.$$

Integrating,

$$\frac{is}{H} = z - \left(1 - \frac{\alpha i}{bg}\right) \left\{ \frac{1}{6} \log_e \frac{z^2 + z + 1}{(z - 1)^2} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2z + 1}{\sqrt{3}} + c \right\}, \quad (1)$$

c being a constant of integration.

This equation may be written

$$\frac{2s}{H} = z - \left(1 - \frac{\alpha i}{bg}\right) \{ \phi(z) + c \}, \quad . \quad . \quad . \quad (2)$$

and between any two points $\frac{y}{H}$ and $\frac{y'}{H'}$

$$\frac{i}{H}(s - s') = z - z' - \left(1 - \frac{\alpha i}{bg}\right) \{ \phi(z) - \phi(z') \}, \quad . \quad (3)$$

the argument being $\frac{y}{H} = \frac{\text{change in depth}}{\text{original depth}}$.

In the case of a dam built across a channel in which the water had previously flowed with a uniform motion, Case I, Art. 13, in the limit,

$$si = h = zH = \infty,$$

and therefore, by eqs. 1 and 2,

$$\phi(z) + c = 0 = \frac{1}{6} \log_e 1 + \frac{1}{\sqrt{3}} \tan^{-1} \infty + c = \frac{1}{\sqrt{3}} \frac{\pi}{2} + c,$$

and

$$c = -.9069.$$

The following Table, calculated by Tutton, gives the value of the backwater function, $\phi(z)$, *in the case of a dam* :

$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$
0.000	∞	.072	.7812	.285	.3860	.68	.1945
.001	2.1837	.074	.7727	.290	.3816	.69	.1918
.002	1.9530	.076	.7644	.295	.3773	.70	.1892
.003	1.8181	.078	.7564	.300	.3730	.71	.1867
.004	1.7225	.080	.7486	.305	.3689	.72	.1843
.005	1.6485	.082	.7410	.310	.3649	.73	.1819
.006	1.5881	.084	.7336	.315	.3609	.74	.1795
.007	1.5379	.086	.7264	.320	.3570	.75	.1772
.008	1.4928	.088	.7194	.325	.3532	.76	.1749
.009	1.4539	.090	.7125	.330	.3495	.77	.1727
.010	1.4191	.092	.7058	.335	.3458	.78	.1705
.011	1.3877	.094	.6993	.340	.3422	.79	.1684
.012	1.3586	.096	.6929	.345	.3387	.80	.1663
.013	1.3327	.098	.6866	.350	.3352	.81	.1642
.014	1.3082	.100	.6805	.355	.3318	.82	.1622
.015	1.2855	.105	.6658	.360	.3285	.83	.1602
.016	1.2644	.110	.6518	.365	.3252	.84	.1583
.017	1.2446	.115	.6387	.370	.3220	.85	.1564
.018	1.2258	.120	.6260	.375	.3189	.86	.1546
.019	1.2081	.125	.6139	.380	.3158	.87	.1528
.020	1.1913	.130	.6024	.385	.3127	.88	.1510
.021	1.1754	.135	.5913	.390	.3097	.89	.1492
.022	1.1602	.140	.5807	.395	.3068	.90	.1475
.023	1.1457	.145	.5706	.400	.3039	.91	.1458
.024	1.1319	.150	.5608	.41	.2992	.92	.1441
.025	1.1186	.155	.5514	.42	.2928	.93	.1425
.026	1.1059	.160	.5423	.43	.2875	.94	.1409
.027	1.0936	.165	.5335	.44	.2824	.95	.1393
.028	1.0817	.170	.5251	.45	.2774	.96	.1377
.029	1.0704	.175	.5169	.46	.2726	.97	.1362
.030	1.0595	.180	.5090	.47	.2680	.98	.1347
.032	1.0387	.185	.5014	.48	.2634	.99	.1332
.034	1.0191	.190	.4939	.49	.2590	1.00	.1318
.036	1.0007	.195	.4867	.50	.2548	1.05	.1250
.038	.9833	.200	.4798	.51	.2506	1.10	.1187
.040	.9669	.205	.4730	.52	.2465	1.15	.1128
.042	.9512	.210	.4664	.53	.2426	1.20	.1074
.044	.9364	.215	.4600	.54	.2388	1.25	.1024
.046	.9223	.220	.4538	.55	.2351	1.30	.0979
.048	.9087	.225	.4478	.56	.2314	1.35	.0936
.050	.8957	.230	.4419	.57	.2279	1.40	.0894
.052	.8833	.235	.4363	.58	.2245	1.45	.0856
.054	.8714	.240	.4306	.59	.2212	1.50	.0821
.056	.8599	.245	.4251	.60	.2179	1.55	.0788
.058	.8488	.250	.4198	.61	.2147	1.60	.0758
.060	.8382	.255	.4145	.62	.2116	1.65	.0728
.062	.8279	.260	.4096	.63	.2086	1.70	.0700
.064	.8179	.265	.4046	.64	.2056	1.75	.0674
.066	.8083	.270	.3998	.65	.2027	1.80	.0650
.068	.7990	.275	.3951	.66	.1999	1.85	.0626
.070	.7899	.280	.3905	.67	.1972	1.90	.0604

$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$
1.95	.0584	3.4	.0260	7.0	.0078	20.0	.0011
2.00	.0564	3.5	.0248	8.0	.0062	25.0	.0007
2.1	.0527	3.6	.0237	9.0	.0050	30.0	.0005
2.2	.0494	3.8	.0218	10.0	.0041	35.0	.0004
2.3	.0464	4.0	.0201	11.0	.0035	40.0	.0003
2.4	.0437	4.2	.0185	12.0	.0030	45.0	.0002
2.5	.0412	4.4	.0172	13.0	.0026	50.0	.0002
2.6	.0389	4.6	.0160	14.0	.0022	99.0	.0001
2.7	.0368	4.8	.0149	15.0	.0019	100.0	.0001
2.8	.0349	5.0	.0139	16.0	.0017	∞	.0000
2.9	.0331	5.5	.0118	17.0	.0016		
3.0	.0314	6.0	.0101	18.0	.0014		
3.2	.0285	6.5	.0089	19.0	.0013		

NOTE.—The corresponding Table, deduced by Bresse, whose argument is $\frac{H+y}{H} = 1 + \frac{y}{H}$, may be at once obtained from the above by adding 1 to Tutton's argument.

In the case of a fall, Case II, Art. 13, in the limit

$$si = h = zH = 0,$$

and therefore, by eqs. (1) and (2),

$$\begin{aligned}\phi(z) + c &= 0 = \frac{1}{6} \log_e 1 + \frac{1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} + c \\ &= \frac{1}{\sqrt{3}} \frac{\pi}{6} + c,\end{aligned}$$

and

$$c = -\frac{1}{\sqrt{3}} \frac{\pi}{6} = -.3023.$$

The following Table, calculated by Tutton, gives the value of the backwater function, $\phi(z)$, *in the case of a fall* :

$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$	$\frac{y}{H}$	$\phi(z)$
0.	∞	.040	1.5448	.20	.9504	.45	.5753
.001	2.7876	.042	1.5279	.21	.9303	.46	.5633
.002	2.5562	.044	1.5117	.22	.9109	.47	.5515
.003	2.4207	.046	1.4962	.23	.8922	.48	.5398
.004	2.3244	.048	1.4813	.24	.8741	.49	.5282
.005	2.2497	.050	1.4670	.25	.8566	.50	.5167
.006	2.1885	.055	1.4335	.26	.8395	.51	.5054
.007	2.1368	.060	1.4027	.27	.8229	.52	.4941
.008	2.0920	.065	1.3743	.28	.8068	.53	.4829
.009	2.0525	.070	1.3479	.29	.7910	.54	.4717
.010	2.0171	.075	1.3231	.30	.7756	.55	.4607
.012	1.9554	.080	1.2999	.31	.7606	.56	.4497
.014	1.9036	.085	1.2779	.32	.7458	.57	.4388
.016	1.8584	.090	1.2571	.33	.7313	.58	.4279
.018	1.8185	.095	1.2372	.34	.7172	.59	.4171
.020	1.7827	.100	1.2185	.35	.7033	.60	.4064
.022	1.7502	.11	1.1831	.36	.6896	.65	.3536
.024	1.7206	.12	1.1504	.37	.6762	.70	.3019
.026	1.6936	.13	1.1201	.38	.6629	.75	.2510
.028	1.6678	.14	1.0918	.39	.6499	.80	.2004
.030	1.6441	.15	1.0651	.40	.6371	.90	.1001
.032	1.6219	.16	1.0399	.41	.6244	1.00	.0000
.034	1.6010	.17	1.0160	.42	.6119		
.036	1.5813	.18	.9931	.43	.5995		
.038	1.5626	.19	.9713	.44	.5873		

NOTE.—Bresse uses the same value $-.9069$ for the constant c both for a dam and for a *fall*. His argument in the latter case is $\frac{H-y}{H} = 1 - \frac{y}{H}$, and to obtain Bresse's Table from the above, the argument adopted by Tutton is subtracted from 1, and .6046 from the value of $\phi(z)$.

Dupuit, again, uses the argument $\frac{y}{H}$, and his Tables may be obtained from those given by Tutton by equating his back-water function to

$$1.4158 + \frac{y}{H} - \phi(z) \text{ for a rise,}$$

and to

$$2.0204 - \frac{y}{H} - \phi(z) \text{ for a fall.}$$

Dupuit neglects the term $\frac{\alpha c^2 i}{g}$ and includes in his back-water function, which may be designated $f(z)$, the term $z - z' \left(= \frac{y - y'}{H} \right)$ in equation (3), so that his formula becomes

$$\frac{is}{H} = f(z) - f(0.01),$$

considering that when $\frac{y}{H} = z = .01$, accurate measurement is no longer possible. Rühlmann gives the same rule.

17. Change of Section.—CASE I., Fig. 160. A channel of slope i , and in which the flow is steady, gradually contracts from a width $AA = B_1$ to a width $CC = B_2$, the surface of

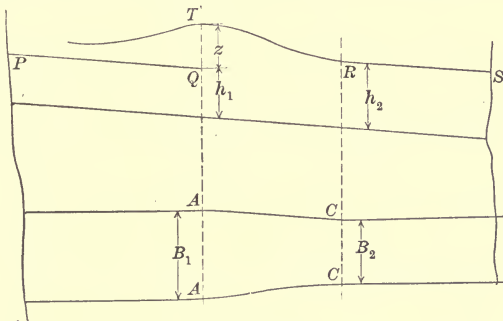


FIG. 160.

steady motion being PQ above AA , and RS below CC . On approaching AA the surface gradually rises and reaches its greatest height $QT = z$ above PQ at AA . This is followed by a gradual fall to the surface of steady motion RS at CC .

Let $h_1, h_2 (> h_1)$ be the depths corresponding to steady motion above AA and below BB , respectively.

“ m_1, m_2 be the mean hydraulic depths above AA and below BB , respectively.

“ u_1, u_2 be the mean velocities of flow above AA and below BB , respectively.

Then, disregarding the effect of surface resistance between AA and CC ,

$$z + h_1 + \frac{u_1^2}{2g} = h_2 + \frac{u_2^2}{2g},$$

or

$$z = h_2 - h_1 + \frac{u_2^2 - u_1^2}{2g}.$$

If the section is a rectangle,

$$h_1 B_1 u_1 = h_2 B_2 u_2.$$

But

$$\frac{b u_1^2}{m_1} = i = \frac{b u_2^2}{m_2}.$$

Therefore

$$h_2 = h_1 \frac{B_1 u_1}{B_2 u_2} = h_1 \sqrt{\frac{B_1^2 m_1}{B_2^2 m_2}}$$

and

$$z = h_1 \left(\sqrt{\frac{B_1^2 m_1}{B_2^2 m_2}} - 1 \right) + \frac{u_2^2 - u_1^2}{2g}.$$

If the width is great as compared with the depth,

$$m_1 = \frac{h_1}{2} \quad \text{and} \quad m_2 = \frac{h_2}{2}, \text{ approximately.}$$

Therefore

$$\frac{u_2^2}{u_1^2} = \frac{m_2}{m_1} = \frac{h_2}{h_1} = \frac{B_1 u_1}{B_2 u_2} = \sqrt[3]{\frac{B_1^2}{B_2^2}}$$

and

$$z = h_1 \left(\sqrt[3]{\frac{B_1^2}{B_2^2}} - 1 \right) + \frac{u_2^2 - u_1^2}{2g}.$$

CASE II. A channel of slope i , in which the flow is steady, $PQRS$ being the surface of steady motion, gradually contracts from a width $AA = B_1$ to a narrower width at CC . The

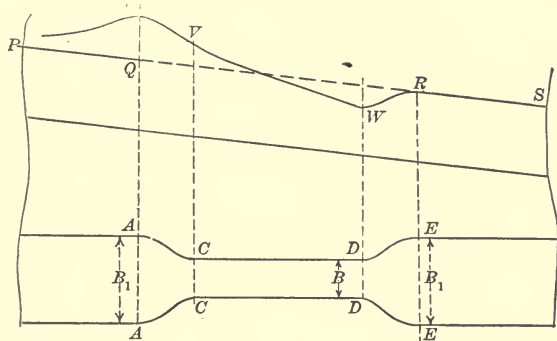


FIG. 161.

channel remains narrow for a limited distance CD and then gradually enlarges to its original size at E . On approaching AA the surface rises, attains its greatest height QT above PQ at A , falls to V at C , then to a point W below $PQRS$ at D , and finally suddenly rises from W to the surface of steady motion at R .

Let z be the depression of W below PS .

“ B, B_1 be the widths at D and E .

“ u, u_1 be the mean velocities at D and E .

Then

$$z = \alpha \frac{u^2 - u_1^2}{2g},$$

where α may be taken = 1.1.

If the section is a rectangle,

$$B(h_1 - z)u = B_1 u_1 h_1.$$

Therefore,

$$z = \alpha \frac{u_1^2}{2g} \left(\frac{B_1^2 h_1^2}{B^2 (h_1 - z)^2} - 1 \right),$$

a cubic equation giving z .

The surface DE may now be plotted, and QT may be found as in Case I.

These expressions also give, approximately, the depression below the surface $PQRS$ of steady motion when the channel has its section suddenly changed by such obstructions as bridge-piers.

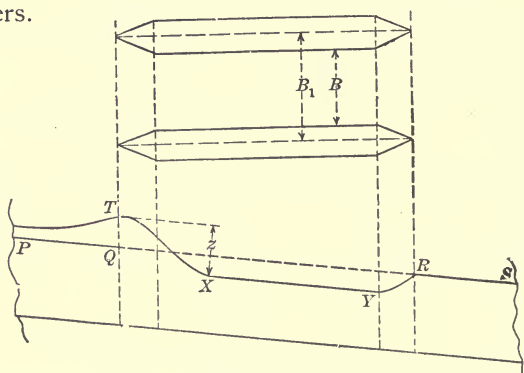


FIG. 162.

On approaching the pier ends the water-surface gradually rises to the maximum height T above PS , then falls to XY below PS between the piers, and finally rises again to the surface of steady motion on passing into the open channel.

Let B_1 , B be the distances between the axes and the inner faces of the piers.

Let H be the depth below XY .

Let z be the fall from T to X .

Then, according to Bresse, the value of z is given by the empirical formula

$$* z = \frac{\alpha Q^2}{2g} \left\{ \frac{1}{c_c^2 B^2 H^2} - \frac{1}{B_1^2 (H + z)^2} \right\},$$

c_c being a coefficient of contraction and having an average value of about .8. Also, Q is the discharge for the width B_1 of the channel.

* This formula, although generally adopted, is open to question. Bresse considers that an equally correct approximation is obtained at a distance of $20(B_1 - B)$ from the contraction by taking $z = 20iB \left(\frac{B_1}{B} - 1 \right)^2$.

18. Gauging of Streams and Watercourses.—The amount of flow Q in cubic feet per second across a transverse section of A sq. ft. in area is given by the expression

$$Q = Au,$$

u being the mean velocity of flow in the section in feet per second.

If the longitudinal profile and several transverse sections of a channel can be plotted, the volume of flow may be calculated by means of eq. (1), p. 275.

Let $u_1, u_2, \dots u_n$ be the mean velocities, $A_1, A_2, \dots A_n$ the areas, and $P_1, P_2, \dots P_n$ the wetted perimeters of n sections of the channel at the specified distances $l_1, l_2, \dots l_{n-1}$ apart. Then z , the fall in the free-surface level, which may be found from the longitudinal profile, is given by the equation

$$z = \alpha \frac{u_n^2 - u_1^2}{2g} + \int_0^l \frac{f}{m} \frac{u^2}{2g} ds,$$

in which

$$l = l_1 + l_2 + \dots + l_{n-1}, \quad \frac{f}{2g} = \frac{mi}{u^2} = b = \frac{1}{c^2},$$

and α may be taken = 1.1.

But $A_1 u_1 = Au = Q = A_2 u_2 = \dots = A_n u_n$, and $m = \frac{A}{P}$.

Therefore

$$z = \frac{\alpha Q^2}{2g} \left(\frac{1}{A_n^2} - \frac{1}{A_1^2} \right) + \frac{f Q^2}{2g} \int_0^l \frac{P}{A^3} ds,$$

and Q can be calculated as soon as the integration has been effected, which may be possible if P and A are known functions of s . An approximate value of the integral may be found graphically as follows:

Plot, as ordinates, the values of $\frac{P}{A^3}$ at the n sections, and join the upper ends of those ordinates. The area between the

extreme ordinates, the axis, and the line thus determined is the value required.

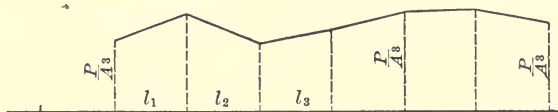


FIG. 163.

Generally speaking, however, the above method of gauging the flow of a stream is not very accurate, on account of the errors in the values of P , A , and the integral. More correct results are obtained by determining the mean velocity.

19. Determination of the Mean Velocity u . METHOD I.

The most convenient method for gauging small streams, canals, etc., is by means of a temporarily constructed weir, which usually takes the form of a rectangular notch. The greatest care should be exercised to insure that the crest of the weir is truly level and properly formed, and that the sides are truly vertical. The difference of level between the crest of the weir and the surface of the water at a point where it has not begun to slope down towards the weir is best estimated by means of Boyden's hook-gauge, Fig. 164.

This gauge consists of a carefully graduated rod, or of a rod with a scale attached, having at the lower end a hook with a thin flat body and a fine point. The rod slides in vertical supports, and a slow vertical movement is given by means of a screw of fine pitch. A stiff vertical rod, with a sharp point, having been placed at 5 to 8 ft. from the back of the weir, with the point on a level with the weir crest, water is run into the flume until it rises slightly above the crest, producing a capillary elevation at the point. The water is then allowed to subside until this elevation is barely

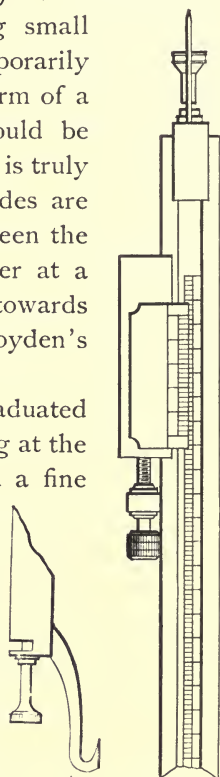


FIG. 164.

perceptible, when the rod may be removed. A hook-gauge is next placed in the same position, and the hook is slowly raised until a capillary elevation is produced over its point. The hook is then slowly lowered until the elevation becomes almost imperceptible, when a reading is taken corresponding to the level of the crest of the weir. More water now flows into the flume and over the weir. As soon as the motion has become steady, the hook is raised and the point adjusted at the surface in the manner just described. A second reading is taken and the difference between the two readings is the head of water over the crest.

In ordinary light, differences of level as small as the one-thousandth of a foot can be easily detected by the hook-gauge, while with a favorable light it is said that an experienced observer can detect a difference of two ten-thousandths of a foot. Such differences, however, cannot be measured under the ordinary conditions of practical work.

METHOD II. A portion of the stream which is tolerably straight and of approximately uniform section is defined by two transverse lines O_1AB , O_2CD at any distance S ft. apart.

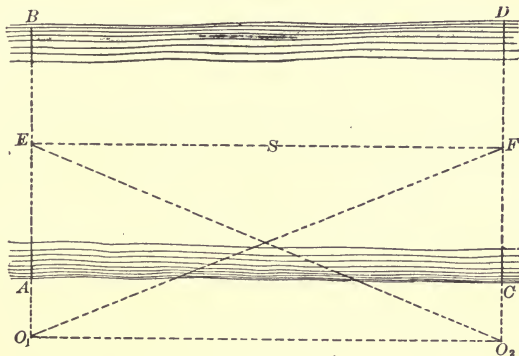


FIG. 165.

The base-line O_1O_2 is parallel to the thread EF of the stream, and observers with chronometers and theodolites (or

sextants) are stationed at O_1 , O_2 . The time T and path EF taken by a float between AB and CD can now be determined. At the moment the float leaves AB the observer at O_1 signals the observer at O_2 , who measures the angle O_1O_2E , and each marks the time. On reaching CD the observer at O_2 signals the observer at O_1 , who measures the angle O_2O_1F , and each again marks the time.

Experience alone can guide the observer in fixing the distance S between the points of observation. It should be remembered that although the errors of time observations are diminished by increasing S , the errors due to a deviation from lines parallel to the thread of the stream are increased.

A number of floats may be sent along the same path, and their velocities $\left(\frac{S}{T}\right)$ are often found to vary as much as 20 per cent and even more.

Having thus found the velocities along any required number of paths in the width of the stream, the mean velocity for the whole width can be at once determined.

Surface-floats are small pieces of wood, cork, or balls of wax, hollow metal and wood, colored so as to be clearly seen, and ballasted so as to float nearly flush with the water-surface and to be little affected by the wind.

Subsurface-floats.—A subsurface float consists of a heavy float with a comparatively large intercepting area, maintained

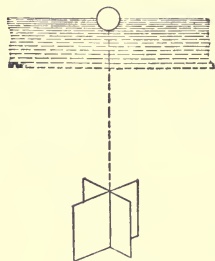


FIG. 166

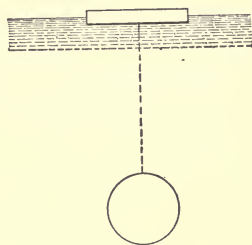


FIG. 167.

at any required depth by means of a very fine and nearly

vertical cord attached to a suitable surface-float of minimum immersion and resistance. Fig. 166 shows such a combination, the lower float consisting of two pieces of galvanized iron soldered together at right angles, the upper float being merely a wooden ball.

Another combination of a hollow metal ball with a piece of cork is shown by Fig. 167.

The motion of the combination is sensibly the same as that of the submerged float, and gives the velocity at the depth to which the heavy float is submerged.

Twin-floats.—Two equal and similar floats (Fig. 168), one denser and the other less dense than water, are connected by a fine cord. The velocity (v_t) of the combination is approximately the mean of the surface velocity (v_s) and of the velocity (v_d) at the depth to which the heavier float is submerged. Thus

$$v_t = \frac{v_s + v_d}{2},$$

and therefore

$$v_d = 2v_t - v_s,$$

so that v_d can be determined as soon as the value of v_t has been observed and the value of v_s found by surface-floats.

Velocity-rod.—This is a hollow cylindrical rod of adjustable

length and ballasted so as to float nearly vertical. It sinks almost to the bed of the stream, and its velocity (v_m) is approximately the mean velocity for the whole depth.

Francis gives the following empirical formula connecting the mean velocity (v_m) with the observed velocity (v_r) of the rod:

$$v_m = v_r \left(1.012 - .116 \sqrt{\frac{d'}{d}} \right),$$

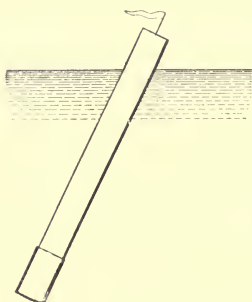


FIG. 169.

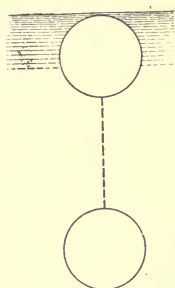


FIG. 168.

d being depth of stream, and d' the depth of water below bottom of rod; but d' should not exceed about one fourth of d .

METHOD III. *Pitot Tube and Darcy Gauge*.—A Pitot tube (Figs. 170 to 172) in its simplest form is a glass tube with a right-angled bend. When the tube is plunged vertically into the stream to any required depth z below the free surface, with its mouth pointing up-stream and normal to the direction of flow,

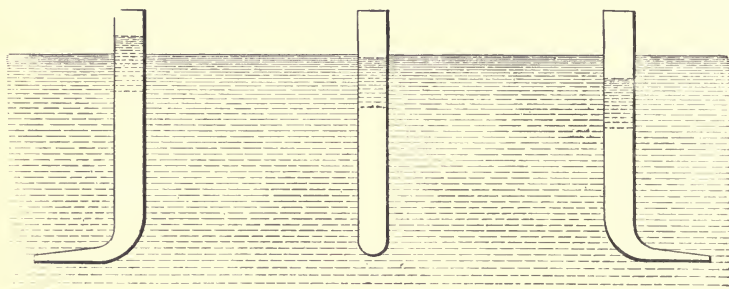


FIG. 170.

FIG. 171.

FIG. 172

the water rises in the tube to a height h above the outside surface, and the weight of the column of water, $z + h$ high, is balanced by the impact of the stream on the mouth. Hence (Chap. V)

$$wA(z + h) = wAz + kwA\frac{u^2}{2g},$$

and therefore

$$h = k\frac{u^2}{2g},$$

A being the sectional area of the tube, u the velocity of flow at the given depth, and k a coefficient to be determined by experiment.

A mean value of k is 1.19. With a funnel-mouth or a bell-mouth Pitot found k to be 1.5. This form of mouth, however, interferes with the stream-lines, and the velocity in front of the mouth is probably a little different from that in the unobstructed stream.

The advantages of tubes of small section are that the disturbance of the stream-lines is diminished and the oscillations

of the column of water are checked. Darcy found by careful measurement that the difference of level between the surfaces of the water-column in a tube of small section placed as in Fig. 170, and of the water-column placed as in Fig. 171 with its mouth parallel to the direction of flow, is almost exactly equal to $\frac{u^2}{2g}$.

When the tube is placed as in Fig. 172 with its mouth pointing downstream and normal to the direction of flow, the level of the surface of the water in the tube is at a depth h' below the outside surface, and

$$h' = k' \frac{u^2}{2g},$$

where k' is a coefficient to be determined by experiment and a little less than unity.

In this case the tube again obstructs the streamlines. Pitot's tube does not give measurable indications of very low velocities. A serious objection

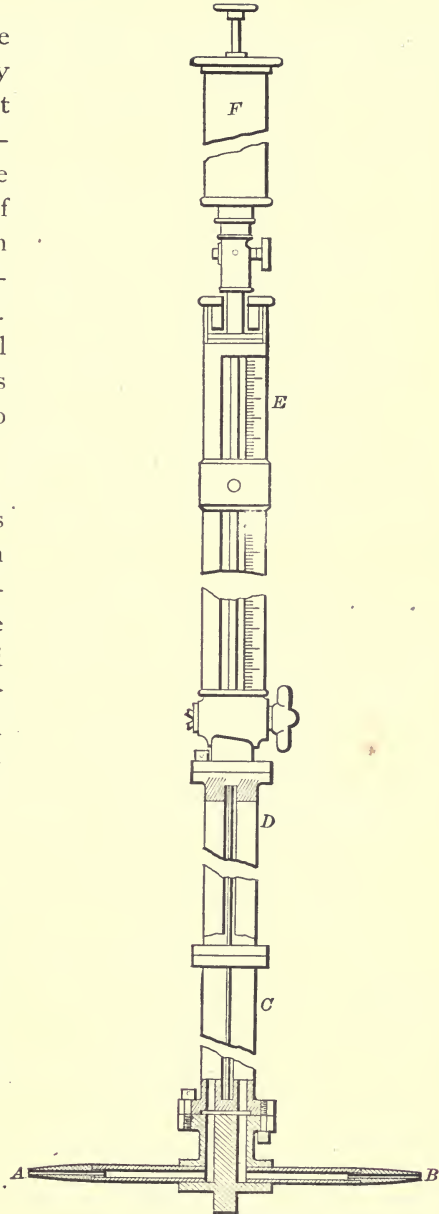


FIG. 173.

to the simple Pitot tube is the difficulty of obtaining accurate readings near the surface of the stream. This objection is removed in the case of Darcy's gauge, shown in the accompanying sketch, Fig. 173.

A and *B* are the water-inlets; *C* and *D* are two double tubes; *E* is a brass tube containing two glass pipes which communicate at the bottom with the water-inlets and at the top with each other, and with a pump *F* by which the air can be drawn out of the glass pipes, thus allowing the water to rise in them to any convenient height.

Thus Darcy's gauge really consists of two Pitot tubes connected by a bent tube at the top and having their mouths at right angles or pointing in opposite directions. If *h* is the difference of level between the water-surfaces in the tubes when the mouths are at right angles, then

$$k \frac{u^2}{2g} = h,$$

and Darcy's experiments indicate that *k* does not sensibly differ from unity.

When the mouths point in opposite directions, let *h*₁, *h*₂ be the differences of level between the stream-surface and the surfaces of the water in the tube pointing up-stream and the tube pointing down-stream, respectively. Then

$$h_1 = k_1 \frac{u^2}{2g},$$

$$h_2 = k_2 \frac{u^2}{2g},$$

and therefore

$$\begin{aligned} h_1 + h_2 &= (k_1 + k_2) \frac{u^2}{2g} \\ &= k \frac{u^2}{2g}, \end{aligned}$$

where $k = k_1 + k_2$.

h having been determined experimentally once for all, the difference of level ($= h_1 + h_2$) between the columns for any given case can be measured on the gauge and the value of u can then be found.

A cock may be inserted in the bend connecting the two tubes, and through this cock air may be exhausted and a partial vacuum created in the upper portion of the gauge. The water-columns will thus rise to higher levels, but the difference between them will remain constant. Thus the surface of the column in the down-stream tube may be brought above the level of the outside surface, and the reading is then easily made.

Sometimes the gauge is furnished with cocks at the lower parts of the tubes, and if these cocks are closed when the measurement is to be made, the gauge may be removed from the stream for the readings to be taken.

METHOD IV. *Current-meters*.—The velocity of flow in large streams and rivers is most conveniently and most accurately ascertained by means of the current-meter. The earliest form of meter, the Woltmann mill, is merely a water-mill with flat vanes, similar in theory and action to the wind-mill. When the Woltmann is plunged into a current, a counter registers the number of revolutions made in a given interval of time, and the corresponding velocity can then be determined. This form of meter has gone out of use and has been replaced by a variety of meters of greater accuracy, of finer construction, and much better suited to the work. In its simplest form the present meter consists of a screw-propeller wheel (Fig. 174), or a wheel with three or more vanes mounted on a spindle and connected by a screw-gearing with a counter which registers the number of revolutions. The meter is put in or out of gear by means of a string or wire. When a current velocity at any given point is to be found, the reading of the counter is noted, the meter is sunk to the required position, and is then set and kept in gear for any specified interval of

time. At the end of the interval the meter is put out of gear and is raised to the surface, when the reading of the counter is again noted. The difference between the readings gives the number of revolutions made during the interval, and the velocity is given by an empirical formula connecting the velocity and the number of revolutions in a unit of time.

The vane V is introduced to compel the meter to take a direction perpendicular to that of the stream-lines, but this may not necessarily be perpendicular to the axis of the stream. The slight error due to this discrepancy is usually disregarded in practice.

In order to prevent the mechanism of the meter from being

FIG. 174.

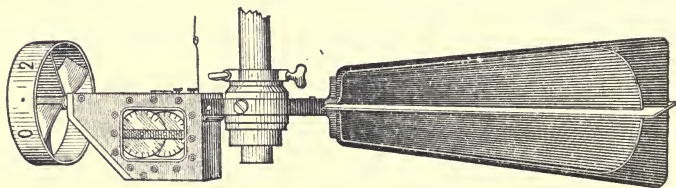
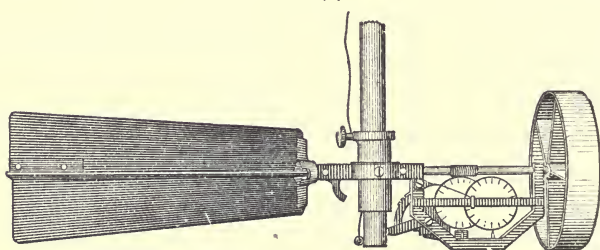


FIG. 175.

injuriously affected by floating particles of detritus, Revy enclosed the counter in a brass box, Fig. 175, with a glass face, and filled the box with pure water so as to insure a constant coefficient of friction for the parts which rub against each other. In the best meters, however, the record of the number of revolutions is kept by means of an electric circuit, Fig. 176,

which is made and broken once, or more frequently, each revolution, and which actuates the recording apparatus. The time at which an experiment begins and ends is noted, and the revolutions made in the interval are read on the counter, which may be kept in a boat or on the shore, as the circumstances of the case may require. The meter is usually attached to a

FIG. 176.

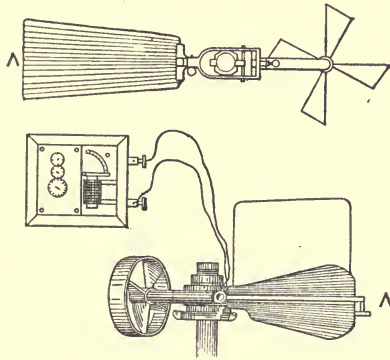


FIG. 177.

suitably graduated pole, so that the depth of the meter below the water-surface can be directly read. In deep and rapid water the meter must be held by a wire cord which will usually require to be guyed to a forward line. The mean velocity for the whole depth at any point of a stream may be found by moving the meter vertically down and then up, at a uniform rate. The mean of the readings at the two surface positions and at the bottom position will be the number of revolutions corresponding to the mean velocity required. The mean velocity for the whole cross-section may also be determined by moving the meter uniformly over all parts of the section.

The meter should be rated both before and after it is used. This is done by driving the meter at different uniform speeds.

through still water. Experiment shows that the velocity v and the number of revolutions n are approximately connected by the formula

$$v = an + b,$$

where a and b are coefficients to be determined by the method of least squares or otherwise.

Exner gives the formula

$$v^2 = c^2 n^2 + v_0^2,$$

v_0 being the velocity at which the meter just ceases to revolve.

OTHER METHODS.—Many other pieces of apparatus for the measurement of current velocities have been designed.

Perrodil's hydrodynamometer, for example, gives the velocity directly in terms of the angle through which a vertical torsion-rod is twisted, and in this respect is superior to the current-meter.

The tachometer or hydrometric pendulum (Fig. 178),

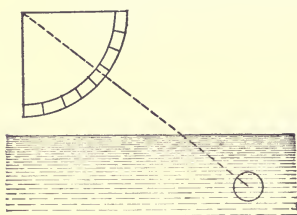


FIG. 178.

again, connects the velocity with the angular deviation from the vertical of a heavy ball suspended by a string in the current.

Hydrometric and torsion balances have also been devised, but they must be regarded rather as curiosities than as being of

any real practical use.

Having found the maximum surface velocity, v_s , at any point in a watercourse, by one of the above methods, then (Art. 10, p. 259) the mean velocity of the whole section is given by the empirical relation

$$u = \frac{4}{5} v_s.$$

If the transverse section of the waterway, at the point in ques-

tion, is plotted and its area, A , measured, the discharge, Q , may be at once calculated by means of the formula

$$Q = \frac{4}{5} A v_s.$$

Again, selecting an approximately straight length of channel, let x be the distance from the origin of a particle in the surface filament of maximum velocity. Then the velocity of this particle is $\frac{dx}{dt}$, and therefore

$$Q = Au = \frac{4}{5} A \frac{dx}{dt}.$$

Hence

$$Qt = \frac{4}{5} \int_0^s A dx,$$

t being the time in which the float passes over the distance s .

If this distance is now divided into n equal divisions, and if $A_0, A_1, A_2, A_3, \dots, A_n$ are the areas of the waterway at the commencement, at the $(n-1)$ intermediate division points and at the end of the length s , then, by Simpson's rule,

$$\int_0^s A dx = \frac{s}{3n} \left\{ A_0 + A_n + 2(A_2 + A_n + \dots + A_{n-2}) \right. \\ \left. + 4(A_1 + A_3 + \dots + A_{n-1}) \right\}.$$

The integration may also be at once effected if A is given as a function of x .

Again, if H is the depth of steady motion,

$$\frac{mi}{b} = v^2 = \frac{Q^2}{A^2},$$

and if the width B of the channel is large as compared with H , $m = H$, approximately, and $A = BH$.

Therefore

$$Q = B\sqrt{\frac{i}{b}}H^{\frac{3}{2}}.$$

At any given point in the stream B may be considered constant, i is also constant, and a coefficient m may be substituted for $B\sqrt{\frac{i}{b}}$. The actual depth h , which may be read on a fixed vertical scale at the point in question, differs from H by a certain quantity n . Thus the last equation may be written in the form

$$Q = m(h + n)^{\frac{3}{2}},$$

a convenient expression which is sometimes used to determine the volume of flow in wide rivers. The coefficients m and n are constant at the same point for all depths, but vary from point to point.

TABLE GIVING THE VALUES OF m AND n , THE UNIT BEING A METRE OR A FOOT.

Locality.	m		n		Authority.
	Metres.	Feet.	Metres.	Feet.	
Mantes bridge on the Seine.....	95	565	.7	2.3	Cuvinot
Roanne bridge on the Loire	180	1070	.25	.82	Graeff
Côme bridge on the Adda	100-3.2 h.	594-5.8 h.	.0	.0	Lombardini

NOTE.—From an examination of a large number of gaugings Bresse infers that $n = .85v_s$ gives better average results than $n = .8v_s$ (Art. 10 and p. 259). The latter, however, is equally safe unless it is necessary to provide against floods. (*Ann. des Ponts et Chaussées*, 1897.)

TABLE GIVING THE VALUES OF b AND c IN THE FORMULÆ $mi = bv^3$ AND $v = c\sqrt{mi}$, IN ACCORDANCE WITH BAZIN'S RESULTS.—Continued.

Value of μ , the Unit being a	VERY SMOOTH FACE, the Unit being a			SMOOTH FACE, the Unit being a			FACES NOT VERY SMOOTH, the Unit being a			BANKS IN EARTH, the Unit being a							
	Metre. b	Metre. c	Foot. b	Metre. b	Metre. c	Foot. b	Metre. b	Metre. c	Foot. b	Metre. b	Metre. c	Foot. b	Foot. c				
.25	.820	.000168	77.2	.0000512	139.5	.000243	64.1	.0000741	116.1	.000480	45.6	.000146	82.6	.001680	24.4	.0005120	44.2
.26	.853	.000167	77.3	.0000509	140.1	.000241	64.4	.0000735	116.5	.000471	46.1	.000144	83.5	.001626	24.8	.0004956	44.9
.27	.886	.000167	77.5	.0000509	140.5	.000239	64.7	.0000728	117.2	.000462	46.5	.000141	84.2	.001576	25.2	.0004804	45.6
.28	.919	.000166	77.6	.0000506	140.7	.000237	64.9	.0000722	117.6	.000454	46.9	.000138	84.9	.001530	25.6	.0004663	46.4
.29	.951	.000166	77.7	.0000506	140.9	.000236	65.1	.0000719	117.9	.000447	47.3	.000136	85.7	.001487	25.9	.0004532	46.9
.30	.984	.000165	77.9	.0000503	141.1	.000234	65.3	.0000713	118.3	.000440	47.7	.0001341	86.4	.001447	26.3	.0004410	47.5
.31	1.017	.000165	78.0	.0000503	141.3	.000233	65.5	.0000710	118.6	.000434	48.0	.0001323	86.9	.001409	26.6	.0004295	48.2
.32	1.050	.000164	78.1	.0000503	141.5	.000232	65.7	.0000707	119.0	.000428	48.4	.0001305	87.7	.001374	27.0	.0004188	48.9
.33	1.083	.000164	78.2	.0000503	141.7	.000230	65.9	.0000701	119.4	.000422	48.7	.0001286	88.2	.001341	27.3	.0004087	49.4
.34	1.116	.000163	78.3	.0000497	141.9	.000229	66.1	.0000698	119.7	.000416	49.0	.0001278	88.7	.001309	27.6	.0003980	50.0
.35	1.15	.000163	78.4	.0000497	142.0	.000228	66.2	.0000695	119.9	.000411	49.3	.0001253	89.3	.001280	28.0	.0003901	50.7
.36	1.18	.000163	78.5	.0000497	142.2	.000227	66.4	.0000692	120.3	.000407	49.6	.0001241	89.8	.001252	28.3	.0003813	51.3
.37	1.214	.000162	78.5	.0000494	142.2	.000226	66.5	.0000689	120.5	.000402	49.9	.0001225	90.3	.001226	28.6	.0003737	51.8
.38	1.247	"	78.6	"	142.4	.000225	66.7	.0000686	120.8	.000398	50.1	.0001213	90.7	.001201	28.9	.0003660	52.3
.39	1.280	"	78.7	"	142.5	.000224	66.8	.0000683	121.0	.000394	50.4	.0001201	91.3	.001177	29.2	.0003587	52.9
.40	1.312	.000161	78.8	.0000491	142.8	.000223	66.9	.0000680	121.2	.000390	50.6	.0001189	91.7	.001155	29.4	.0003520	53.2
.41	1.345	"	"	"	"	.000222	67.1	.0000677	121.5	.000386	50.9	.0001177	92.2	.001134	29.7	.0003456	53.8
.42	1.378	"	78.9	"	142.9	"	67.2	"	121.7	.000383	51.1	.0001167	92.6	.001113	30.0	.0003392	54.3
.43	1.411	.000160	"	.0000488	"	.000221	67.3	.0000674	121.9	.000380	51.3	.0001158	92.9	.001094	30.2	.0003334	54.7
.44	1.444	"	79.0	"	143.0	.000220	67.4	.0000671	122.1	.000376	51.5	.0001147	93.3	.001075	30.5	.0003277	55.2
.45	1.476	"	79.1	"	143.2	"	67.5	"	122.3	.000373	51.8	.0001136	93.8	.001058	30.7	.0003225	55.6
.46	1.509	"	"	"	"	"	67.6	"	122.4	.000370	52.0	.0001128	94.2	.001041	31.0	.0003173	56.1
.47	1.542	"	79.2	"	143.4	.000218	67.7	.0000664	122.6	.000368	52.2	.0001122	94.6	.001025	31.2	.0003124	56.5
.48	1.575	.000159	79.2	.0000485	143.4	.000218	67.8	.0000664	122.8	.000365	52.3	.00011.3	94.7	.001009	31.5	.0003075	52.0

TABLE GIVING THE VALUES OF b AND c IN THE FORMULÆ $mi = bc^2$ AND $v = c\sqrt{mi}$, IN ACCORDANCE WITH BAZIN'S RESULTS.—Continued.

Value of m , the Unit being a	VERY SMOOTH FACE, the Unit being a			SMOOTH FACE, the Unit being a			FACES NOT VERY SMOOTH, the Unit being a			BANKS IN EARTH, the Unit being a					
	Metre. b	c	Foot. b c	Metre. b c	Foot. b c	Metre. b c	Foot. b c	Metre. b c	Foot. b c	Metre. b c	Foot. b c				
.49	1.608	.000159	143.6	.000217	67.9	.0000661	123.0	.000362	52.5	.0001103	95.1	.000994	31.7	.0003030	57.4
.50	1.641	"	"	"	"	"	"	.000360	52.7	.0001097	95.5	.000980	31.9	.0002987	57.7
.51	1.673	"	143.8	.000216	68.0	.0000658	123.2	.000358	52.9	.0001091	95.8	.000966	32.2	.0002944	58.3
.52	1.706	"	"	"	68.1	"	123.4	.000355	53.0	.0001082	96.0	.000953	32.4	.0002905	58.7
.53	1.739	.000158	"	.000215	68.2	.0000655	123.5	.000353	53.2	.0001076	96.4	.000940	32.6	.0002865	59.0
.54	1.772	"	144.0	"	68.3	"	123.7	.000351	53.4	.0001070	96.7	.000928	32.8	.0002829	59.4
.55	1.805	"	"	.000214	"	.0000652	"	.000349	53.5	.0001064	96.9	.000916	33.0	.0002792	59.7
.56	1.837	"	"	"	68.4	"	123.9	.000347	53.7	.0001058	97.3	.000905	33.2	.0002758	60.1
.57	1.870	"	144.2	.000213	68.5	.0000649	124.1	.000345	53.8	.0001052	97.4	.000894	33.4	.0002725	60.5
.58	1.903	"	"	"	"	"	"	.000343	54.0	.0001045	97.8	.000883	33.6	.0002691	60.8
.59	1.936	"	"	"	68.6	"	124.3	.000342	54.1	.0001042	98.0	.000873	33.8	.0002661	61.2
.60	1.969	"	144.4	.000212	68.7	.0000646	124.4	.000340	54.2	.0001036	98.2	.000863	34.0	.0002630	61.5
.61	2.001	.000157	"	"	"	"	"	.000338	54.4	.0001030	98.5	.000854	34.2	.0002603	61.9
.62	2.034	"	"	.000211	68.8	.00006431	124.6	.000337	54.5	.0001027	98.7	.000845	34.4	.0002576	62.5
.63	2.067	"	144.5	"	"	"	"	.000335	54.6	.0001021	98.9	.000836	34.6	.0002548	62.6
.64	2.100	"	"	"	68.9	"	124.8	.000334	54.7	.0001018	99.1	.000827	34.8	.0002521	63.0
.65	2.132	"	"	"	"	"	"	.000332	54.9	.0001012	99.4	.000818	35.0	.0002493	63.4
.66	2.165	"	144.7	.000210	69.0	.0000640	125.0	.000331	55.0	.0001009	99.6	.000810	35.1	.0002469	63.6
.67	2.198	"	"	"	"	"	"	.000330	55.1	.0001006	99.8	.000802	35.2	.0002444	63.8
.68	2.231	"	"	"	69.1	"	125.2	.000328	55.2	.0001001	100.0	.000795	35.5	.0002423	64.3
.69	2.26	"	"	.000209	"	.0000637	"	.000327	55.3	.0000997	100.1	.000787	35.7	.0002399	64.6
.70	2.30	.000156	144.9	"	69.2	"	125.3	.000326	55.4	.0000994	100.3	.000780	35.8	.0002377	64.8
.71	2.33	"	"	"	"	"	"	.000325	55.5	.0000991	100.5	.000773	36.0	.0002356	65.2
.72	2.37	"	"	.000208	69.3	.0000634	125.5	.000323	55.6	.0000984	100.7	.000766	36.1	.0002335	65.4

TABLE GIVING THE VALUES OF b AND c IN THE FORMULÆ $mi = b\bar{c}^2$ AND $v = c^4 \bar{m}i$, IN ACCORDANCE WITH BAZIN'S RESULTS.—*Continued.*

Value of <i>m</i> , the Unit being a	VERY SMOOTH FACE, the Unit being a			SMOOTH FACE, the Unit being a			FACES NOT VERY SMOOTH, the Unit being a			BANKS IN EARTH, the Unit being a							
	Metre. <i>b</i>	Metre. <i>c</i>	Foot. <i>b</i>	Metre. <i>b</i>	Metre. <i>c</i>	Foot. <i>b</i>	Metre. <i>b</i>	Metre. <i>c</i>	Foot. <i>b</i>	Metre. <i>b</i>	Metre. <i>c</i>	Foot. <i>b</i>	Foot. <i>c</i>				
.73	2.40	.000156	80.0	.0000475	144.9	.000208	69.3	.0000634	125.5	.000322	55.7	.0000981	100.9	.000759	36.3	.0002313	65.7
.74	2.43	"	"	"	"	"	"	"	"	.000321	55.8	.0000978	101.1	.000753	36.4	.0002295	65.9
.75	2.46	"	80.1	"	145.1	"	69.4	"	125.7	.000320	55.9	.0000975	101.2	.000747	36.6	.0002277	66.3
.76	2.49	"	"	"	"	"	"	"	"	.000319	56.0	.0000972	101.4	.000741	36.8	.0002259	66.3
.77	2.53	"	"	"	"	.000207	69.5	.0000631	125.9	.000318	56.1	.0000969	101.6	.000735	36.9	.0002340	66.8
.78	2.56	"	"	"	"	"	"	"	"	.000317	56.2	.0000966	101.8	.000729	37.0	.0002222	67.0
.79	2.59	"	"	"	"	"	"	"	"	.000316	56.3	.0000963	102.0	.000723	37.2	.0002204	67.4
.80	2.62	"	80.2	"	145.3	"	69.6	"	126.1	.000315	"	.0000960	"	.000718	37.3	.0002188	67.5
.81	2.66	"	"	"	"	.000206	"	.0000628	"	.000314	56.4	.0000957	102.1	.000712	37.5	.0002170	67.9
.82	2.69	.000155	"	.0000472	"	"	"	"	"	.000313	56.5	.0000954	102.3	.000707	37.6	.0002155	68.1
.83	2.72	"	"	"	"	"	69.7	"	126.3	.000312	56.6	.0000951	102.5	.000702	37.8	.0002140	68.4
.84	2.76	"	"	"	"	"	"	"	"	.000311	56.7	.0000948	102.7	.000697	37.9	.0002124	68.6
.85	2.79	"	"	"	"	"	"	"	"	"	56.8	"	102.9	.000692	38.0	.0002109	68.8
.86	2.82	"	80.3	"	145.4	.000205	69.8	.0000625	126.4	.000310	"	.0000945	"	.000687	38.2	.0002094	69.2
.87	2.86	"	"	"	"	"	"	"	"	.000309	56.9	.0000942	103.0	.000682	38.3	.0002079	69.3
.88	2.89	"	"	"	"	"	"	"	"	.000308	57.0	.0000939	103.2	.000678	38.4	.0002067	69.5
.89	2.92	"	"	"	"	"	69.9	"	126.6	.000307	"	.0000936	"	.000673	38.5	.0002055	69.7
.90	2.95	"	"	"	"	"	"	"	"	"	57.1	"	103.4	.000669	38.7	.0002039	70.1
.91	2.98	"	"	"	"	"	"	"	"	.000306	57.2	.0000933	103.6	.000665	38.8	.0002027	70.2
.92	3.01	"	80.4	"	145.6	.000204	"	.0000622	"	.000305	"	.0000930	"	.000660	38.9	.0002012	70.4
.93	3.04	"	"	"	"	"	70.0	"	126.8	"	57.3	"	103.7	.000656	39.0	.0001999	70.6
.94	3.08	"	"	"	"	"	"	"	"	.000304	57.4	.0000927	103.9	.000652	39.2	.0001987	71.0
.95	3.12	"	"	"	"	"	"	"	"	.000303	"	.0000924	"	.000648	39.3	.0001975	71.1
.96	3.15	"	"	"	"	"	"	"	"	"	57.5	"	104.1	.000645	39.4	.0001966	71.3

TABLE GIVING THE VALUES OF b AND c IN THE FORMULÆ $mi = br^2$ AND $v = c\sqrt{mi}$, IN ACCORDANCE WITH BAZIN'S RESULTS.—*Continued.*

Value of m , the Unit being a	VERY SMOOTH FACE, the Unit being a			SMOOTH FACE, the Unit being a			FACES NOT VERY SMOOTH, the Unit being a			BANKS IN EARTH, the Unit being a			
	Metre. b	c	Foot. b	Metre. b	c	Foot. b	Metre. b	c	Foot. b	Metre. b	c	Foot. b	c
1.42	4.66	.000153	146.4	80.8	.000199	128.2	.000282	59.6	.0000860	.000526	43.6	.0001603	79.0
1.44	4.72	"	"	"	"	"	"	"	"	.000523	43.7	.0001594	79.2
1.46	4.79	"	"	"	70.9	128.4	.000281	59.7	.0000857	.000520	43.9	.0001585	79.5
1.48	4.86	"	"	"	"	"	"	59.8	"	.000516	44.0	.0001573	79.7
1.50	4.92	"	"	"	"	"	.000280	"	.0000854	.000513	44.1	.0001564	79.9
1.52	4.99	"	"	"	"	"	.000279	59.9	.0000851	.000510	44.3	.0001554	80.2
1.54	5.15	"	"	80.9	"	128.6	"	"	"	.000507	44.4	.0001545	80.4
1.56	5.12	"	"	"	71.0	"	.000278	60.0	.0000847	.000504	44.5	.0001536	80.6
1.58	5.18	"	"	"	"	"	"	"	"	.000502	44.7	.0001530	80.8
1.60	5.25	"	"	"	.000198	.0000604	.000277	"	.0000844	.000499	44.8	.0001521	81.2
1.62	5.32	"	"	"	"	"	"	60.1	"	.000496	44.9	.0001512	81.4
1.64	5.38	"	"	"	"	"	"	"	"	.000493	45.0	.0001503	81.5
1.66	5.45	"	"	"	71.1	128.8	.000276	60.2	.0000841	.000491	45.1	.0001497	81.7
1.68	5.51	"	"	"	"	"	"	"	"	.000488	45.3	.0001487	82.0
1.70	5.58	"	"	"	"	"	.000275	60.3	.0000838	.000486	45.4	.0001481	82.2
1.72	5.65	"	"	"	"	"	"	"	"	.000483	45.5	.0001472	82.4
1.74	5.71	"	"	"	"	"	.000274	60.4	.0000835	.000481	45.6	.0001466	82.6
1.76	5.78	"	146.7	81.0	"	"	"	"	"	.000479	45.7	.0001460	82.8
1.78	5.84	"	"	"	"	"	"	"	"	.000477	45.8	.0001454	82.9
1.80	5.91	"	"	"	.000197	.0000600	.000273	60.5	.0000832	.000474	46.9	.0001445	83.1
1.82	5.98	.000152	"	"	"	"	"	"	"	.000472	46.0	.0001439	83.3
1.84	6.04	"	.000463	"	"	"	"	60.6	"	.000470	46.1	.0000432	83.5
1.86	6.10	"	"	"	"	"	.000272	"	.0000829	.000468	46.2	.0001426	83.7
1.88	6.17	"	"	"	"	"	"	"	"	.000466	46.3	.0001420	83.8

TABLE GIVING THE VALUES OF b AND c IN THE FORMULÆ $mi = bv^2$ AND $v = c\sqrt{mi}$, IN ACCORDANCE WITH BAZIN'S RESULTS.—*Continued.*

Value of m , the Unit being a	VERY SMOOTH FACE, the Unit being a			SMOOTH FACE, the Unit being a			FACES NOT VERY SMOOTH, the Unit being a			BANKS IN EARTH, the Unit being a		
	Metre. b	c	Foot. b c	Metre. b c	b c	Foot. b c	Metre. b c	b c	Foot. b c	Metre. b c	b c	Foot. b c
3.90 12.79	.000151	81.3	.0000460	147.3	.000193	71.9	.0000588	130.2	.000255	62.6	.0000777	113.4
4.00 13.12	"	"	"	"	"	"	"	"	"	"	"	"
4.25 13.94	"	81.4	"	147.5	"	72.0	"	130.4	.000254	62.7	.0000774	113.5
4.50 14.76	"	"	"	"	"	"	"	"	.000253	62.8	.0000771	113.7
4.75 15.58	"	"	"	"	"	"	"	"	"	62.9	"	"
5.00 16.40	"	"	"	"	"	"	"	"	.000252	63.0	.0000768	114.1
5.25 17.22	"	"	"	"	"	72.1	"	130.6	.000251	63.1	.0000765	114.3
5.50 18.04	"	"	"	"	.000192	"	.0000585	"	"	"	"	"
5.75 18.86	"	"	"	"	"	"	"	"	.000250	63.2	.0000762	114.5
6.00 19.68	"	"	"	"	"	"	"	"	"	"	"	"
										.000338	54.4	.0001030
										.000370	52.0	.0001128
										.000368	52.2	.0001122
										.000362	52.5	.0001103
										.000358	52.9	.0001091
										.000354	53.2	.0001078
										.000350	53.5	.0001067
										.000347	53.7	.0001058
										.000344	53.9	.0001049
										.000341	54.2	.0001039
										.000338	54.4	.0001030

VALUES OF b AND c , FOR THE SIX CLASSES I TO VI, P. 250,
 IN BAZIN'S NEW FORMULA, $bv^2 = mi$, OR $v = \sqrt[4]{mi}$, WHERE
 $c\left(1 + \frac{y}{\sqrt{m}}\right) = 87 \sqrt[4]{mi}$, OR $= 157.6 \sqrt[4]{mi}$, ACCORDING AS THE
 UNIT IS A METRE OR A FOOT.

Value of m , the Unit being a		CLASS I, the Unit being a				CLASS II, the Unit being a				CLASS III, the Unit being a			
		Metre.		Foot.		Metre.		Foot.		Metre.		Foot.	
Metre	Foot.	b	c	b	c	b	c	b	c	b	c	b	c
.05	.16	.0146	68.5	.00445	124.1	.0197	50.7	.00600	91.8	.0352	28.4	.0107 ³	51.1
.06	.20	.0143	69.8	.00436	126.4	.0190	52.6	.00579	95.3	.0331	30.2	.0100 ⁹	54.5
.07	.23	.0141	70.9	.00430	128.4	.0185	54.2	.00564	98.2	.0315	31.7	.0096 ⁰	56.3
.08	.26	.0139	71.8	.00424	130.0	.0180	55.6	.00549	100.7	.0302	33.1	.0092 ⁰	59.9
.09	.30	.0138	72.5	.00421	131.3	.0176	56.7	.00536	102.7	.0287	34.4	.0088 ⁷	62.3
.10	.33	.0137	73.1	.00418	132.4	.0173	57.7	.00527	104.5	.0282	35.5	.0085 ⁶	64.3
.11	.36	.0136	73.6	.00415	133.3	.0170	58.7	.00518	106.3	.0274	36.5	.0083 ⁵	66.1
.12	.39	.0135	74.1	.00411	134.2	.0168	59.5	.00512	108.3	.0263	37.4	.0081 ⁷	67.7
.13	.43	.0134	74.6	.00408	135.1	.0166	60.2	.00506	109.0	.0262	38.2	.0079 ⁹	69.2
.14	.46	.0133	75.0	.00405	135.8	.0164	60.9	.00500	110.3	.0256	39.0	.0078 ⁰	70.6
.15	.49	"	75.3	"	136.3	.0163	61.5	.00497	111.4	.0252	39.7	.0076 ⁸	71.9
.16	.52	.0132	75.6	.00402	136.9	.0161	62.1	.00491	112.5	.0247	40.5	.0075 ³	73.4
.17	.56	"	75.9	"	137.4	.0160	62.7	.00488	113.6	.0243	41.2	.0074 ¹	74.7
.18	.59	.0131	76.2	.00399	138.0	.0158	63.2	.00482	114.5	.0240	41.8	.0073 ²	75.8
.19	.62	"	76.5	"	138.5	.0157	63.6	.00477	115.2	.0236	42.4	.0071 ⁹	76.9
.20	.65	.0130	76.7	.00396	138.9	.0156	64.1	.00476	116.1	.0233	42.9	.0071 ⁰	77.7
.21	.69	"	76.9	"	139.3	.0155	64.5	.00473	116.8	.0230	43.5	.0070 ¹	78.8
.22	.72	"	77.1	"	139.6	.0154	64.9	.00470	117.6	.0228	44.0	.0069 ⁵	79.7
.23	.75	.0129	77.3	.00393	140.0	.0153	65.2	.00467	118.1	.0225	44.4	.0068 ⁶	80.1
.24	.79	"	77.5	"	140.4	"	65.5	"	118.6	.0223	44.8	.0068 ⁰	81.2
.25	.82	"	77.6	"	140.5	.0152	65.9	.00463	119.3	.0221	45.3	.0067 ⁴	81.7
.26	.85	"	77.8	"	140.9	.0151	66.2	.00460	119.9	.0219	45.7	.0066 ⁸	82.8
.27	.88	.0128	78.0	.00390	141.3	.0150	66.5	.00457	120.4	.0217	46.1	.0066 ²	83.5
.28	.92	"	78.1	"	141.5	"	66.8	"	121.0	.0215	46.5	.0065 ⁴	84.2
.29	.95	"	78.3	"	141.8	.0149	67.0	.00454	121.4	.0213	46.9	.0064 ⁹	84.9
.30	.98	"	78.4	"	142.0	"	67.3	"	121.9	.0211	47.3	.0064 ³	85.7
.31	1.02	"	78.5	"	142.2	.0148	67.6	.00451	122.5	.0210	47.6	.0063 ⁹	86.2
.32	1.05	.0127	78.6	.00387	142.4	"	67.8	"	122.9	.0209	47.9	.0063 ⁷	86.7
.33	1.08	"	78.8	"	142.7	.0147	68.0	.00446	123.2	.0208	48.2	.0063 ⁴	87.3
.34	1.12	"	78.9	"	142.9	"	68.2	"	123.5	.0206	48.5	.0062 ⁸	87.8
.35	1.15	"	79.0	"	143.1	.0146	68.4	.00445	123.9	.0204	48.8	.0062 ²	88.3
.36	1.18	"	79.1	"	143.3	"	68.6	"	124.2	.0203	49.2	.0061 ⁹	89.1
.37	1.21	.0126	79.2	.00384	143.5	.0145	68.8	.00441	124.6	.0202	49.5	.0061 ⁶	89.6
.38	1.25	"	"	"	143.5	"	69.0	"	125.0	.0201	49.8	.0061 ³	90.2
.39	1.28	"	79.3	"	143.6	.0144	69.2	.00438	125.4	.0200	50.1	.0060 ⁹	90.7
.40	1.31	"	79.4	"	143.8	"	69.4	"	125.7	.0199	50.4	.0060 ⁶	91.2
.41	1.34	"	79.5	"	144.1	"	69.6	"	126.1	"	50.6	"	91.5
.42	1.38	"	79.6	"	144.2	.0143	69.7	.00436	126.2	.0197	50.9	.0060 ⁰	92.2
.43	1.41	"	79.7	"	144.4	"	69.9	"	126.7	.0196	51.1	.0059 ⁷	92.6
.44	1.44	.0125	"	.00381	"	"	70.1	"	127.0	.0195	51.4	.0059 ⁵	93.1
.45	1.47	"	79.8	"	144.5	.0142	70.2	.00433	127.2	.0194	51.6	.0059 ⁹	93.5
.46	1.51	"	79.9	"	144.7	"	70.4	"	127.4	.0193	51.8	.0058 ⁸	93.8
.47	1.54	"	80.0	"	144.9	"	70.5	"	127.7	.0192	52.0	.0058 ⁵	94.2
.48	1.57	"	"	"	"	"	70.6	"	127.9	.0191	52.3	.0058 ²	94.7
.49	1.61	"	80.1	"	145.1	.0141	70.8	.00430	128.2	"	52.5	"	95.1
.50	1.64	"	80.2	"	145.3	"	70.9	"	128.4	.0190	52.7	.0057 ⁰	95.5
.55	1.80	.0124	80.4	.00378	145.6	.0140	71.5	.00427	129.5	.0186	53.7	.0056 ⁷	97.3
.60	1.97	"	80.7	"	146.2	.0139	72.1	.00424	130.6	.0183	54.6	.0055 ⁸	98.9
.65	2.13	"	80.9	"	146.6	.0138	72.6	.00421	131.5	.0181	55.4	.0055 ²	100.3
.70	2.30	.0123	81.1	.00375	146.9	.0137	73.0	.00418	132.2	.0178	56.1	.0054 ³	101.6
.75	2.46	"	81.3	"	147.3	.0136	73.4	.00415	132.9	.0176	56.8	.0053 ⁷	102.9
.80	2.62	"	81.5	"	147.6	"	73.8	"	133.6	.0174	57.4	.0053 ⁰	104.0
.85	2.79	.0122	81.7	.00372	148.0	.0135	74.1	.00411	134.2	.0172	58.0	.0052 ⁴	105.1
.90	2.95	"	81.8	"	148.2	.0134	74.4	.00408	134.8	.0171	58.6	.0052 ¹	106.1

VALUES OF b AND c , FOR THE SIX CLASSES I TO VI, P. 250,
 IN BAZIN'S NEW FORMULA, $bv^2 = mi$, OR $v = c\sqrt{mi}$, WHERE
 $c\left(1 + \frac{r}{\sqrt{m}}\right) = 87\sqrt{mi}$, OR $= 157.6\sqrt{mi}$, ACCORDING AS THE
 UNIT IS A METRE OR A FOOT.

Value of m , the Unit being a		CLASS I, the Unit being a				CLASS II, the Unit being a				CLASS III, the Unit being a			
		Metre.		Foot.		Metre.		Foot.		Metre.		Foot.	
Metre	Foot.	b	c	b	c	b	c	b	c	b	c	b	c
.95	3.12	.0122	81.9	.00372	148.4	.0134	74.7	.00408	135.1	.0160	59.1	.00515	107.0
1.00	3.28	"	82.0	"	148.5	.0133	75.0	.00405	135.8	.0168	59.6	.00512	107.9
1.10	3.61	"	82.2	"	148.9	"	75.4	"	136.5	.0165	60.5	.00503	109.6
1.20	3.94	.0121	82.4	.00369	149.3	.0132	75.9	.00402	137.4	.0163	61.3	.00497	111.0
1.30	4.26	"	82.6	"	149.6	.0131	76.3	.00399	138.2	.0161	62.0	.00491	112.4
1.40	4.59	"	82.8	"	150.0	"	"	"	139.0	.0160	62.6	.00488	113.4
1.50	4.91	"	82.9	"	150.2	.0130	76.9	.00396	139.3	.0158	63.2	.00482	114.5
1.60	5.25	.0120	83.0	.00366	150.4	"	77.2	"	139.8	.0157	63.8	.00470	115.6
1.70	5.58	"	83.1	"	150.5	.0129	77.5	.00393	140.3	.0156	64.3	.00476	116.5
1.80	5.91	"	83.2	"	150.7	"	77.7	"	140.7	.0154	64.8	.00470	117.4
1.90	6.24	"	83.3	"	150.9	.0128	77.9	.00390	141.1	.0153	65.2	.00466	118.1
2.00	6.57	"	83.4	"	151.1	"	78.1	"	141.5	.0152	65.6	.00463	118.9
2.20	7.22	"	83.6	"	151.4	.0127	78.5	.00387	142.2	.0151	66.4	.00460	120.3
2.40	7.87	.0119	83.7	.00363	151.6	"	78.8	"	142.8	.0149	67.1	.00454	121.5
2.60	8.53	"	83.8	"	151.8	.0126	79.1	.00384	143.3	.0148	67.7	.00451	122.6
2.80	9.19	"	83.9	"	152.0	"	79.4	"	143.9	.0147	68.2	.00448	123.5
3.00	9.84	"	84.0	"	152.2	"	79.6	"	144.2	.0146	68.7	.00445	124.4
3.20	10.50	"	84.1	"	152.4	.0125	79.8	.00381	144.6	.0145	69.2	.00442	125.3
3.40	11.15	"	84.2	"	152.5	"	80.0	"	144.9	.0144	69.6	.00439	126.0
3.60	11.81	"	84.3	"	152.7	"	80.2	"	145.3	.0143	70.0	.00436	126.8
3.80	12.47	"	84.4	"	152.9	.0124	80.4	.00378	145.6	.0142	70.4	.00433	127.5
4.00	13.12	.0118	"	.00360	152.9	"	80.5	"	145.8	.0141	70.7	.00430	128.1
4.50	14.76	"	84.6	"	153.3	"	80.9	"	146.5	.0140	71.5	.00427	129.5
5.00	16.4	"	84.7	"	153.4	.0123	81.2	.00375	147.0	.0139	72.1	.00424	130.6
5.50	18.04	"	84.8	"	153.6	.0123	81.4	"	146.6	.0138	72.7	.00421	131.7
6.00	19.69	"	84.9	"	153.8	"	81.6	"	147.8	.0137	73.2	.00418	132.6
6.50	21.33	"	85.0	"	154.0	.0122	81.8	.00372	148.2	.0136	73.7	.00415	133.5
7.00	22.96	"	"	"	"	"	82.0	"	148.5	.0135	74.1	.00412	134.2
7.50	23.61	"	85.1	"	154.2	"	82.2	"	148.9	.0134	74.5	.00408	134.9
8.00	26.25	"	85.2	"	154.3	"	82.3	"	149.1	"	74.8	"	135.5
8.50	27.89	.0117	"	.00356	"	.0121	82.4	.00369	149.3	.0133	75.1	.00405	136.0
9.00	29.53	"	85.3	"	154.5	"	82.6	"	149.6	"	75.4	"	136.5
9.50	31.17	"	"	"	"	"	82.7	"	149.8	.0132	75.7	.00402	137.0
10.00	32.81	"	"	"	"	"	82.8	"	150.0	"	75.9	"	137.4
11.00	36.09	"	85.4	"	154.7	"	83.0	"	150.4	.0131	76.4	.00399	138.4
12.00	39.37	"	85.5	"	154.9	.0120	83.1	.00366	150.5	.0130	76.8	.00396	139.1
13.00	42.65	"	"	"	"	"	83.3	"	150.9	"	77.1	"	139.7
14.00	45.93	"	85.6	"	155.0	"	83.4	"	151.1	.0129	77.4	.00393	140.2
15.00	49.21	"	"	"	"	"	83.5	"	151.3	"	77.7	"	140.7
16.00	52.49	"	85.7	"	155.2	"	83.6	"	151.4	.0128	78.0	.00390	141.2
17.00	55.77	"	"	"	"	.0119	83.7	.00363	151.6	"	78.3	"	141.8
18.00	59.06	"	"	"	"	"	83.8	"	151.8	.0127	78.5	.00387	142.2
19.00	62.34	"	85.8	"	155.4	"	83.9	"	152.0	"	78.7	"	142.6
20.00	65.62	"	"	"	"	"	84.0	"	152.2	"	78.8	"	142.8

VALUES OF b AND c , FOR THE SIX CLASSES I TO VI, P. 250,

IN BAZIN'S NEW FORMULA, $bv^2 = mi$, OR $v = c\sqrt{mi}$, WHERE

$$c\left(1 + \frac{\gamma}{\sqrt{m}}\right) = 87\sqrt{mi}, \text{ OR } = 157.6\sqrt{mi}, \text{ ACCORDING AS THE}$$

UNIT IS A METRE OR A FOOT.

Value of m , the Unit being a		CLASS IV, the Unit being a				CLASS V, the Unit being a				CLASS VI, the Unit being a			
		Metre.		Foot.		Metre.		Foot.		Metre.		Foot.	
Metre	Foot.	b	c	b	c	b	c	b	c	b	c	b	c
.05	.16	.0552	18.1	.01682	32.8	.0784	12.8	.02390	23.2	.1015	9.9	.03094	17.9
.06	.20	.0514	19.4	.01567	35.1	.0725	13.8	.02210	25.0	.0937	10.7	.02856	19.4
.07	.23	.0484	20.6	.01475	37.3	.0680	14.7	.02073	26.6	.0876	11.4	.02670	20.6
.08	.26	.0461	21.7	.01405	39.3	.0644	15.5	.01963	28.1	.0827	12.1	.02521	21.9
.09	.30	.0441	22.7	.01345	41.1	.0613	16.3	.01868	29.5	.0786	12.7	.02366	23.0
.10	.33	.0424	23.6	.01292	42.7	.0588	17.0	.01792	30.8	.0751	13.3	.02239	24.1
.11	.36	.0410	24.4	.01250	44.2	.0566	17.7	.01725	32.1	.0722	13.9	.02201	25.2
.12	.39	.0397	25.2	.01210	45.6	.0547	18.3	.01667	33.2	.0696	14.4	.02121	26.1
.13	.43	.0386	25.9	.01177	46.9	.0530	18.9	.01615	34.2	.0673	14.9	.02055	27.0
.14	.46	.0376	26.7	.01146	47.5	.0515	19.4	.01570	35.1	.0653	15.3	.01990	27.7
.15	.49	.0367	27.2	.01119	49.3	.0501	19.9	.01544	36.0	.0625	15.8	.01936	28.6
.16	.52	.0359	27.8	.01094	50.3	.0489	20.4	.01490	36.9	.0618	16.2	.01884	29.3
.17	.56	.0352	28.4	.01073	51.4	.0478	20.9	.01457	37.9	.0603	16.6	.01839	30.1
.18	.59	.0345	29.0	.01052	52.5	.0467	21.4	.01423	38.8	.0589	17.0	.01795	30.8
.19	.62	.0339	29.5	.01033	53.4	.0458	21.8	.01396	39.5	.0577	17.3	.01759	31.4
.20	.65	.0334	30.0	.01018	54.3	.0449	22.3	.01369	40.4	.0565	17.7	.01722	32.1
.21	.69	.0328	30.5	.00997	55.2	.0441	22.7	.01345	41.1	.0554	18.1	.01689	32.8
.22	.72	.0323	30.9	.00985	56.0	.0434	23.1	.01324	41.8	.0544	18.4	.01658	33.7
.23	.75	.0319	31.4	.00972	56.9	.0427	23.4	.01302	42.4	.0535	18.7	.01631	33.9
.24	.79	.0315	31.8	.00960	57.6	.0420	23.8	.01280	43.1	.0526	19.0	.01603	34.4
.25	.82	.0310	32.2	.00945	58.3	.0414	24.2	.01262	43.8	.0518	19.3	.01579	34.9
.26	.85	.0307	32.6	.00936	59.0	.0408	24.5	.01244	44.4	.0510	19.6	.01555	35.5
.27	.88	.0303	33.0	.00924	59.8	.0403	24.8	.01229	44.9	.0502	19.9	.01528	36.0
.28	.92	.0300	33.4	.00915	60.5	.0397	25.2	.01210	45.6	.0495	20.2	.01507	36.5
.29	.95	.0297	33.7	.00905	61.0	.0393	25.5	.01198	46.2	.0489	20.5	.01489	37.1
.30	.98	.0293	34.1	.00893	61.8	.0388	25.8	.01183	46.8	.0482	20.7	.01468	37.4
.31	1.02	.0291	34.3	.00887	62.1	.0383	26.1	.01167	47.4	.0476	21.0	.01450	38.0
.32	1.05	.0288	34.7	.00878	62.9	.0379	26.4	.01155	48.0	.0471	21.2	.01435	38.4
.33	1.08	.0285	35.1	.00869	63.6	.0375	26.7	.01143	48.6	.0465	21.5	.01417	38.9
.34	1.12	.0283	35.4	.00863	64.1	.0371	26.9	.01131	48.7	.0460	21.7	.01402	39.3
.35	1.15	.0280	35.7	.00853	64.7	.0368	27.2	.01122	49.3	.0455	22.0	.01387	39.9
.36	1.18	.0278	36.0	.00847	65.2	.0364	27.5	.01110	49.9	.0450	22.2	.01372	40.1
.37	1.21	.0276	36.3	.00841	65.7	.0361	27.7	.01100	50.2	.0446	22.4	.01360	40.6
.38	1.25	.0274	36.6	.00835	66.3	.0357	28.0	.01088	50.8	.0441	22.7	.01344	41.1
.39	1.28	.0272	36.8	.00829	66.6	.0354	28.2	.01079	51.1	.0437	22.9	.01331	41.4
.40	1.31	.0270	37.1	.00823	67.1	.0351	28.5	.01070	51.7	.0433	23.1	.01319	41.8
.41	1.34	.0268	37.4	.00817	67.7	.0349	28.7	.01064	52.0	.0429	23.3	.01307	42.2
.42	1.38	.0266	37.6	.00811	68.1	.0346	28.9	.01055	52.3	.0426	23.5	.01298	42.5
.43	1.41	.0264	37.9	.00805	68.6	.0343	29.2	.01046	52.9	.0422	23.7	.01286	42.9
.44	1.44	.0262	38.1	.00799	69.1	.0340	29.4	.01036	53.2	.0418	23.9	.01274	43.3
.45	1.47	.0261	38.4	.00796	69.6	.0338	29.6	.01030	53.6	.0415	24.1	.01265	43.7
.46	1.51	.0259	38.6	.00789	69.9	.0335	29.8	.01021	54.0	.0412	24.3	.01256	44.0
.47	1.54	.0258	38.8	.00786	70.5	.0333	30.0	.01015	54.3	.0409	24.5	.01247	44.4
.48	1.57	.0256	39.1	.00780	70.8	.0331	30.2	.01009	54.7	.0405	24.7	.01234	44.7
.49	1.61	.0255	39.3	.00777	71.2	.0329	30.4	.01003	55.0	.0403	24.8	.01228	44.9
.50	1.64	.0253	39.5	.00771	71.5	.0326	30.6	.00994	55.4	.0400	25.0	.01189	45.3
.55	1.80	.0247	40.5	.00753	73.4	.0317	31.6	.00966	57.2	.0386	25.9	.01177	46.9
.60	1.97	.0241	41.4	.00734	75.0	.0308	32.5	.00939	58.9	.0375	26.7	.01143	48.3
.65	2.13	.0236	42.3	.00719	76.6	.0300	33.3	.00914	60.3	.0365	27.4	.01112	49.6
.70	2.30	.0232	43.1	.00707	78.1	.0294	34.1	.00896	61.8	.0356	28.1	.01085	50.9
.75	2.46	.0228	43.9	.00695	79.5	.0288	34.8	.00878	63.0	.0347	28.8	.01058	52.1
.80	2.62	.0224	44.6	.00683	80.8	.0282	35.5	.00850	64.3	.0340	29.4	.01036	53.0
.85	2.79	.0221	45.2	.00674	81.9	.0277	36.1	.00841	65.4	.0333	30.0	.01015	54.3

VALUES OF b AND c , FOR THE SIX CLASSES I TO VI, P. 250,
 IN BAZIN'S NEW FORMULA, $bv^2 = mi$, OR $v = c\sqrt{mi}$, WHERE
 $c\left(1 + \frac{r}{\sqrt{mi}}\right) = 87\sqrt{mi}$, OR $= 157.6\sqrt{mi}$, ACCORDING AS THE
 UNIT IS A METRE OR A FOOT.

Value of m , the Unit being a		CLASS IV, the Unit being a				CLASS V, the Unit being a				CLASS VI, the Unit being a			
		Metre.		Foot.		Metre.		Foot.		Metre.		Foot.	
Metre	Foot.	b	c	b	c	b	c	b	c	b	c	b	c
.90	2.95	.0218	45.9	.00663	83.1	.0273	36.7	.00829	66.5	.0327	30.6	.00997	55.4
.95	3.12	.0215	46.5	.00654	84.2	.0267	37.3	.00821	67.6	.0321	31.1	.00979	56.3
1.00	3.28	.0213	47.0	.00649	85.1	.0265	37.8	.00795	68.5	.0316	31.6	.00963	57.2
1.10	3.61	.0208	48.0	.00634	86.9	.0258	38.8	.00786	70.3	.0307	32.6	.00936	59.0
1.20	3.94	.0204	48.9	.00622	88.6	.0251	39.7	.00765	71.9	.0299	33.5	.00911	60.7
1.30	4.26	.0201	49.8	.00613	90.2	.0246	40.6	.00749	73.5	.0291	34.3	.00887	62.1
1.40	4.59	.0198	50.6	.00604	91.6	.0241	41.4	.00734	74.9	.0285	35.1	.00869	63.6
1.50	4.91	.0195	51.3	.00595	92.9	.0237	42.2	.00722	76.3	.0279	35.8	.00850	64.8
1.60	5.25	.0192	52.0	.00585	94.2	.0233	42.9	.00710	77.7	.0274	36.5	.00835	66.2
1.70	5.58	.0190	52.6	.00579	95.3	.0230	43.6	.00701	79.0	.0269	37.1	.00820	67.2
1.80	5.91	.0188	53.2	.00573	96.3	.0226	44.2	.00689	80.1	.0265	37.7	.00808	68.3
1.90	6.24	.0186	53.8	.00567	97.4	.0223	44.8	.00680	81.2	.0261	38.3	.00796	69.4
2.00	6.57	.0184	54.3	.00561	98.4	.0221	45.3	.00674	81.7	.0257	38.9	.00784	70.5
2.20	7.22	.0181	55.3	.00552	100.2	.0216	46.4	.00659	84.0	.0251	39.9	.00765	72.3
2.40	7.87	.0178	56.2	.00545	101.1	.0212	47.3	.00646	85.7	.0245	40.8	.00747	73.9
2.60	8.53	.0175	57.0	.00534	103.1	.0208	48.1	.00634	87.1	.0240	41.7	.00732	75.1
2.80	9.19	.0173	57.7	.00528	104.4	.0204	48.9	.00622	88.5	.0235	42.5	.00717	77.0
3.00	9.84	.0171	58.3	.00521	105.6	.0201	49.7	.00613	89.9	.0231	43.3	.00705	78.4
3.20	10.50	.0170	58.9	.00518	106.7	.0199	50.4	.00607	91.2	.0227	44.0	.00693	79.7
3.40	11.15	.0168	59.5	.00512	107.8	.0196	51.0	.00598	92.3	.0224	44.6	.00683	80.8
3.60	11.81	.0167	60.1	.00509	108.9	.0194	51.6	.00592	93.5	.0221	45.2	.00674	81.9
3.80	12.47	.0165	60.6	.00503	109.8	.0192	52.2	.00585	94.6	.0218	45.8	.00665	83.0
4.00	13.12	.0164	61.0	.00500	110.5	.0190	52.7	.00579	95.5	.0216	46.4	.00657	84.0
4.50	14.76	.0161	62.1	.00491	112.5	.0186	53.9	.00567	97.6	.0210	47.6	.00639	86.2
5.00	16.40	.0159	63.0	.00485	114.1	.0182	55.0	.00555	99.6	.0205	48.8	.00624	88.3
5.50	18.04	.0157	63.8	.00479	115.5	.0179	56.0	.00546	101.4	.0201	49.8	.00613	90.2
6.00	19.69	.0155	64.6	.00473	116.9	.0176	56.8	.00536	102.9	.0197	50.7	.00600	91.8
6.50	21.33	.0153	65.2	.00466	118.1	.0174	57.6	.00530	104.3	.0194	51.6	.00591	93.5
7.00	22.96	.0152	65.8	.00463	119.2	.0172	58.3	.00524	105.6	.0191	52.3	.00582	94.7
7.50	23.61	.0151	66.4	.00460	120.3	.0170	58.9	.00518	106.7	.0189	53.0	.00576	96.0
8.00	26.25	.0150	66.9	.00457	121.2	.0168	59.5	.00512	107.8	.0186	53.7	.00567	97.3
8.50	27.89	.0149	67.4	.00454	122.1	.0166	60.1	.00506	108.9	.0184	54.3	.00561	98.4
9.00	29.53	.0148	67.8	.00451	122.8	.0165	60.7	.00503	109.9	.0182	54.9	.00555	99.4
9.50	31.17	.0147	68.2	.00448	123.5	.0163	61.2	.00497	110.8	.0180	55.6	.00549	100.7
10.00	32.81	.0146	68.5	.00445	124.0	.0162	61.6	.00494	111.6	.0179	56.0	.00545	101.4
11.00	36.09	.0144	69.2	.00438	125.3	.0160	62.5	.00488	113.2	.0176	57.0	.00536	103.2
12.00	39.37	.0143	69.9	.00436	126.6	.0158	63.3	.00482	113.6	.0173	57.8	.00527	104.6
13.00	42.65	.0142	70.4	.00433	127.5	.0156	63.9	.00476	113.9	.0171	58.6	.00521	105.0
14.00	45.93	.0141	70.9	.00430	128.4	.0155	64.5	.00473	116.8	.0169	59.3	.00515	107.4
15.00	49.21	.0140	71.3	.00427	129.1	.0154	65.1	.00470	117.0	.0167	59.9	.00509	108.5
16.00	52.49	.0139	71.7	.00424	129.8	.0152	65.6	.00463	118.8	.0165	60.5	.00503	109.6
17.00	55.77	"	72.1	"	130.5	.0151	66.1	.00460	119.7	.0164	61.1	.00500	110.7
18.00	59.06	.0138	72.5	.00421	131.3	.0150	66.6	.00457	120.6	.0162	61.6	.00494	111.6
19.00	62.34	.0137	72.8	.00418	131.8	.0149	67.0	.00454	121.4	.0161	62.1	.00491	112.5
20.00	65.62	"	73.0	"	132.2	.0148	67.3	.00451	121.9	"	62.5	"	113.2

VALUE OF c IN GANGUILLET AND KUTTER'S FORMULA $v = c\sqrt{mi}$, ART. 8, THE UNIT BEING A METRE OR A FOOT.

Value of n .	The Slope per 40,000 being															
	Value of m , the Unit being a		1		2		4		8		16		40		100	
	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.
I. $n = .01$.05	.16	38	69	44	79	51	92	54	98	56	101	57	103	58	105
	.10	.33	49	89	56	101	61	110	65	118	68	123	70	127	71	128
	.20	.66	63	112	70	127	74	134	77	139	78	141	79	143	80	145
	.30	.98	72	130	77	139	81	147	84	152	85	154	86	156	86	156
	.50	1.64	83	150	86	156	88	159	90	163	91	165	91	165	91	165
	1.00	3.28	100	181	100	181	100	181	100	181	100	181	100	181	100	181
	2.00	6.56	115	208	111	201	109	197	107	194	106	192	105	190	105	190
	3.00	9.84	124	224	117	212	113	205	111	201	110	199	109	197	108	195
	5.00	16.40	134	243	123	223	118	214	115	208	113	205	112	203	111	201
	15.00	49.20	151	273	135	244	125	226	121	219	118	214	117	212	116	210
II. $n = .013$.05	.16	28	51	31	56	35	63	38	69	40	72	41	74	42	76
	.10	.33	36	65	40	72	44	79	47	85	49	89	50	91	51	92
	.20	.66	46	83	50	91	53	97	56	101	58	105	59	107	59	107
	.30	.98	53	97	57	103	60	109	63	114	64	116	64	116	65	118
	.50	1.64	62	112	65	118	67	121	69	125	69	125	70	127	70	127
	1.00	3.28	77	139	77	139	77	139	77	139	77	139	77	139	77	139
	2.00	6.56	90	163	87	158	85	154	84	152	83	150	82	148	82	148
	3.00	9.84	99	179	94	170	89	161	88	159	87	158	86	156	85	154
	5.00	16.40	108	195	100	181	93	168	91	165	90	163	89	161	88	159
	*15.00	49.20	125	226	114	206	102	185	98	177	96	174	94	170	92	167

VALUE OF c IN GANGUILLET AND KUTTER'S FORMULA $v = c\sqrt{m}i$, ART. 8, THE UNIT BEING A METRE OR A FOOT.

Value of n .	Value of m , the Unit being a	The Slope per 40,000 being											
		1		2		4		8		16		40	
		Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.
$n = .017$.05	19	34	22	40	24	43	26	47	28	51	29	53
	.10	25	45	29	53	32	58	34	62	35	63	36	65
	.20	34	62	37	67	39	71	41	74	42	76	42	76
	.30	40	72	43	78	45	81	46	83	47	85	47	85
	.50	47	85	49	89	50	91	51	92	51	92	52	94
	1.00	58	105	58	105	58	105	58	105	58	105	58	105
	2.00	71	128	69	125	67	121	66	119	65	118	64	116
	3.00	78	141	74	134	71	128	70	127	69	125	68	123
	5.00	87	158	79	143	75	136	73	132	72	130	71	128
	15.00	105	190	90	163	83	150	79	143	77	139	76	137
$n = .020$.05	15	27	18	33	20	36	21	38	23	42	23	42
	.10	21	38	23	42	25	45	28	51	29	53	29	53
	.20	21	38	23	42	25	45	28	51	29	53	29	53
	.30	33	60	35	63	37	67	38	69	39	71	40	72
	.50	40	72	41	74	42	76	43	78	43	78	44	79
	1.00	50	91	50	91	50	91	50	91	50	91	50	91
	2.00	61	110	59	107	57	103	56	101	56	101	55	100
	3.00	69	125	64	116	61	110	59	107	59	107	58	105
	5.00	76	137	70	127	66	119	63	114	62	112	61	110
	15.00	94	170	81	147	73	132	70	127	68	123	67	119

VALUE OF c IN GANGLUILLET AND KUTTER'S FORMULA $v = c\sqrt{mz}$, ART. 8, THE UNIT BEING A METRE OR A FOOT.

Value of m , the Unit being a		The Slope per 40,000 being															
		1		2		4		8		16		40		100			
		Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.		
$n = .025$.05	.16	12	22	13	24	15	27	16	29	17	31	18	33	18	33	
	.10	.33	17	31	18	33	19	34	20	36	21	38	22	40	22	40	
	.20	.66	22	40	23	42	24	43	25	45	26	47	27	49	27	49	
	.30	.98	26	47	28	51	29	53	30	54	30	54	31	56	31	56	
	.50	1.64	31	56	32	58	33	60	34	61	34	62	35	63	35	63	
	1.00	3.28	40	72	40	72	40	72	40	72	40	72	40	72	40	72	
	2.00	6.56	50	91	48	87	47	85	46	83	45	92	45	92	45	92	
	3.00	9.84	56	101	53	97	51	92	49	89	48	87	47	85	47	85	
	5.00	16.40	64	116	59	107	54	98	53	97	52	94	51	92	50	91	
	15.00	49.20	81	147	71	128	63	114	59	107	57	103	56	101	55	100	
$n = .030$.05	.16	10	18	11	20	12	22	13	24	13	24	14	25	14	25	
	.10	.33	13	24	14	25	15	27	16	29	17	31	18	33	18	33	
	.20	.66	18	33	19	34	19	34	20	36	21	38	22	40	22	40	
	.30	.98	21	38	22	40	23	42	24	43	24	43	25	45	25	45	
	.50	1.64	25	45	26	47	27	49	27	49	28	51	29	53	29	53	
	1.00	3.28	33	60	33	60	33	60	33	60	33	60	33	60	33	60	
	2.00	6.56	42	76	41	74	40	72	40	72	39	71	38	69	38	69	
	3.00	9.84	48	87	45	81	43	78	42	76	42	76	41	74	41	74	
	5.00	16.40	56	101	51	92	47	85	45	81	44	80	43	78	43	78	
	15.00	49.20	72	130	62	112	55	100	52	94	51	92	49	89	48	87	

VALUE OF c IN GANGUILLET AND KUTTER'S FORMULA $v = c\sqrt{mi}$, ART. 8, THE UNIT BEING A METRE OR A FOOT.

Value of n .		Value of m , the Unit being a		The Slope per 10,000 being											
				1		2		4		8		16		40	
				Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.
Value of n .	.035	.05	.16	8	14	9	16	9	16	10	18	10	18	11	20
		.10	.33	11	20	12	22	12	22	13	24	13	24	14	25
		.20	.66	15	27	16	29	16	29	17	31	17	31	18	33
		.30	.98	18	33	19	34	19	34	20	36	20	36	21	38
		.50	1.64	22	40	23	42	23	42	24	43	24	43	24	43
		1.00	3.28	29	53	29	53	29	53	29	53	29	53	29	53
		2.00	6.56	36	65	35	63	34	62	34	62	33	60	33	60
		3.00	9.84	42	76	40	72	38	69	37	67	36	65	36	65
		5.00	16.40	49	89	45	81	43	78	42	76	41	74	40	72
		15.00	49.20	65	118	56	101	51	92	47	85	45	81	44	80
Value of n .	.040	.05	.16	6	11	7	13	7	13	8	14	8	14	9	16
		.10	.33	9	16	10	18	11	20	11	20	12	22	12	22
		.20	.66	13	24	14	25	14	25	15	27	15	27	16	29
		.30	.98	15	27	16	29	17	31	18	33	18	33	18	33
		.50	1.64	19	34	19	34	20	36	20	36	21	38	21	38
		1.00	3.28	25	45	25	45	25	45	25	45	25	45	25	45
		2.00	6.56	32	58	31	56	31	56	30	54	30	54	29	53
		3.00	9.84	37	67	35	63	34	62	33	60	33	60	32	58
		5.00	16.40	44	79	41	74	39	71	38	69	37	67	36	65
		15.00	49.20	59	107	52	94	46	83	43	78	42	76	41	74

MANNING'S VALUES OF c IN THE FORMULA $v = c\sqrt{mi}$, THE UNIT BEING A METRE OR A FOOT.

Value of m .		Very Smooth Surface.		Smooth Surface.		Surface not very Smooth.		Rough Surface.	
Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.
.05	.16	61	110	47	85	36	65	30	54
.10	.33	68	123	52	94	40	72	34	62
.20	.66	76	137	59	107	45	81	38	69
.30	.98	82	148	63	114	48	87	41	74
.50	1.64	89	161	69	125	52	94	45	81
1.00	3.28	100	181	77	139	59	107	50	91
2.00	6.56	112	203	86	156	66	119	56	101
3.00	9.84	120	217	92	167	71	128	60	109
5.00	16.40	131	237	101	183	77	139	65	118
15.00	49.20	157	284	121	219	92	167	79	143

Value of m .		Surface in Earth.		Surface in Gravelly Soil.		Irregular Surface.		Very Irregular Surface.	
Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.	Metre.	Foot.
.05	.16	24	43	20	36	17	31	15	27
.10	.33	27	49	23	42	19	34	17	31
.20	.66	31	56	25	45	22	40	19	34
.30	.98	33	60	27	49	23	42	20	36
.50	1.64	36	65	30	54	25	45	22	40
1.00	3.28	40	72	33	60	29	53	25	45
2.00	6.56	45	81	37	67	32	58	28	51
3.00	9.84	48	87	40	72	34	62	30	54
5.00	16.40	52	94	44	79	37	67	33	60
15.00	49.20	63	114	52	94	45	81	63	114

EXAMPLES.

1. What fall must be given to a canal 2600 ft. long, 7 ft. wide at the top, 3 ft. wide at the bottom, $1\frac{1}{2}$ ft. deep, and conveying 40 cu. ft. of water per second? ($f = \frac{1}{64}$.) *Ans.* 1 in 135.

2. Determine the fall of a canal 1500 ft. long, of 2 ft. lower, 8 ft. upper breadth, and 4 ft. deep, which is to convey 70 cu. ft. of water per second? ($f = .008$.) *Ans.* 1 in 1088.4.

3. For a distance of 300 ft. a brook with a mean water perimeter of 40 ft. has a fall of 9.6 ins.; the area of the upper transverse profile is 70 sq. ft., that of the lower 60 sq. ft. Find the discharge. ($f = .008$.)

Ans. 352.12 cu. ft. per sec.

4. In a horizontal trench 5 ft. broad and 800 ft. long it is desired to carry off 20 cu. ft. discharge and to let it flow in at a depth of 2 ft.; what must be the depth at the end of the canal? ($f = .008$.)

Ans. 1.36 ft.

5. Water flows along an open channel 12 ft. wide and 4 ft. deep, at the rate of 2 ft. per second. What is the fall? A dam 12 ft. by 3 ft. high is formed across the channel; how high will the water rise over the crest of the dam?

Ans. 1 in 480, f being .08; 1.899 ft.

6. A stream is rectangular in section, 12 ft. wide, 4 ft. deep, and falls 1 in 100. Determine the discharge (1) with an air-perimeter; (2) without air-perimeter. ($f = .008$.)

Ans. (1) 646 cu. ft. per sec.

(2) 665.088 cu. ft. per sec.

7. A canal 20 ft. wide at the bottom and having side slopes of $1\frac{1}{2}$ to 1 has 8 ft. of water in it; find the hydraulic mean depth.

Ans. 5.163 ft.

8. The water in a semicircular channel of 10 ft. radius when full flows with a velocity of 2 ft. per second; the fall is 1 in 400. Find the coefficient of friction.

Ans. .2.

9. Calculate the flow per minute across a given section of a rectangular canal 20 ft. deep, 45 ft. wide, the slope of the bed being 22 ins. per mile and the coefficient of friction per square foot = .008.

Ans. 292.856 cu. ft.

10. Why does the water of a river rise on the formation of the ice?

11. Find the depth and width of a rectangular stream of 900 sq. ft. sectional area, so that the flow might be a maximum; also find the flow, f being .008 and the slope 22 ins. per mile.

Ans. 21.21 ft.; 42.42 ft.; 4885 cu. ft. per sec.

12. The section of an aqueduct is a trapezium with a bottom width of 6.56 ft., a top width of 7.546 ft., and a depth of 7.874 ft., the slope is 6 per 1000, and the faces of the aqueduct are of brickwork. Determine

the discharge in cubic feet per second when the depth of the water is 4.92 ft., using the coefficient given by (a) Bazin; (b) Kutter; (c) Manning.

Ans. (a) 471.276; (b) 494.5484; (c) 487.6973.

13. An aqueduct of rectangular section is to convey 9504 (Imp.) gallons of water per hour at the maximum velocity of flow. Assuming as a first approximation that $b = .000114$, and that the slope is .33 per 1000, find the proper width and slope. Also find the corresponding velocity of flow.

Ans. 1.01 ft. and 3 in 10,000; .828 ft. per sec.

14. What head is required to give a velocity 4 ft. per second in a semicircular channel of 3 ft. diameter and 5000 ft. long, f being .0064?

Ans. $10\frac{11}{16}$ ft.

15. The section of a length of La Roche Canal in rock has a bottom width of .7 m., one vertical face and the other face inclined to the horizon at an angle $\tan^{-1} 2$. The mean velocity of flow, when the water runs .5 m. deep, is .514 m. per second. Find the slope, a suitable value for the coefficient b or c being selected from the Tables.

Ans. .002.

16. A section of the La Roche Canal in earthwork has its sides sloped at 45° and has a bottom width of .3 m. When the depth of the water is 0.5 m. the discharge is at the rate of 205 litres per second. Determine the slope, a suitable value for the coefficient b being selected from the Tables. Also show that, according to Bazin's formula, the maximum surface and the bottom filament velocities are .816 m. and .49 m., respectively.

Ans. Slope = .002.

17. Water flows along a symmetrical channel, 20 ft. wide at top and 8 ft. wide at bottom; the friction at the sides varies as the square of the velocity, and is 1 lb. per square foot for a velocity of 16 ft. per second. Find the proper slope so that the water may flow at the rate of 2 ft. per second when its depth is 6 ft.

Ans. 1 in 3445.

18. Calculate the flow across the vertical section of a stream 4 ft. deep, 18 ft. wide at top, 6 ft. wide at bottom, the slope of the surface being 18 in. per mile. ($f = .008$.)

Ans. 110.9376 cu. ft. per sec.

19. The waterway in a channel of a regular trapezoidal section, has a sectional area of 100 sq. ft. If the banks slope at 40° to the horizontal, what will be the best dimensions for the section?

Ans. Bottom width = 5.25 ft.; depth of water = 7.22 ft.

20. The sides of an open channel of given inclination slope at 45° and the bottom width is 20 ft. Find the depth of water which will make the velocity of flow across a vertical section a maximum.

Ans. 6.73 ft.

21. The banks of a channel slope at 45° ; the flow across a transverse section is to be at the rate of 100 cubic feet at a maximum velocity of 5 ft. per second. Determine the dimensions of the transverse profile.

Ans. 11.05 ft. wide at bottom; 2.28 ft. deep.

22. What dimensions must be given to the transverse profile of a

canal whose banks slope at 40° , and which has to conduct away 75 cubic feet with a mean velocity of 3 ft. per second?

Ans. Depth = 3.6 ft.; width at bottom = 2.62 ft.

23. The section of a canal is a regular trapezoid; its slope is 1 in 500; its width at the bottom is 8 ft.; the sides are inclined at 30° to the vertical. On one occasion when the water was 4 ft. deep a wind was blowing up the canal, causing an air-resistance for each unit of free surface equal to one fifth of that for like units at the bottom and sides, where the coefficient of friction may be taken to be .08. Determine the discharge.

Ans. 75.34 cu. ft. per sec.

24. A canal is 20 ft. wide at the bottom, its side slopes are $1\frac{1}{2}$ to 1, its longitudinal slope is 1 in 360; calculate H.M.D. and the flow per minute across any given vertical section when there is a depth of 8 ft. of water in the canal. (Coeff. of friction = .008.)

If a weir 2 ft. high were built across the canal, what would be the increase in the depth of the water?

Ans. 5.24 ft.; 2762.7776 cu. ft. per sec.; 2.79 ft.

25. In the Ourcq canal the earthen banks slope at $\cot^{-1} 1\frac{1}{2}$, and the bottom width is 3.5. Find the depth of the water when the discharge is 3000 litres per second, the slope of the canal being .1236 per 1000. Also find the mean velocity.

Ans. 1.5 m. to 1.4 m.; .4 m. per sec.

26. The banks of a canal slope at 45° , the section being a trapezium. The discharge is to be 1200 litres per second at the rate of .5 m. per second. Find the best bottom width and depth and also the slope.

Ans. .94 m.; 1.14 m.; .0004 according to Bazin and .0003 according to Manning, the mean being .00035.

27. In the transverse section $ABCD$ of an open channel with a vertical slope of 1 in 300, the bottom width is 20 ft., the angle $ABC = 90^\circ$ and the angle $BCD = 45^\circ$. Find the height to which the water will rise so that the velocity of flow may be a maximum; also find the discharge across the section, f being .008.

Ans. 11.715 ft.; 1584 cu. ft. per second.

28. The sewers in Vancouver are square in section and are laid with one diagonal vertical. To what height should the water rise so that (a) the velocity of flow may be a maximum; (b) the discharge may be a maximum? (A side of the square = 12 in.)

Ans. (a) .292 ft. above horizontal diameter.

(b) .5797 ft. " " "

29. The section of a channel is a rhombus with a diagonal vertical. How high must the water rise in the channel (a) to give a maximum of flow, and (b) to give a maximum discharge?

Ans. If D is the length of the horizontal diameter, and if θ is the inclination of a side to the vertical, the water must rise above the horizontal diameter to the height $D \cot \theta \times .207$ in (a) and to the height $D \cot \theta \times .4099$ in (b).

30. An aqueduct has a given slope and a square section with a diag-

onal vertical. Show that the discharge at maximum velocity, the discharge when running full and the maximum discharge are in the ratios of 1 to 1.115 to 1.140, and that the corresponding mean depths are .293*a*, .25*a*, and .27*a*, *a* being a side of the square.

31. An aqueduct, with a section in the form of an isosceles right-angled triangle of height *h*, is laid with its base horizontal. Compare the quantities of water conveyed (*a*) when running full; (*b*) when the velocity is a maximum; (*c*) when the quantity conveyed is a maximum, and find the corresponding mean depths.

$$\begin{array}{lll} \text{Ans. Quantities,} & (a) \frac{c\sqrt{h^5i}}{2.1973}; & (b) \frac{c\sqrt{h^5i}}{2.0906}; & (c) \frac{c\sqrt{h^5i}}{2.0484}. \\ \text{Mean depths,} & (a) .207h; & (b) .2288h; & (c) .218h. \end{array}$$

32. A length of a circular aqueduct of waterway *A* and mean depth *m* has to be replaced by a length of an equivalent rectangular aqueduct. If the depth of the water is *y* and the width of the rectangular section *x*, show that

$$b'x^3y^3 = bA^2m(x + 2y),$$

the value of $\frac{m^2}{v^2}$ being *b'* for the pipe and *b* for the rectangular aqueduct.

Note.—In first approximations take *b* = *b'*.

33. Taking the coefficient *b* for a given open channel to be .00010058 and the corresponding coefficient $\left(= \frac{2f}{g} \right)$ for pipe-flow to be .00012485, show that, approximately, if the volume of flow under the same head is the same both for the channel and the pipe,

$$d^5P = 8A^3,$$

A being the sectional area of the waterway in the channel, *P* the wetted perimeter and *d* the diameter of the pipe.

34. Using the same coefficients as in the preceding example, show that the loss of head per unit of length in a pipe is nearly 88 per cent greater than the loss in an open semicircular channel of an equal waterway and giving the same discharge.

(*Note.*—*Since the whole of a pipe-surface develops resistance to flow, it is evident a priori that the loss of head per unit of length must be much greater than in the case of the open channel.*)

35. The Dhuis aqueduct, which supplies Pau with water, has a slope of 1 in 10,000. Its section is egg-shaped, the lowest portion being a semicircle of .7 m. radius. The aqueduct conveys, normally, 200 litres per second. Find the angle subtended at the centre of the semicircle by the water-line, and hence find the sectional area of the waterway, its depth and the velocity of flow. *Ans.* 154°; .55 sq. m.; .54 m.; 36 m. per sec.

36. Deduce the flow formula for a circular aqueduct of radius r , when the wetted perimeter subtends an angle of 240° at the centre.

$$\text{Ans. } r^3 i = .261 b Q^2.$$

37. A circular aqueduct of 6.56 ft. diam. conveys 49.44 cu. ft. of water per sec. The slope is 1 in 10,000. Find (a) the angle subtended at the centre by the water-line; (b) the clear head above the water surface; (c) the velocity of flow.

$$\text{Ans. (a) } 240^\circ 30'; (b) 1.63 \text{ ft.}; (c) 1.815 \text{ ft. per sec.}$$

38. The Avre circular aqueduct conveys 2.05 cm. per second, and in one length the slope is 4 in 10,000. Its water-line subtends 120° at the centre. Find the radius, taking $b = .0002$ as a first approximation.

The surface has a very smooth coat of cement .02 in. thick; determine the actual waterway, the wetted perimeter, the mean depth, the velocity of flow, and the clear height above the water-line.

$$\text{Ans. Radius} = .88 \text{ m.}; 1813 \text{ sq. m.}; 3.549 \text{ m.}; .51 \text{ m.}; 1.13 \text{ m. per sec.}; .445 \text{ m.}$$

39. The Potomac aqueduct, which is faced with brick, has a diameter of 9.0225 ft. and a slope of .143 in 10,000. The water-line subtends an angle of 240° at the centre. Taking $b = .0000609$, determine quantity of water conveyed in gallons per day.

$$\text{Ans. } 69,997,071 \text{ Imp. gallons.} \\ 84,019,066 \text{ U. S. gallons.}$$

40. Taking $b = .0000609$, find the angle subtended at the centre by the water-line and also find the free height above the water-surface in the Vanne aqueduct when conveying 49.442 cu. ft. per second, the diameter of the aqueduct being 6.562 ft., and the slope 1 in 10,000.

$$\text{Ans. } 240^\circ 30'.$$

41. Show that the quantities of water conveyed by a circular aqueduct of radius r , when the water-line subtends an angle of 240° at the centre, when the velocity of flow is greatest, when running full, and when the quantity conveyed is a maximum, are in the ratios of 1 to 1.086 to 1.131 to 1.188, and find the angles subtended at the centre by the water-lines in the three last cases. Also determine the mean hydraulic depths.

$$\text{Ans. Angles, } 257^\circ 27'; 360^\circ; 308^\circ.$$

$$\text{Mean depths, } .603r; .608r; .5r; .573r.$$

42. For a small tachometer the velocities are .163, .205, .298, .366, .61 metre; the number of revolutions per second are .6, .835, 1.467, 1.805, 3.142. Find the constants corresponding to the wheel.

$$\text{Ans. } .169; .061.$$

43. Assuming (1) that a river flows over a bed of uniform resistance to source; (2) that to maintain stability the velocity is constant from source to mouth; (3) that the river sections at all points are similar; (4) that the discharge increases uniformly in consequence of the supply from affluents—determine the longitudinal section of such a river.

$$\text{Ans. A parabola.}$$

44. In an aqueduct with a slope of 1 in 10,000, the depth of water corresponding to a condition of uniform steady motion is 1.77 ft. At a certain point the depth is increased to 4.43 ft. by a weir 3.77 ft. in height. Find the distance to which the "rise" extends along the aqueduct. *Ans.* 50,038 ft.

45. The channel of a river 328 ft. wide is narrowed by the abutments of a bridge to a width of 42.65 ft. The depth of the water under the bridge is 12.63 ft., and the quantity of flow per hour is 2,406.250 gallons. Find the height of swell. *Ans.* .104 ft.

46. In a broad channel of approximately rectangular section there is a small change of $n\%$ in the depth. Show that the corresponding changes in the velocity of flow and in the discharge are $\frac{1}{2}n\%$ and $1\frac{1}{2}n\%$ respectively. Also, if the banks slope at an angle θ , show that the changes become $\frac{nhv}{100}\left(\frac{b}{2A} - \frac{1}{P \sin \theta}\right)$ and $\frac{nhQ}{100}\left(\frac{3b}{2A} - \frac{1}{P \sin \theta}\right)$ respectively, h , b , A , P , v , and Q being the initial depth, breadth, area of waterway, wetted perimeter, velocity of flow, and discharge, respectively.

CHAPTER IV.

RAMS, PRESSES, ACCUMULATORS, WATER-PRESSURE ENGINES.

1. Hydraulic Rams.—By means of the hydraulic ram a quantity of water falling through a vertical distance h_1 is made to force a smaller weight of water to a higher level.

The water is brought from a reservoir through a supply-pipe S . At the end of this pipe there is a valve opening into an air-chamber C , which is connected with a discharge-pipe D . At E there is a weighted check- or clack-valve opening inwards, and the length of its stem (or the stroke) is regulated by means of a nut or cottar. When the waste-valve at E is open the water begins to escape with a velocity due to the head h_1 and suddenly closes the valve. The momentum of

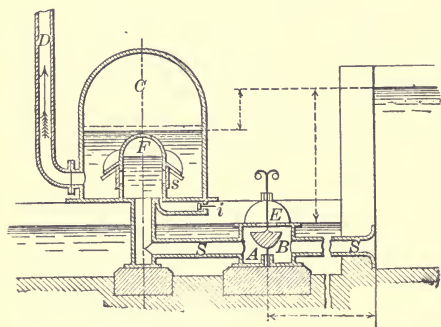


FIG. 179.

the water in the pipe opens the valve at B , and a portion of the water is discharged into the air-vessel. From this vessel it passes into the discharge-pipe in consequence of the reaction of the compressed air. At the end of a very short interval of time the momentum of the water has been destroyed, the valve

opening into the chamber C closes, the waste-valve again opens, and the action commences as before. It is found that

the efficiency of the ram is increased by introducing the small air-vessel F . The wave-motion started up in the supply-pipe by the opening and closing of the valve opening into the chamber C , has been utilized in driving a piston so as to pump up water from some independent source.

Let v be the velocity of flow in the supply-pipe at the moment when the valve at E is closed.

Let W_1 be the weight of the mass of water in motion.

Then $\frac{W_1 v^2}{g \cdot 2}$ is the energy of the mass, and this energy is expended in opening the valve at B , forcing the water into the air-chamber, compressing the air, and finally causing the elevation of a weight W_2 of the water through a vertical distance h' .

Let h_f be the head consumed in frictional and other hydraulic resistances.

Then

$$W_2(h' + h_f) = \text{the actual work done} = \frac{W_1 v^2}{g \cdot 2}.$$

This equation shows that, however great h' may be, W_2 has a definite and positive value, and therefore water may be raised to any required height by the hydraulic ram.

The efficiency of the machine $= \frac{W_2 h'}{W_1 h_1}$, and may be as much as 66 per cent if the machine is well made. According to d'Aubuisson,

$$\frac{W_2 h'}{W_1 h_1} = 1.42 - .28 \sqrt{\frac{h'}{h_1}}.$$

2. Hydraulic Press.—The hydraulic press is a machine by means of which great pressures can be exerted and heavy weights lifted, the energy being transmitted through water. It consists essentially of a strong cast-iron or cast-steel chamber or cylinder containing a plunger or ram which is acted

upon by water pumped through piping into the chamber by a single-acting force-pump, which may be either worked by hand or by power.

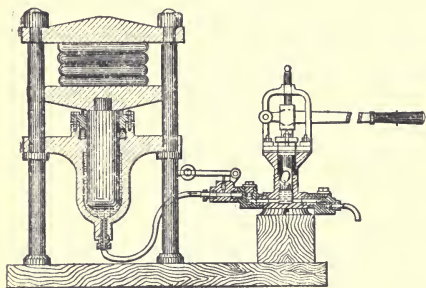


FIG. 180.

The action of the press depends on the principle that fluids press equally in all directions and thus the pressure per square inch on the ram is equal to the pressure per square inch on the pump-plunger. Originally discovered by Pascal, the press was first made of

practical utility by Bramah, who made the moving parts water-tight by the introduction of cup-leather packing.

The ram is packed with a leather collar of \cap form which is fitted into a recess turned out in the neck of the cylinder and is kept in place by the cylinder-cover gland.

According to experiments made by Hick, the friction at the collar increases directly with the diameter of the ram and with the pressure, but is independent of the depth of the collar. Hick's law of friction is expressed by the following formula:



FIG. 181.

$$\text{the total frictional resistance} = .0314dp \text{ or } .0471dp,$$

according as the leather is in good condition and well lubricated or is new and badly lubricated.

The friction is about 1 per cent of the pressure for a 4-in. ram.

At low pressures *hemp packing* is invariably used, and sometimes also for pressures as great as 2000 lbs. per sq. in., but, generally speaking, it is rarely used for pressures exceeding about 700 lbs. per sq. in. The ram is driven forwards by the pressure of the water through the tight collar, and is capable of lifting a weight or exerting a pressure which is limited in

magnitude only by the strength of the chamber and connections and by the capacity of the pump.

Let L be the stroke of the ram.

Let W be the weight on the ram, including the weight of the ram.

$$\text{Then the work done} = WL = \frac{\pi}{4} d^2 p L.$$

Let Q be the axial force on the plunger produced by a force P on the pump-lever at a distance p from the fulcrum.

Let q be the distance between the fulcrum and the axis of the plunger. Then, disregarding fluid friction, the friction at the fulcrum, and the leather or "packing" friction,

$$Pp = Qq.$$

$$\text{But } Q = \frac{\pi D^2}{4} p = \frac{\pi D^2}{4} \frac{W}{\frac{\pi d^2}{4}} = \frac{D^2}{d^2} W = \frac{Pp}{q},$$

$$\text{or } W = P \frac{p d^2}{q D^2}.$$

If r_1 , r_0 are the internal and external radii of a press, and if p_1 , p_0 , and f are the intensities of pressure at the internal and external surfaces and the intensity of stress at the radius r , then

$$f = \frac{p_0 r_0^2 - p_1 r_1^2}{r_0^2 - r_1^2} + \frac{p_0 - p_1}{r^2} \frac{r_0^2 r_1^2}{r_0^2 - r_1^2}.$$

(See Appendix, "Th. of Structures," Bovey.)

Hydraulic presses of different designs, but which are all more or less modifications of the Bramah, are employed for a variety of pressing and lifting operations. For example, they are used in making lead pipes, in expressing oil from seeds, in baling cotton, in pressing yarn, in packing hay, etc., while the modern systems of punching, riveting, stamping, forging,

shearing, welding, and bending depend upon the peculiar advantages of hydraulic power for such purposes.* Hydraulic presses for forging have largely superseded the steam-hammer,

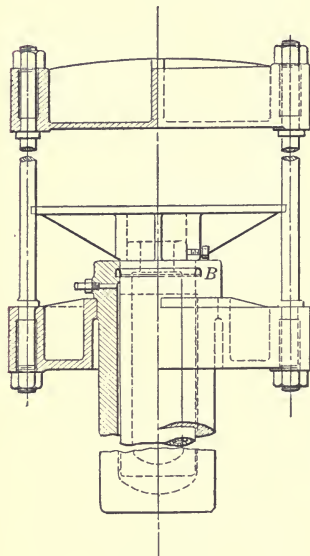


FIG. 182.
Hydraulic Press.

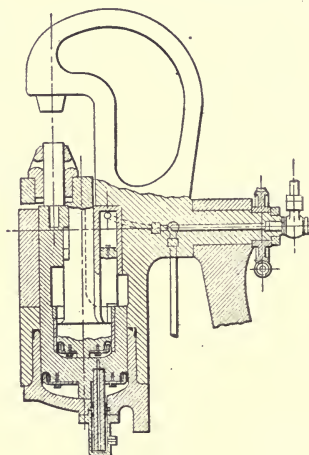


FIG. 183.
Portable Riveter.

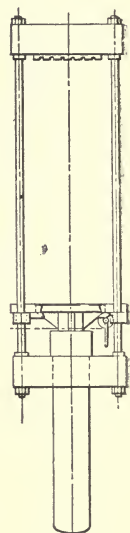


FIG. 184.
Baling-press.

and it is now common to find presses with capacities ranging from 4000 to 10,000 tons, the working intensity of pressure being as great as 3 tons per sq. in.

The hydraulic jack, Fig. 185, is a portable machine for raising heavy weights through short distances. It is a compact combination of a force-pump and a press. The ram *Q* fits the press *S* and is made water-tight by the cup-leather *D*. The pump is worked by the up-and-down movement of a lever which presses upon a cam or is connected with other suitable gearing and communicates motion to the pump-plunger *R*. The water in the chamber is thus forced through a valve into the hydraulic cylinder, developing a pressure which causes the ram to rise and to lift the load resting on the head *H*.

* In America compressed air is largely used for punching, riveting, etc.

As the pump-plunger rises a partial vacuum is produced in the pump-chamber, and the pressure in the reservoir *B* overcomes the resistance of the spring on the inlet-valve and opens a passage for the water into the pump-chamber. To lower the jack, a relief-valve is unscrewed, and the water returns to the reservoir *B* while the ram falls. The ram

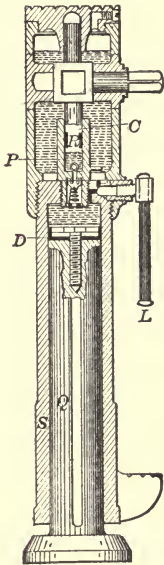


FIG. 185.

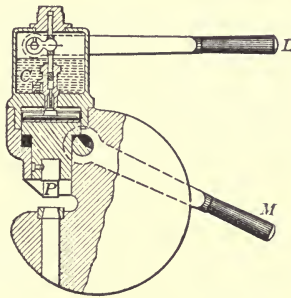


FIG. 186.

may be prevented from turning round by means of a steel set-pin screwed on the side of the press and fitting a vertical slot in the ram.

The construction and action of the punching-bear, Fig. 186, are essentially the same as in the hydraulic jack. By actuating the lever *L*, the water passes into the hydraulic cylinder *C* and by its action forces the punch *P* down. The punch is raised by first opening a relief-valve and then lowering the lever *M*, which causes the cam to raise the hydraulic ram, and the water from the hydraulic cylinder flows back into the reservoir. The relief-valve is now closed and the punching operation may be again repeated.

3. Accumulator.—Low pressures of 170 lbs. (= 392 ft.) to 250 lbs. (= 576 ft.) per sq. in. can sometimes be obtained from a natural supply or from a reservoir, but the higher

pressures of 700 lbs. (= 1612 ft.) to 1000 lbs. (= 2304 ft.) per sq. in. and upwards, which are almost exclusively adapted to the working of intermittent machines, must be artificially produced by means of pumping-engines. In a direct supply the capacity of these engines must be sufficient to meet the maximum demand at any moment, but the fluctuation in the demand upon the mains for cranes, capstans, elevators, etc., was soon found to be so great as to render imperative some method of *storing* energy. This has been effected by the introduction of the *accumulator*, which, in its simplest form, consists of an annular cylinder (Fig. 187) partially or wholly filled with scrap, slag, or other heavy material, or of a series of trays (Fig. 188) loaded with pig iron or lead, supported by a cross-head on the top of a ram working in a cylinder with a

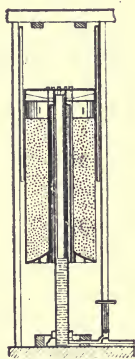


FIG. 187.

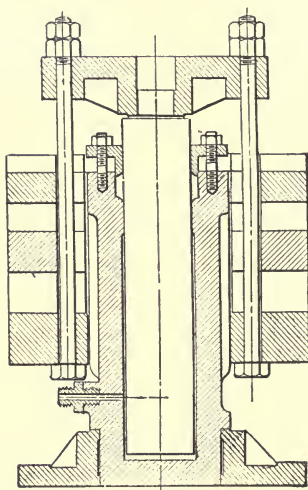


FIG. 188.

stuffing-box and gland at the upper end. The pressure-water is admitted by a branch pipe at the lower end and raises the ram together with the weight it carries. Thus, if W tons are lifted through a vertical distance s and if the water-pressure on

the ram of d in. diameter is p lbs. per sq. in., the total store of energy in foot-pounds

$$= 2240Ws = \frac{\pi d^2}{4} sp.$$

When the accumulator has reached the highest point it actuates a lever which shuts off the steam so that the engines cease to work and the accumulator falls. When it has reached the lowest point it again actuates a lever which opens a valve and admits steam. The engines again commence to work and the accumulator rises.

In small plants the accumulator fully provides for the *storage* of sufficient energy to meet the momentary fluctuations of demand for the power necessary to work machines which are intermittent in action, and without the accumulator pumping-engines of greater capacity would be required. In large plants, as in the cities of London, Manchester, and Glasgow, the total accumulator storage capacity is a very small fraction of the total supply, and at the times when the demand is heavy the accumulators are usually almost stationary. In such cases they may be considered rather as *regulators* of pressure. They are also of great importance in automatically facilitating the control of the plant, and act as buffers in preventing breakage and shocks. If lack of space prevents the use of an accumulator of the type just described, an *intensifier*, Fig. 189, may be employed. Water at a pressure of p lbs. per sq. in. is admitted from the water-mains or from a tank at a suitable elevation to the lower side of a piston of diameter D ins., working in an hydraulic cylinder. The piston-rod of diameter d ins. forms the ram of the accumulator B , and works through a water-tight neck. Thus the pressure in the accumulator in lbs. per sq. in.

$$= \frac{\frac{\pi D^2}{4} p}{\frac{\pi d^2}{4}} = \frac{D^2}{d^2} p,$$

and this is also the intensity of the pressure in the hydraulic mains *C*.

Tweddell's *differential* accumulator, Fig. 190, is also designed for cases in which space is of importance. A heavy cylinder *A*, with the usual glands and cup-leathers at the top

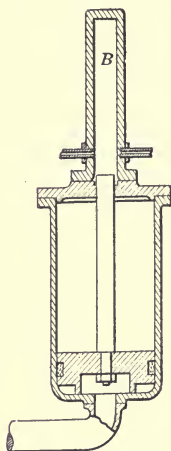


FIG. 189.

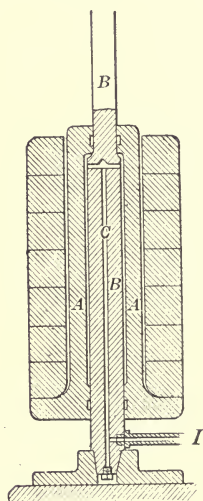


FIG. 190.

and bottom, is loaded with a number of lead or cast-iron weights *W*, fitted into each other, and slides upon a ram *B*, fixed at the upper end by a bracket and at the lower by a step. A brass liner is shrunk upon the lower portion of the ram* so that its diameter is slightly greater than that of the upper portion. A hollow passage *C* is drilled axially along the ram and connects with a cross-passage just above the brass liner. The water is pumped through the inlet-pipe *I*, fills these passages and exerts an upward pressure over an effective area equal to the *difference* between the areas of the lower and upper portions of the ram. Thus very heavy pressures, up to 2000 lbs. per sq. in., or more, can be readily obtained with a comparatively small weight. But the volume of water is

* The ram, however, is usually solid steel.

small, and any large demand for power will cause the loaded cylinder to fall rapidly, so that when it is brought to rest a considerable increase of pressure is developed which is of advantage in punching, riveting, etc. The uppermost weight is connected by means of a chain with a relief-valve which enables the limiting positions of the cylinder to be automatically regulated.

Let W be the *total* dead weight lifted.

Let F be the friction of each of the cup-leathers.

Let d_1, d_2 be the diameters of the lower and upper portions of the ram.

With the cylinder at the height x above its lowest position, let p_1 be the intensity of pressure in the inlet-pipe I when the cylinder is rising, and p_2 the intensity when it is falling. Then

$$p_1 = wx + \frac{W + 2F}{\frac{\pi}{4}(d_1^2 - d_2^2)},$$

$$p_2 = wx + \frac{W - 2F}{\frac{\pi}{4}(d_1^2 - d_2^2)}.$$

Hence an approximate measure of the variation of the intensity of pressure is

$$p_1 - p_2 = \frac{16F}{\pi(d_1^2 - d_2^2)},$$

and the value of this variation is ordinarily from about 1 per cent of the pressure for a 16-in. ram to about 4 per cent for a 4-in. ram.

Experiment has shown the efficiency of an accumulator to be as high as 98 per cent, 1 per cent being lost in charging and 1 per cent in discharging. Its total store of energy is comparatively small and it cannot maintain a supply for any length of time, but it possesses the great advantage of being able to use its energy at a high rate for a short period.

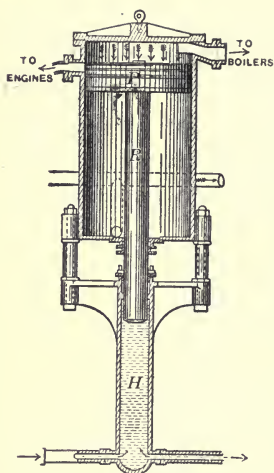


FIG. 191.

being drawn off. This accumulator is specially for use on ships.

4. Water-pressure Engines.—In these engines water under pressure is admitted into a strong chamber or cylinder, and acts upon a piston or plunger in precisely the same manner as in the case of the steam-engine. The cylinder is made of gun-metal or of cast iron, and its thickness t , which is relatively large on account of the wear, may be calculated from the formula

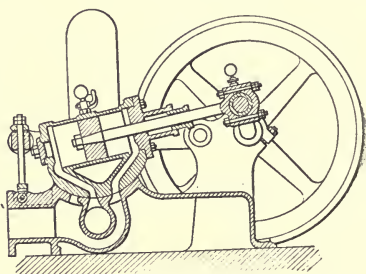


FIG. 192.

$$t \text{ ins.} = .0024 p_a d + 1.25 \text{ ins.},$$

p_a being the pressure in atmospheres, and d the diameter in inches.

The frictional resistances and the possibility of severe shocks are increased by rapid motion and reversals of motion. Hence the velocity of flow in the supply-pipe should not exceed 10 ft.

per second, and preferably should be limited to 6 ft. per second (Art. 11, p. 156), while the plunger should have a long stroke. In practice the stroke is usually from $2\frac{1}{2}$ to 6 times

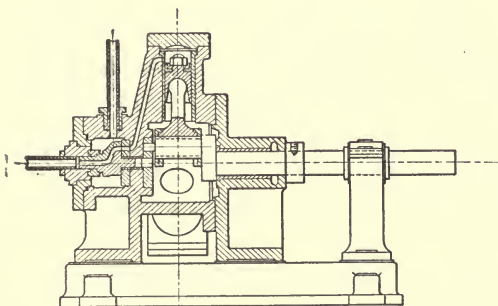


FIG. 193.—Sectional Elevation.

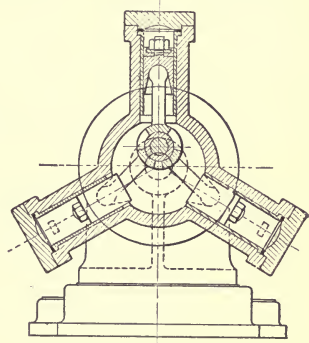


FIG. 194.—Cross-section.

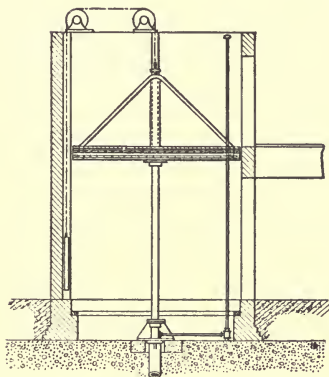


FIG. 195.—Freight-hoist.

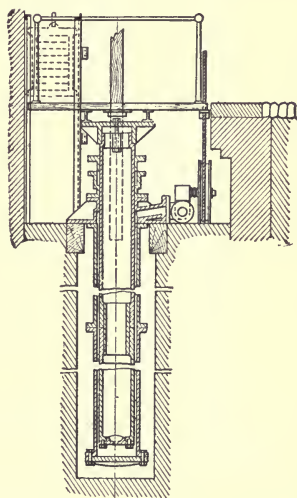


FIG. 196.—Balanced-ram Lift.

the diameter of the cylinder, and the mean velocity of the plunger is about 1 ft. per second, rarely exceeding 80 ft. per minute. As the water is practically incompressible, its free and immediate passage should be insured by means of large

and wide-open ports. An important advantage connected with this property of incompressibility is that the hydraulic resistances may be indefinitely increased by simply closing a valve. Thus no brakes are required, but the water contains within itself its own brake, and an absolute control is provided which secures the highest degree of safety.

The water-pressure engine is necessarily a slow-moving machine, and is both cumbrous and costly unless actuated by pressures of great intensity. These engines are advantageously employed in working cranes, hoists, elevators, capstans, dock-gates, presses, and other machinery in which the action is of an intermittent character.

The hydraulic-ram lift, Fig. 197, more completely utilizes

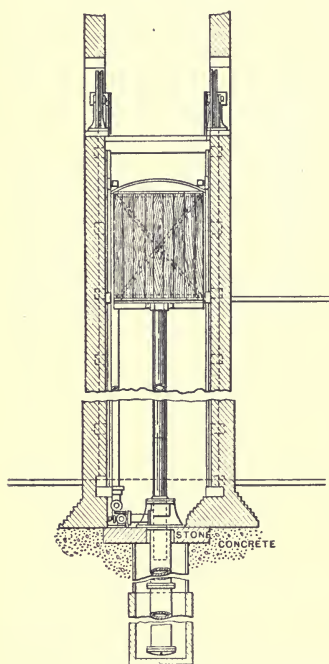


FIG. 197.

than any other the properties of incompressibility and direct pressure, and, owing to its greater safety, its adoption is sometimes recommended for elevators of considerable height. Under a full load its efficiency may be as great as 95 per cent. The speed of a suspended lift is rarely less than 100 ft. per minute and often exceeds 500 or 600 ft. per minute. Between such limits a large variation in the efficiency might be expected, and although the efficiency under a full load, even when the ram-stroke is multiplied 8 or 10 times, may be 75 or 80 per cent, it may also fall below 40 per cent when the load is light.

The chief loss of efficiency is due to the fact that the same quantity of pressure-water, and therefore of energy, is used

whether the load is heavy or light. Various devices have been adopted to remedy this evil: the length of stroke may be automatically proportioned, as in the Hastie engine, to the work to be done; the pressure-water may be admitted for a part of the stroke only, the remainder being provided by the discharge-water; cranes and elevators are often provided with a large cylinder for heavy loads and a small cylinder for light loads, and for the same purpose a single cylinder with a differential piston is sometimes used.

Other important losses of efficiency are due to (a) pipe friction; (b) elbows, curves, etc., and abrupt changes of section; (c) the friction of mechanism.

Let p_m be the mean intensity of the pressure in the cylinder.

Let s be the stroke.

Let v_m be the mean velocity of the plunger.

Then

$$\text{the work done per stroke} = \frac{\pi d^2}{4} p_m s;$$

the quantity of motive water used per stroke

$$= \frac{\pi d^2}{4} v_m, \quad \text{or} \quad \frac{1}{2} \frac{\pi d^2}{4} v_m,$$

according as the engine is of the double- or single-acting type.

Analysis.—In a direct-acting pressure-engine let A be the sectional area of the working cylinder (Fig. 198).

Let a be the sectional area of the supply-pipe.

Let $A = na$.

Let W be the weight of the water, piston, and other reciprocating parts in the working cylinder.

Let l be the length of the supply-pipe.

Let f be the acceleration of the piston. Then nf is the acceleration of the water in the supply-pipe.

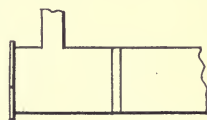


FIG. 198.

The force required to accelerate the piston

$$= \frac{W}{g}f,$$

and the corresponding pressure in feet of water

$$= \frac{W}{wA} \frac{f}{g}.$$

The force required to accelerate the water in the supply-pipe

$$= \frac{wal}{g}nf,$$

and the corresponding pressure in feet of water

$$= nl \frac{f}{g}.$$

Similarly, if l' is the length of the discharge-pipe and $\frac{A}{n}$ its sectional area, the pressure-head due to the inertia of the discharge-water

$$= n'l' \frac{f}{g}.$$

Hence the total pressure in feet of water required to overcome inertia in the supply-pipe and cylinder

$$= \frac{f}{g} \left(\frac{W}{wA} + nl \right).$$

The quantity $\frac{W}{wA} + nl$ has been designated the length of working cylinder equivalent to the inertia of the moving parts. Let the engine drive a crank of radius r , and assume that the velocity V of the crank-pin is approximately constant. Then the acceleration of the plunger when it is at a distance x from its central position

$$= f = \frac{V^2}{r^2}x$$

and the pressure due to inertia

$$= \frac{V^2}{gr^2} \left(\frac{W}{wA} + nl \right) x.$$

Let v be the velocity of the plunger in the working cylinder.

Let u be the velocity of the water in the supply-pipe.

Let h be the vertical distance between the accumulator-ram and the motor.

Let p_0 be the unit pressure at the accumulator-ram.

Let p be the unit pressure in the working cylinder.

Then

$$\frac{p_0}{w} + \frac{u^2}{2g} + h = \frac{p}{w} + \frac{v^2}{2g} + \left\{ \begin{array}{l} \text{losses due to friction, sudden} \\ \text{changes of section, etc.} \end{array} \right.$$

Thus

$$\frac{p_0 - p}{w} = \frac{v^2 - u^2}{2g} - h + \text{losses.}$$

The term $\frac{v^2 - u^2}{2g} + \text{losses}$ may be approximately expressed in the form $K \frac{v^2}{2g}$, K being the coefficient of hydraulic resistance. Hence

$$\frac{p_0 - p}{w} = K \frac{v^2}{2g} = \frac{KV^2}{2gr^2} (r^2 - x^2), \quad . \quad . \quad . \quad (1)$$

the term h being disregarded, as it is usually very small as compared with $\frac{p_0}{w}$.

Thus the total pressure-head in feet required to overcome inertia and the hydraulic resistances

$$= \frac{V^2}{gr^2} \left\{ \left(\frac{W}{wA} + nl \right) x + \frac{K}{2} (r^2 - x^2) \right\}, \quad . \quad . \quad . \quad (2)$$

and is represented by the ordinate between the parabola ced and the line ab in Fig. 199, in which $afgb$ is a rectangle, ab representing the stroke $2r$,

$$ae = bd = \frac{V^2}{gr} \left(\frac{W}{wA} + nl \right),$$

the pressure due to inertia at the end of the stroke, and

$$oc = K \frac{V^2}{2g},$$

the pressure required to overcome the hydraulic resistances at the centre of the stroke.

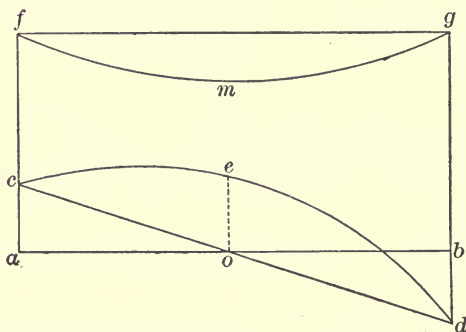


FIG. 199.

The ordinate between the parabola fmg and the line fg represents the back pressure, which is necessarily proportional to the square of the piston-velocity, i.e., to $\frac{V^2}{r^2}(r^2 - x^2)$. Hence the effective pressure-head on the piston, transmitted to the crank-pin, is represented by the ordinate between the curves fmg and ced . The diagram shows that the pressure at the end of the stroke is very large and may become excessive. It is therefore usual to introduce relief-valves or air-vessels to prevent violent shocks. In certain cases, however, as, e.g., in a riveting-machine, a heavy pressure at the end of the

stroke, just where it is most needed to close the rivet, is of great advantage, and therefore the inertia effect is increased by the use of a supply-pipe of small diameter and an accumulator with a small water section (Fig. 197).

By equation (1),

$$v^2 = \frac{2g}{wK}(p_0 - p). \quad . \quad . \quad . \quad . \quad (3)$$

This speed v can be regulated at will by the turning of a cock, as in this manner the hydraulic resistances may be indefinitely increased.

Let the engine be working steadily under a pressure P , and let v_0 be the speed of steady motion. Then

$$v_0^2 = \frac{2g}{wK}(p_0 - P),$$

and

$$P = \left\{ \begin{array}{l} \text{useful resistance overcome by the piston} \\ + \text{friction between piston and accumulator-cylinder.} \end{array} \right.$$

If P is diminished, the speed v_0 will be slightly increased,

but in no case can it exceed $\sqrt{\frac{2gp_0}{wK}}$.

5. Losses of Energy.—The losses may be enumerated as follows:

(a) *The Loss L_1 due to Piston-friction.*—It may be assumed that piston-friction consumes from 10 to 20 per cent of the total available work.

(b) *The Loss L_2 due to Pipe-friction.*—The loss of head in the supply-pipe of diameter d_1

$$= \frac{4fl}{d_1} \frac{(wv)^2}{2g}.$$

The loss of head in the discharge-pipe of diameter d_2

$$= \frac{4fl'}{d_2} \frac{(n'v)^2}{2g}.$$

Hence the total loss of head in pipe-friction is

$$L_2 = 4f \left(\frac{n^2 l}{d_1} + \frac{(n')^2 l'}{d_2} \right) \frac{v^2}{2g} = f_2 \frac{v^2}{2g}.$$

The loss in the relatively short working cylinder is very small and may be disregarded.

(c) *The Loss L_3 due to Inertia.*—The work expended in moving the water in the supply-pipe

$$= \frac{wA}{gn} l \frac{v^2}{2},$$

and in moving the water in the discharge-pipe

$$= \frac{wA}{gn'} l' \frac{v^2}{2}.$$

The total work thus expended

$$= wA \left(\frac{l}{n} + \frac{l'}{n'} \right) \frac{v^2}{2g},$$

and it may be assumed that nearly the whole of this is wasted.

Hence the corresponding loss of head is

$$L_3 = \frac{wA}{A2r} \left(\frac{l}{n} + \frac{l'}{n'} \right) \frac{v^2}{2g} = \frac{w}{2r} \left(\frac{l}{n} + \frac{l'}{n'} \right) \frac{v^2}{2g} = f_3 \frac{v^2}{2g}.$$

(d) *The Loss L_4 due to Curves and Elbows.*—The losses due to curves and elbows may be expressed in the form

$$L_4 = f_4 \frac{v^2}{2g} \text{ (Chap. II, Art. 14).}$$

(e) *The Loss L_5 due to Sudden Changes of Section.*—The loss of head in the passage of the water through the ports may be expressed in the form $f' \frac{v^2}{2g}$.

The loss occasioned by valves may also be expressed by $f'' \frac{v^2}{2g}$.

Thus the total loss is

$$L_5 = (f' + f'') \frac{v^2}{2g} = f_5 \frac{v^2}{2g}.$$

The coefficient f'' may be given any desired value between 0 and ∞ by turning a valve, so that any excess of pressure may be destroyed and the speed regulated at will.

(f) *The Loss L_6 due to the Velocity with which the Water leaves the Discharge-pipe.*

$$L_6 = \frac{(n'v)^2 v^2}{2g} = f_6 \frac{v^2}{2g}.$$

Hence

$$\text{the effective head} = \frac{h_0}{\tau v} - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6),$$

$$\text{and the efficiency} = 1 - \frac{\tau v}{h_0} (L_1 + L_2 + L_3 + L_4 + L_5 + L_6).$$

6. Brakes.—Hydraulic resistances absorb energy which is proportional to the square of the speed. This property has been taken advantage of in the design of hydraulic brakes for arresting the motion of a rapidly moving mass, as a gun or a train, of weight W . In Fig. 200 the fluid is allowed to pass

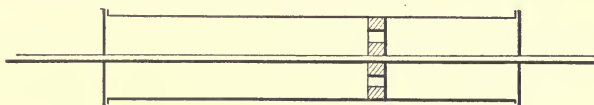


FIG. 200.

from one side of the piston to the other through orifices in the piston.

Let m be the ratio of the area of the piston to the effective area of the orifices.

Let v be the velocity of the piston when moving under a force P .

Let A be the sectional area of the cylinder.

Then

$$\begin{aligned} \text{the work done per second} &= Pv \\ &= \text{the kinetic energy produced} \\ &= wAv \frac{(m-1)^2 v^2}{2g}, \end{aligned}$$

and therefore

$$P = wA(m-1)^2 \frac{v^2}{2g},$$

which is the force required to overcome the hydraulic resistance at the speed v .

Let V be the initial value of v , and P_1 the maximum value of P . Then

$$P_1 = wA(m-1)^2 \frac{V^2}{2g}.$$

Let F be the friction of the slide. Then

$$P + F = wA(m-1)^2 \frac{v^2}{2g} + F,$$

and $P_1 + F$ is the maximum retarding force. It would certainly be an advantage if the retarding force could be constant. In order that this might be the case $(m-1)v$ must be constant, and therefore as v diminishes m should increase and consequently the orifice area diminish. Various devices have been adopted to produce this result.

Assuming the retarding force to be constant, let x be the piston's distance from the end of the stroke when its velocity is v . Then

$$\frac{wv^2}{2g} = (P + F)x,$$

and therefore v^2 is proportional to x .

But $(m-1)v$ is constant.

Therefore $(m-1)$ is inversely proportional to \sqrt{x} .

EXAMPLES.

1. A 4-ton hydraulic jack with a 2-in. ram and a 1-in. plunger is to lift a weight of 1 ton, and is worked by a handle with a leverage of 12 to 1. If the efficiency of the jack is 80 per cent, what force must be applied to the handle? *Ans.* $52\frac{1}{8}$ lbs.

2. The ram of an hydraulic press has a sectional area 50 times as great as the pump-plunger. The mechanical advantage of the lever is 10 to 1. If a force of 50 lbs. is exerted on the handle, find the pressure on the ram. *Ans.* 25,000 lbs.

3. A force of P lbs. is required to punch a hole of d ins. diameter. Find the diameter of the ram, the available fluid pressure being p lbs per square inch. If this pressure is developed by a steam-intensifier with a steam-piston area n times that of the intensifier's ram, find the required steam-pressure. *Ans.* $\sqrt{\frac{14}{11}} \frac{P}{p}$; $\frac{p}{n}$.

4. In a steel hydraulic press the fluid pressure is 6000 lbs. per square inch, and the maximum allowable stress in the metal is 18,000 lbs. per square inch. If the internal diameter of the press is 12 ins., what must the thickness of the metal be? If the thickness of the metal is 3 ins., what must the internal diameter be? *Ans.* 2.485 ins.; 14.485 ins.

5. A straight-line law is found experimentally to connect the weight W to be lifted and the effort E on the handle. Find the law from the following data: when $W = 1605$ lbs., $E = 10$ lbs., and when $W = 6805$ lbs., $E = 50$ lbs. A pressure-gauge gives the fluid pressure as 1932 lbs. per square inch, when $W = 7000$ lbs.; find the frictional loss at the leather, and if there is the same percentage of loss at the two leathers find the law connecting E and the force P on the plunger. The experiments were made on a jack with a $2\frac{1}{4}$ -in. ram, a $\frac{3}{4}$ -in. plunger, and a lever with a velocity ratio of 30. (Perry.)

Ans. $W = 305 + 130 E$; 9.1 per cent; $P = 41 + 17.5 E$.
(Perry's "Applied Mechanics.")

6. An accumulator-ram is 8.8 ins. in diameter and has a stroke of 21 ft. Find the store of energy in foot-pounds when the ram is at the top of its stroke and is loaded till the pressure is 750 lbs. per square inch. *Ans.* 958,320 ft.-lbs.

7. In a differential accumulator the diameters of the spindle are 7 ins. and 5 ins; the stroke is 10 ft. Find the store of energy when full and loaded to 2000 lbs. per square inch. *Ans.* 377,000 ft.-lbs.

8. The pressure on a 5-in. ram is to be 1000 lbs. per square inch, and

the supply comes from a tank 100 ft. high. Find the necessary diameter of the piston in the intensifier. *Ans.* 24 ins.

9. In a differential press the diameters of the upper and lower portions of the ram are 6 ins. and 8 ins. respectively. The pressure is 1000 lbs. per square inch, and the stroke is 10 ft. Find the load on the accumulator, the maximum store of energy, and the store of water.

Ans. 22,000 lbs.; 220,000 ft.-lbs.; $1\frac{1}{8}$ cu. ft.

10. What load must be applied to a differential accumulator to give a pressure of 1600 lbs. per square inch? The upper and lower diameters of the ram are 3 and $3\frac{1}{4}$ ins. respectively, and the friction of the cup-leathers may be taken as 5 per cent of the gross load.

Ans. 6062 lbs.; 6700 lbs.

11. Find the weight which will give an average fluid pressure of 750 lbs. per square inch in an accumulator with a 14-in. ram and a stroke of 16 ft. How much energy can be stored up? Find the friction at each cup-leather, assuming that between slow rising and falling the pressure fluctuates between 780 and 738 lbs. per square inch. If the pressure is 750 lbs. per square inch at mid-lift, find the actual fluctuation.

Ans. 115,500 lbs.; 1,848,000 ft.-lbs.; 3234 lbs.; 3769 lbs.

12. An accumulator, loaded to a pressure of 750 lbs. per square inch, has a ram of 21 ins. diameter, with a stroke of 24 ft. How much H.P. can be obtained for a period of 50 seconds?

Ans. 226.8.

13. An accumulator under a load of 200,000 lbs. is to transmit 100 H.P. through a 4-in. pipe 1 mile long with a loss of 10 per cent. What should be the diameter of the ram, the coefficient of pipe friction being .006?

Ans. 17.33 ins.

14. A steam-accumulator has to develop a total force of 66,000 lbs. upon the ram of a punch. The piston area is 15 times that of the hydraulic-cylinder, which has a diameter of 10 inches. Find the intensities of the steam and the water-pressure.

Ans. 56 lbs.; 840 lbs.

15. The piston and ram areas of a steam-accumulator are in the ratio of 10 to 1. Find their diameters so that a steam-pressure of 100 lbs. per sq. in. may develop a total load on the ram of 38,500 lbs.

Ans. 22.136 ins.; 7 ins.

16. A Brotherhood engine with a 4-in. cylinder and a 3-in. stroke makes 50 revols. per minute. The average motive pressure is 700 lbs. per sq. in., and the average back pressure, due to frictional resistances, etc., is 210 lbs. per sq. inch. Find the H.P. developed, and also determine the diameter of the cylinder if only *one half* of this power is to be developed.

Ans. 7; 2.83 ins.

17. A crane with an hydraulic efficiency of .9 and a mechanical efficiency of .45 is worked by water at a pressure of 750 lbs. per sq. inch. The piston has an effective area of 96 sq. ins. on one side, 48 sq. ins. on the other, and pushes a three-sheave pulley-block. Find the maximum weight which can be lifted and the work done per gallon of

water, *first* when the water presses on one side only, and *second* when it presses on both sides. Also find the work done per gallon of water when the full loads in the two kinds of working are being lifted.

Ans. 4860 lbs.; 6998.4 ft.-lbs.; 2430 lbs.; 3499.2 ft.-lbs.; 6998.4 ft.-lbs.

18. An hydraulic crane with a velocity ratio of 9 and a mechanical efficiency of .75 has to lift a weight of 10,000 lbs. It is worked by water at a pressure of 750 lbs. per sq. in., and the frictional loss of pressure is 91 lbs. per sq. inch. Find the diameter of the ram. *Ans.* 15.2 ins.

19. The two wire ropes from the cage of a ram-lift pass vertically over a pulley to a counterweight, and the ram rises from 100 ft. below to 20 ft. above the level of the supply-pipe. Water-pressures of 500 lbs. and 100 lbs. per sq. in. act upon a $3\frac{1}{2}$ -in. and a 7-in. ram, respectively. Find the weight of the ropes per lineal foot and the lifting force at the top and bottom of the stroke.

Ans. 4.2 lbs., 16.7 lbs.; 5230 lbs., 4729 lbs.; 5521 lbs., 3516 lbs.

20. Find the pressure due to inertia at the end of the out-stroke of a rotary motor with a 4-in. piston and a 7-in. stroke, driven by water in a 4-in. supply-pipe 250 ft. long. The motor makes 125 revs. per minute, and the length of the connecting-rod is 15 inches.

Ans. 20.7 lbs.; 12.9 lbs.

21. A direct-acting lift has a ram 9 inches diameter, and works under a *constant* head of 73 feet, of which 13 per cent is required by ram friction and friction of mechanism. The supply-pipe is 100 feet long and 4 inches diameter. Find the speed of steady motion when raising a load of 1350 lbs., and also the load it would raise at double that speed. ($f = .00672$.)

If a valve in the supply-pipe is partially closed so as to increase the coefficient of resistance by $5\frac{1}{2}$, what would the speed be?

Ans. Speed = 2 ft. per second; load = 150 lbs.

22. Eight cwt. of ore is to be raised from a mine at the rate of 900 feet per minute by a water-pressure engine, which has four single-acting cylinders, 6 inches diameter, 18 inches stroke, making 60 revolutions per minute. Find the diameter of a supply-pipe 230 feet long for a head of 230 feet, disregarding resistances and taking $f = .006$.

Ans. Diameter = 4 inches.

23. If λ be the length equivalent to the inertia of a water-pressure engine, F the coefficient of hydraulic resistance, both reduced to the ram, v_0 the speed of steady motion, find the velocity of ram after moving from rest through a space x against a constant useful resistance. Also find the time occupied.

$$\text{Ans. } v^2 = v_0^2 \left(1 - e^{-\frac{F}{\lambda} x} \right); \quad t = \frac{\lambda}{F v_0} \log_e \frac{v_0 + v}{v_0 - v}.$$

24. An hydraulic motor is driven from an accumulator, the pressure

in which is 750 lbs. per square inch, by means of a supply-pipe 900 feet long, 4 inches diameter; what would be the maximum power theoretically attainable, and what would be the velocity in the pipe corresponding to that power? Find approximately the efficiency of transmission at half power, $f = .007$.

Ans. H.P. = 250; $v = 22$ ft.; efficiency = .66 nearly.

25. A gun recoils with a maximum velocity of 10 feet per second. The area of the orifices in the compressor, after allowing for contraction, may be taken as one twentieth the area of the piston. Find the initial pressure in the compressor in feet of liquid.

Assuming the weight of the gun to be 12 tons, friction of slide 3 tons, diameter of compressor 6 inches, fluid in compressor water, find the recoil.

Find the mean resistance to recoil. Compare the maximum and mean resistances, each exclusive of friction of slide.

Ans. 621; 4 ft, $2\frac{1}{2}$ in.; total mean resistance = 4.4 tons; ratio = 2.5.

CHAPTER V.

IMPACT, REACTION, IMPACT AND TANGENTIAL TURBINES.

NOTE.—The following symbols are used:

v_1 = the velocity of the jet before impact;

v_2 = “ “ “ “ “ after leaving the vane;

u = “ “ “ “ vane;

V = “ “ “ “ water relatively to the vane;

A = sectional area of the impinging jet;

m = mass of the water reaching the vane per second.

1. Impact of a Jet upon a Flat Vane Oblique to the Direction of the Jet.—Let θ be the angle between the normal to the vane and the direction of the impinging jet, ϕ the angle between the normal to the vane and the direction of the vane's motion, and α the angle between the vane and the vertical.

The jet, moving with its stream-lines parallel, swells out near the vane, over which it spreads and with which it travels along in the direction

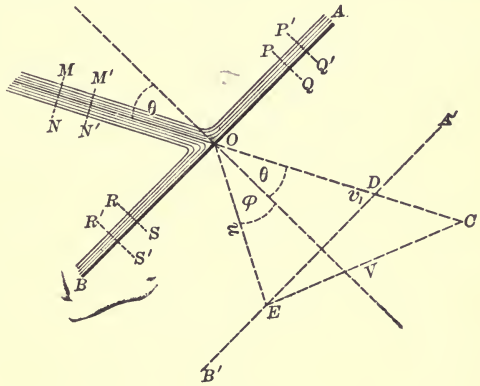


FIG. 201.

of the vane's motion, and finally again flows along with its stream-lines sensibly parallel to the vane.

The problem is still further complicated by the production of eddies and vortices for which allowance can only be made in a purely empirical manner.

Let N be the normal pressure on the vane due to the impact.

Let N' be the total normal pressure on the vane.

Let W be the weight of water on the vane.

Then

$$\begin{aligned} N &= N' - W \sin \alpha = \text{change of momentum in direction of} \\ &\quad \text{the normal} \\ &= mv_1 \cos \theta - mu \cos \phi, \end{aligned}$$

or

$$N = m(v_1 \cos \theta - u \cos \phi), \quad \dots \quad (I)$$

(N.B. The sign in front of $u \cos \phi$ will be plus if the jet and vane move in opposite directions.)

The term $W \sin \alpha$ may be designated the *static* pressure, and the term $m(v_1 \cos \theta - u \cos \phi)$ the *dynamic* pressure, which causes the deviation of the stream-lines.

NOTE.—The pressure when a jet *first* strikes the plane is greater than when the flow has become steady, or a permanent régime is established.

This is made evident by the following consideration:

At any moment let MN , PQ , RS be the bounding planes across which the water is flowing with its stream-lines sensibly parallel.

In a unit of time let the bounding planes of the mass be $M'N'$, $P'Q'$, $R'S'$.

Then, initially, the reaction of the plane must destroy the motion of the mass of the fluid bounded by $M'N'$, $P'Q'$, and $R'S'$.

Take OC to represent v_1 in direction and magnitude.

“ OE “ “ “ “ “ “ “

In one second the vane AB moves parallel to itself into the position $A'B'$. Let $A'B'$ intersect OC in D .

Then

$$\begin{aligned} m &= \frac{w}{g} A \cdot DC = \frac{w}{g} A (v_1 - OD) \\ &= \frac{w}{g} A \left(v_1 - u \frac{\cos \phi}{\cos \theta} \right). \quad \dots \dots \dots (2) \end{aligned}$$

Thus equation (1) becomes

$$N = \frac{w}{g} \frac{A}{\cos \theta} (v_1 \cos \theta - u \cos \phi)^2. \quad \dots \dots (3)$$

Let P be the pressure in the direction of the vane's motion.
Then

$$P = N \cos \phi = \frac{w}{g} A \frac{\cos \phi}{\cos \theta} (v_1 \cos \theta - u \cos \phi)^2, \quad (4)$$

and the *useful work* done on the vane per second

$$= Pu = \frac{w}{g} A \frac{\cos \phi}{\cos \theta} u (v_1 \cos \theta - u \cos \phi)^2. \quad \dots \dots (5)$$

$$\text{The total available work} = \frac{w}{g} A \frac{v_1^3}{2}. \quad \dots \dots \dots (6)$$

$$\begin{aligned} \text{Hence the efficiency} &= \frac{\frac{w}{g} A \frac{\cos \phi}{\cos \theta} u (v_1 \cos \theta - u \cos \phi)^2}{\frac{w}{g} A \frac{v_1^3}{2}} \\ &= 2 \frac{\cos \phi}{\cos \theta} \frac{u}{v_1^3} (v_1 \cos \theta - u \cos \phi)^2. \quad (7) \end{aligned}$$

This is a maximum when

$$v_1 \cos \theta = 3u \cos \phi, \quad \dots \dots \dots (8)$$

and therefore

$$\text{the maximum efficiency} = \frac{8}{27} \cos^2 \theta. \quad \dots \dots (9)$$

If the vane is of small sectional area, a portion of the water will escape over the boundary and the pressure must necessarily be less than that given by equation (3).

Series of Vanes.—Instead of one vane moving before the jet, let a series of vanes be introduced at short intervals at the same point in the path of the jet.

The quantity of water now reaching the vane per second is evidently

$$m = \frac{w}{g} A v_1, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and, by equation (1), the normal pressure

$$N = \frac{w}{g} A v_1 (v_1 \cos \theta - u \cos \phi). \quad . \quad . \quad . \quad (11)$$

Also, the *pressure* in the direction of the motion of the vane

$$= P = N \cos \phi = \frac{w}{g} A v_1 (v_1 \cos \theta - u \cos \phi) \cos \phi. \quad (12)$$

The *useful work* done per second

$$= Pu = \frac{w}{g} A v_1 u (v_1 \cos \theta - u \cos \phi) \cos \phi, \quad . \quad (13)$$

and the *efficiency*

$$\begin{aligned} &= \frac{\frac{w}{g} A v_1 u (v_1 \cos \theta - u \cos \phi) \cos \phi}{\frac{w}{g} A \frac{v_1^3}{2}} \\ &= \frac{2u(v_1 \cos \theta - u \cos \phi) \cos \phi}{v_1^2}. \quad . \quad . \quad . \quad (14) \end{aligned}$$

This is a maximum when $v_1 \cos \theta = 2u \cos \phi$, . . . (15)
and therefore

$$\text{the maximum efficiency} = \frac{\cos^2 \theta}{2}. \quad . \quad . \quad . \quad (16)$$

EX. 1. Let a single vane be at right angles to, and move in the line of, the jet's motion, Fig. 202.

Then $\theta = 0 = \phi$. Hence

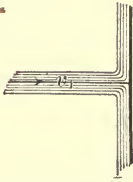


FIG. 202.

$$\text{the pressure} = P = N = \frac{w}{g} A(v_1 - u)^2; \quad \dots \quad (17)$$

$$\text{the useful work} = Pu = \frac{w}{g} Au(v_1 - u)^2; \quad \dots \quad (18)$$

$$\text{the efficiency} = \frac{2u}{v_1^3} (v_1 - u)^2; \quad \dots \quad (19)$$

$$\text{the maximum efficiency} = \frac{8}{27} \quad \dots \quad (20)$$

Again, if $u = 0$, i.e., if the vane be fixed, and if H be the head corresponding to the velocity v_1 , then, by equation (17),

$$P = \frac{w}{g} Av_1^2 = 2wAH$$

= twice the weight of a column of water
of height H and sectional area A .

EX. 2. Let each of a series of vanes be at right angles to, and move in the line of, the jet's motion at the instant of impact.

Then $\theta = 0 = \phi$. Hence

$$\text{the pressure} = N = P = \frac{w}{g} Av_1(v_1 - u); \quad \dots \quad (21)$$

$$\text{the useful work} = Pu = \frac{w}{g} Av_1u(v_1 - u); \quad \dots \quad (22)$$

$$\text{the efficiency} = \frac{2u(v_1 - u)}{v_1^2}; \quad \dots \quad (23)$$

$$\text{the maximum efficiency} = \frac{1}{2} \quad \dots \quad (24)$$

EX. 3. A stream of .125 sq. ft. sectional area delivers 10 cu. ft. of water per second and impinges normally against a flat vane. It is required to find (a) the pressure on the vane if fixed; (b) the pressure and the useful effect if the vane moves in the direction of the jet's motion with a velocity of 40 ft. per second; (c) the pressure and useful effect when the single vane in (b) is replaced by a series of vanes which follow each other at intervals of a second.

The velocity of the jet before impact = $\frac{10}{.125} = 80$ ft. per sec.

(a) The pressure on vane = momentum of jet = $\frac{62\frac{1}{2}}{32} \times 10 \times 80 = 1562\frac{1}{2}$ lbs.

(b) The quantity of water reaching the vane per sec.
= $\frac{1}{8}(80 - 40) = 5$ cu. ft.

The pressure on the vane = momentum of jet

$$= \frac{62\frac{1}{2}}{32} 5(80 - 40) = 390\frac{1}{8} \text{ lbs.}$$

The useful effect = $390\frac{1}{8} \times 40 = 15,625 \text{ ft.-lbs.}$

The total available work = $\frac{62\frac{1}{2}}{32} 10 \cdot \frac{80^2}{2} = 62,500 \text{ ft.-lbs.}$

Therefore the efficiency = $\frac{15625}{62500} = \frac{1}{4}$.

(c) The quantity of water now reaching the vane per second

$$= \frac{1}{8} \times 80 = 10 \text{ cu. ft.}$$

The pressure on the vane = momentum of jet

$$= \frac{62\frac{1}{2}}{32} 10(80 - 40) = 781\frac{1}{4} \text{ lbs.}$$

The useful effect = $781\frac{1}{4} \times 40 = 31,250 \text{ ft.-lbs.}$

The efficiency = $\frac{31250}{62500} = \frac{1}{2}$.

Ex. 4. The jet in the preceding example impinges upon a vane with its normal inclined at 60° to the jet's direction, and is driven with a velocity of 20 ft. per second in a direction making an angle of 30° with the vane's normal. Find (a) the pressure on the vane; (b) the useful effect.

(a) The quantity of water reaching the jet per second

$$= \frac{1}{8} \left(80 - 20 \frac{\cos 30^\circ}{\cos 60^\circ} \right) = \frac{5}{2} (4 - \sqrt{3}) = 5.67 \text{ cu. ft.}$$

The relative velocity in the direction of the normal

$$= 80 \cos 60^\circ - 20 \cos 30^\circ = 10(4 - \sqrt{3}) = 22.68 \text{ ft. per sec.}$$

The normal pressure upon the vane = momentum in direction of normal

$$= 5.67 \times 22.68 = 128.6 \text{ lbs.}$$

The pressure in direction of vane's motion = $128.6 \cos 30^\circ$

$$= 111.35 \text{ lbs.}$$

(b) The useful effect = $111.35 \times 20 = 2227 \text{ ft.-lbs.}$

The efficiency = $\frac{2227}{62500} = .0356$.

5. Jet of Water Impinging upon a Surface of Revolution Moving in the Direction of its Axis and also in the Line of the Jet's Motion.—The relative velocity of the jet is $v_1 - u$ if the jet and surface move in the same direction, Figs. 203 and 204, and $v_1 + u$ if they move in opposite directions, Figs.

205 and 506. This relative velocity, if friction is disregarded, remains unchanged in magnitude as the water flows over the surface, but the stream-line direction is deviated through an angle β .

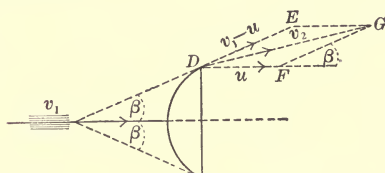


FIG. 203.

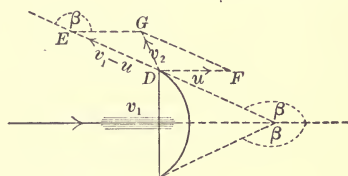


FIG. 204.

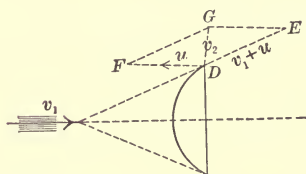


FIG. 205.

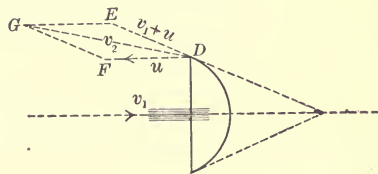


FIG. 206.

Let the water leave the surface at D , and in the direction of the tangent at D take $DE = v_1 - u$, Figs. 203 and 204, and $DE = v_1 + u$, Figs. 205 and 206. Draw DF parallel to the axis and take $DF = u$.

Complete the parallelogram $DEGF$.

Then DG , the diagonal, must represent, in direction and magnitude, the absolute velocity, v_2 , with which the water leaves the surface.

Hence, from Figs. 203 and 204,

$$v_2^2 = u^2 + (v_1 - u)^2 - 2u(v_1 - u) \cos (180^\circ - \beta), \quad (1)$$

and the work done by the water on the surface

$$\begin{aligned} &= m \frac{v_1^2 - v_2^2}{2} \\ &= mu(v_1 - u)(1 - \cos \beta) \\ &= 2 \frac{wA}{g} u(v_1 - u)^2 \sin^2 \frac{\beta}{2}. \quad \dots \quad (2) \end{aligned}$$

From Figs. 205 and 206,

$$v_2^2 = u^2 + (v_1 + u)^2 - 2u(v_1 + u) \cos \beta,$$

and the work done by the surface on the water

$$\begin{aligned} &= m \frac{v_2^2 - v_1^2}{2} \\ &= mu(v_1 + u)(1 - \cos \beta) \\ &= 2 \frac{wA}{g} u(v_1 + u)^2 \sin^2 \frac{\beta}{2}. \quad . \quad . \quad . \quad (3) \end{aligned}$$

Let P be the pressure on the surface in the direction of its motion. Then

$$Pu = \text{work done} = 2 \frac{wA}{g} u(v_1 + u)^2 \sin^2 \frac{\beta}{2},$$

and therefore

$$P = 2 \frac{wA}{g} (v_1 + u)^2 \sin^2 \frac{\beta}{2}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The *efficiency* for the case of Figs. 203 and 204

$$\begin{aligned} &= \frac{2 \frac{wA}{g} u(v_1 - u)^2 \sin^2 \frac{\beta}{2}}{\frac{wA}{g} \frac{v_1^3}{2}} = \frac{4u(v_1 - u)^2 \sin^2 \frac{\beta}{2}}{v_1^3}, \quad . \quad . \quad . \quad (5) \end{aligned}$$

which is a maximum and $= \frac{16}{27} \sin^2 \frac{\beta}{2}$ when $v_1 = 3u$.

Series of Surfaces.—If a number of surfaces are successively introduced at short intervals at the same point in the path of the jet, the quantity of water reaching each surface per second becomes

$$m = \frac{wA}{g} v_1.$$

In this case

$$\text{the work done} = 2 \frac{wA}{g} v_1 u (v_1 \mp u) \sin^2 \frac{\beta}{2}, \quad \dots \quad (6)$$

$$\text{and the pressure} = 2 \frac{wA}{g} v_1 (v_1 \mp u) \sin^2 \frac{\beta}{2}. \quad \dots \quad (7)$$

Also, the efficiency, when the water drives the surface,

$$\begin{aligned} &= \frac{2 \frac{wA}{g} v_1 u (v_1 - u) \sin^2 \frac{\beta}{2}}{\frac{wA}{g} \frac{v_1^3}{2}} \\ &= \frac{4u(v_1 - u) \sin^2 \frac{\beta}{2}}{v_1^2}, \quad \dots \quad (8) \end{aligned}$$

which is a maximum and $= \sin^2 \frac{\beta}{2}$ when $v_1 = 2u$.

With a convex surface $\beta < 90^\circ$, and the coefficient $2 \sin^2 \frac{\beta}{2}$, or $1 - \cos \beta$, is less than unity.

With a concave surface $\beta > 90^\circ$, and the coefficient $2 \sin^2 \frac{\beta}{2}$, or $1 - \cos \beta$, is greater than unity.

If the surface be of the cup type and hemispherical, the maximum efficiency $= \sin^2 \frac{180^\circ}{2} = 1$, since $\beta = 180^\circ$. The water should therefore leave the surface without velocity, and, substituting $v_1 = 2u$ and $\beta = 180^\circ$ in equation (1),

$$v_2^2 = u^2 + (2u - u)^2 - 2u(2u - u) = 0.$$

Ex. A jet of water of .125 sq. ft. sectional area delivers 12 cu. ft. of water and impinges axially upon a 120° cone. Find (a) the pressure on the cone when fixed, and (b) the pressure on the cone and the useful

effect when the cone is driven in the direction of its axis with a velocity of 32 ft. per second.

The velocity of the jet before impact $= \frac{12}{.125} = 96$ ft. per sec.

(a) Pressure on *convex* surface $= 2 \frac{62\frac{1}{2}}{32} \cdot 12.96 \sin^2 \frac{60^\circ}{2} = 1125$ lbs.

Pressure on *concave* surface $= 2 \frac{62\frac{1}{2}}{32} 12.96 \sin^2 \frac{120^\circ}{2} = 3375$ lbs.

(b) When the water impinges on the *convex* surface

the work done $= 2 \frac{62\frac{1}{2}}{32} \frac{1}{8} 32(96 - 32)^2 \sin^2 \frac{60^\circ}{2} = 16,000$ ft.-lbs.,

the pressure $= \frac{16000}{32} = 500$ lbs.

When the water impinges on the *concave* surface

the work done $= 2 \frac{62\frac{1}{2}}{32} \frac{1}{8} 32(96 - 32)^2 \sin^2 \frac{120^\circ}{2} = 48,000$ ft.-lbs.,

the pressure $= \frac{48000}{32} = 1500$ lbs.

6. Impact of a Jet of Water upon a Vane with Borders.

—Let the vane in Art. 1 be provided with borders, Figs. 207 and 208, so as to produce a further deviation of the stream-lines, and let the water finally flow off with a velocity v_2 in a direction making an angle θ' with the normal to the vane.

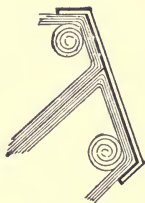


FIG. 207.

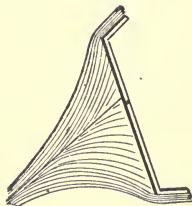


FIG. 208.

Then

the normal pressure $= N$

$$\begin{aligned} &= mv_1 \cos \theta \mp mv_2 \cos \theta' \mp mu \cos \phi \\ &= m(v_1 \cos \theta \mp v_2 \cos \theta' \mp u \cos \phi), \end{aligned}$$

the sign of the second term being plus or minus according to the direction in which the stream-lines are finally deviated.

The effect of the borders is therefore to increase or diminish the normal pressure, and hence also the useful work and the efficiency.

SPECIAL CASE.—Let the vane be at rest, i.e., let $u = 0$, and let the final and initial directions of the jet be parallel.

Also, let $v_1 = v_2$. Then

$$N = m(v_1 \cos \theta + v_1 \cos \theta)$$

$$= 2 \frac{w}{g} A v_1^2 \cos \theta$$

$$= 4wAH \cos \theta.$$

Hence, if $\theta = 0$, the normal pressure $N = 4wAH =$ *four* times the weight of a column of water of height H and sectional area A .

7. Impact Apparatus in Hydraulic Laboratory, McGill University.—This apparatus was constructed for the purpose of determining the force with which jets from orifices, nozzles, etc., impinge upon vanes of different forms and sizes.

A massive cast-iron bracket, Fig. 209, has one end securely bolted to the front of the tank, and the other supported by a vertical tie-rod from one of the oak beams in the ceiling. The upper surface is provided with accurately planed slides, which are set level about 5 ft. above the orifice axis. If, from any cause, the end of the bracket farthest from the tank is found to be too high or too low, the error can be corrected by loosening or tightening the nut on the tie-rod.

The balance proper is carried by a sliding frame which can be moved horizontally into any position along the bracket by means of a rack and pinion actuated by a sprocket-wheel with chain. At one end the frame has two equal arms with a common horizontal axis parallel to the bracket, and each arm has a stop on its lower surface which serves to limit the oscillation of the balance.

The balance, in its mean position, consists of a main trunk with horizontal axis rigidly connected with a vertical slotted arm and with two equal horizontal arms at one end. The common axis of the latter is horizontal and perpendicular to the axis of the main trunk. The hardened-steel knife-edges of the balance are 4 ft. centre to centre and rest in hardened-steel vees inserted in the ends of the sliding frame on each side of the bracket. The bottom of each vee is in the same

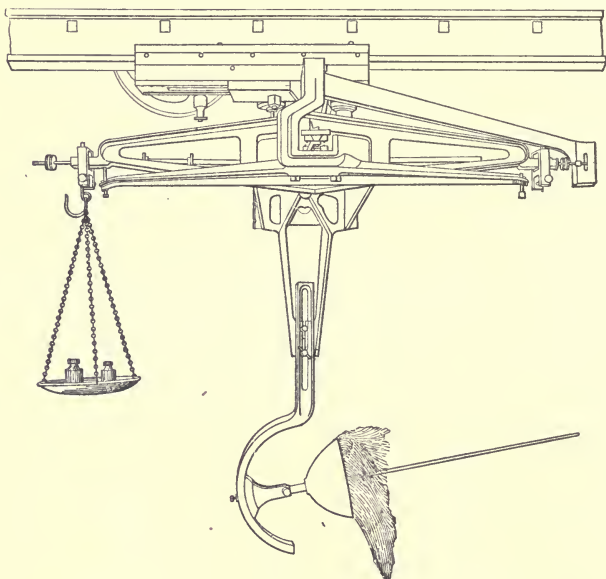


FIG. 209.

horizontal line (called the axis of the vees) at right angles to the bracket.

A bar with the upper portion graduated in inches and tenths has a slot in the lower portion, which is bent into a circular segment of $9\frac{1}{2}$ ins. radius. The bar slides along the slot in the vertical arm of the balance. A radial block, with the holder into which the several vanes are screwed, moves along the slot in the circular segment, and may be clamped in any required position, the angular deviations from the vertical

being shown by graduations on the segment. The centre of this segment in every case coincides with the central point of impact on a vane, is in the vertical axis of the balance-arm, and is also vertically below the axis of the vees. Thus the jet can always be made to strike the vane both centrally and normally.

The scale-pan hangs from a knife-edge at one end of the horizontal arms of the balance, while to the other end is attached a fine pointer, which indicates the angular movement of the balance on a graduated arc fixed to the sliding frame. The balance is in its mid-position when the pointer is opposite the zero mark.

When a vane has been secured in any given position, the preliminary adjustment of the balance is effected by moving a heavy cast-iron disc along a horizontal screw fixed into the main trunk. The sensitiveness of the balance is also increased or diminished by raising or lowering heavy weights on two vertical screws in the top of the trunk.

Assume that the adjustments have all been made and that the jet, Fig. 210, now impinges normally upon a vane.

Let W be the weight required in the scale-pan to bring the balance back into its mid-position.

Let F_a be the *actual* force of impact determined by the balance.

Let F_t be the *theoretical* force of impact deduced by the ordinary formulæ.

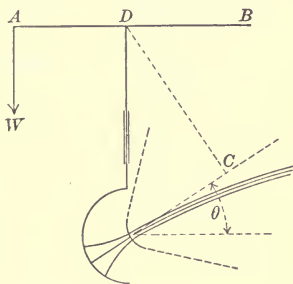


FIG. 210.

Then the ratio $\frac{F_a}{F_t} = c_i$ may be called the coefficient of impact.

Let y be the vertical distance of the central point of impact below the horizontal axis of the orifice, which is 36 ins. below

the axis of the vees. The distance between this axis and the point of suspension of the scale-pan is 24 ins.

Let v be the velocity with which the water issues from the orifice.

Let v' be the velocity of the jet at the point of impact.

Then

$$F_t = 2 \frac{w}{g} Q v' \sin^2 \frac{\beta}{2},$$

Q being the delivery per second and β the angle through which the water is turned on the vane.

If the axis of the jet at the point of impact makes an angle θ with the horizontal, then

$$v' \cos \theta = v = c_v \sqrt{2gh}.$$

Therefore

$$F_t \cos \theta = 2 \frac{w}{g} Q v \sin^2 \frac{\beta}{2}.$$

Again, taking moments about D ,

$$F_a \cos \theta (36 + y) = W \cdot 24.$$

Hence

$$\begin{aligned} c_i = \frac{F_a}{F_t} &= \frac{12 W g}{w Q v \sin^2 \frac{\beta}{2} (36 + y)} \\ &= \frac{6W}{w c_d c_v^2 A h (36 + y) \sin^2 \frac{\beta}{2}} \end{aligned}$$

A being the sectional area of the orifice.

A large number of experiments have been made for the purpose of determining the value of c_i and are described in the Trans. of the Royal Soc. of Can., Vol. II, 1896, and of the Can. Soc. of Civil Engineers, Vol. XII. No definite law of

variation has yet been found, but the following general results have been obtained:

The actual force of impact is always much less than that indicated by theory. Even under the most favorable conditions, with a very large coefficient of velocity, the theoretical force of impact was found to exceed the actual by 3 or 4 per cent.

The coefficient of impact, c_i , increases with the velocity of the jet.

The coefficient rapidly diminishes with the angle through which the stream is deflected. It is also of interest to note that, with small angles of deflection, c_i was greatest with a concave parabolic vane, less with an elliptic, and least with a circular, but that this order was reversed when the deflections were larger.

8. Reaction—Jet Propeller.—The term reaction is employed to denote the pressure upon a surface due to the direction and velocity with which the water leaves the surface. Water, for example, issues under the head h and with the velocity v_1 (at contracted section) from an orifice of sectional area A in the vertical side of a vessel, Fig. 211.

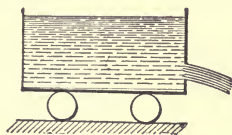


FIG. 211.

Let R be the reaction on the opposite vertical side of the vessel, and let Q be the quantity of water which flows through the orifice per second. Then

$$\begin{aligned}
 R &= \text{horizontal change of momentum} \\
 &= \frac{wQ}{g} v_1 = \frac{w}{g} c_v A v_1^2 = 2wc_v c_v^2 A h = 2wA h, \quad \dots \quad (1)
 \end{aligned}$$

disregarding the contraction and putting $c_v = 1$.

Thus the reaction is double the corresponding pressure when the orifice is closed (Ex. 1, p. 363).

Again, let the vessel be propelled in the opposite direction with a velocity u relatively to the earth.

Then $v_1 - u$ is the velocity of the jet at the contracted section relatively to the earth and

$$R = \text{horizontal change of momentum} \\ = \frac{w}{g} Q (v_1 - u). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The useful work done by the jet

$$= Ru = \frac{w}{g} Qu (v_1 - u). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The energy carried away by the issuing water

$$= \frac{w}{g} Q \frac{(v_1 - u)^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence

$$\begin{aligned} \text{the total energy} &= \frac{w}{g} Qu (v_1 - u) + \frac{w}{g} Q \frac{(v_1 - u)^2}{2} \\ &= \frac{w}{g} Q \frac{v_1^2 - u^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

and

$$\text{the efficiency} = \frac{\frac{w}{g} Qu (v_1 - u)}{\frac{w}{g} Q \frac{v_1^2 - u^2}{2}} = \frac{2u}{v_1 + u}. \quad . \quad . \quad . \quad (6)$$

Thus the more nearly v_1 is equal to u , and therefore the larger the area A of the orifice, the greater is the efficiency.

If the vessel is driven in the same direction as the jet, then $v_1 + u$ is the relative velocity of the jet with respect to the earth, and the reaction is

$$\begin{aligned} R &= \text{horizontal change of momentum} \\ &= \frac{w}{g} Q (v_1 + u) = \frac{w}{g} c_v A v_1 (v_1 + u) \\ &= \frac{w}{g} A v_1 (v_1 + u), \quad . \quad . \quad . \quad . \quad . \quad (7) \end{aligned}$$

disregarding the contraction and putting $c_v = 1$.

9. The Jet Reaction Wheel (Scotch Turbine).—In this form of motor the water enters the centre of the wheel, spreads out radially in tubular passages, and issues from openings at the ends tangentially to the direction of rotation.

FIG. 212.

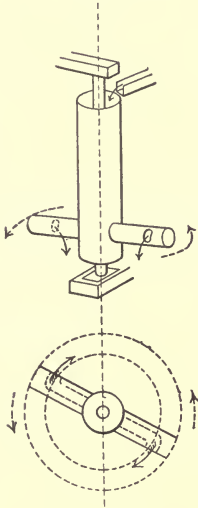


FIG. 213.

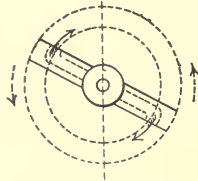


FIG. 214.

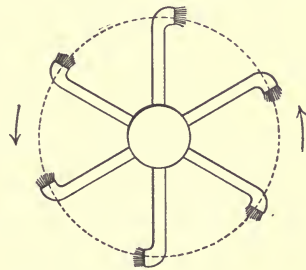


FIG. 215.

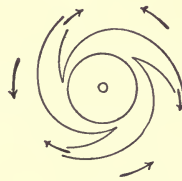


Fig. 212 represents the simplest wheel of this class. In England it is known as Barker's Mill, and in Germany as Segner's Water-wheel.

A reaction wheel may have several tubular passages as in Fig. 214, while the vertical chamber XY may be cylindrical, prismatic, or conical.

The Scotch or Whitelaw's turbine, Fig. 215, does not differ essentially, excepting in the curved arms, from the simple reaction wheel.

Let r be the horizontal distance between axis of orifice and axis of rotation.

“ h “ “ head of water over the orifices when closed,

Let V be the velocity of efflux relatively to the tube when the orifices are open.

“ u “ “ corresponding linear velocity of rotation at the centre of an orifice.

“ v_2 “ “ absolute velocity of efflux $= V - u$.

“ Q “ “ discharge.

“ R “ “ reaction.

Then

$$V^2 = c_v^2(u^2 + 2gh), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

c_v being the coefficient of discharge.

Also,

$$\begin{aligned} \frac{wQ}{g}(V - u) &= \text{horizontal linear change of momentum} \\ &= \text{reaction producing rotation} \\ &= R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

The useful work

$$= Ru = \frac{wQ}{g}(V - u)u. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The efficiency

$$= \frac{Ru}{wQh} = \frac{(V - u)u}{gh} = \frac{2(V - u)u}{\frac{V^2}{c_v^2} - u^2}. \quad . \quad . \quad . \quad . \quad (4)$$

Again, the efficiency

$$\begin{aligned} &= \frac{(V - u)u}{gh} = \frac{u^2}{gh} \left(\frac{V}{u} - 1 \right) \\ &= \frac{u^2}{gh} \left\{ c_v \left(1 + \frac{2gh}{u^2} \right)^{\frac{1}{2}} - 1 \right\} \\ &= \frac{u^2}{gh} \left\{ c_v \left(1 + \frac{gh}{u^2} \right) - \text{terms containing higher powers of } \frac{1}{u} \right\} - 1 \left\{ . \right. \end{aligned}$$

Thus the efficiency must theoretically increase with u , but the value of u is limited by the practical consideration that, even at moderately high speeds, so much of the head is

absorbed by frictional resistance as to sensibly diminish the efficiency.

The serious defects of the reaction wheel are that its speed is most unstable and that it admits of no efficient system of regulation for a varying supply of water.

By equation (4), the efficiency is a maximum, for a given value of u , when

$$V^2 - 2Vu + c_v^2 u^2 = 0,$$

or

$$V = u(1 + \sqrt{1 - c_v^2}). \quad (5)$$

Experiment also indicates that the best effect is produced when the linear speed of rotation (u) is that due to the total head (h), so that

$$u^2 = 2gh,$$

and therefore

$$V^2 = 4c_v^2 gh.$$

Substituting these values in equation (5), it is found that

$$c_v = \frac{2\sqrt{2}}{3},$$

and hence, by equation (4), the *maximum* efficiency = $\frac{2}{3}$.

Thus, one third of the head, i.e., $\frac{h}{3}$, is lost, and of this amount the portion $\frac{v_2^2}{2g} = \frac{(V-u)^2}{2g} = \frac{h}{9}$, is carried away by the effluent water in its energy of motion. The remainder, viz., $\frac{h}{3} - \frac{h}{9} = \frac{2}{9}h$ is lost in frictional resistance, etc.

EX. A reaction wheel with six tubular passages, each of 4 sq. ins. sectional area, passes 112,500 gallons of water per hour and makes 105 revolutions per minute. The distance between the axis of revolution and the axis of an orifice is 2 feet. (Take $c_v = 1$.)

$$V \frac{4}{144} = \frac{1}{6} \frac{112500}{64 \cdot 60 \cdot 60} = \frac{5}{6} \text{ cu. ft. per sec. per orifice.}$$

Therefore $V = 30$ ft. per sec.

Again, $u = \frac{2 \cdot \pi \cdot 2.105}{60} = 22$ ft. per sec.

Hence, if h is the head over the orifices,

$$30^2 = 22^2 + 2.32 \cdot h,$$

and $h = 6\frac{1}{2}$ ft.

The reaction on each tube $= \frac{62\frac{1}{2}}{32} \cdot \frac{5}{6} (30 - 22) = 13\frac{1}{8}$ lbs.

The useful work $= 6 \times 13\frac{1}{8} \times 22 = 1718\frac{1}{4}$ ft.-lbs.
 $= 3\frac{1}{8}$ H.P.

The efficiency $= \frac{1718\frac{1}{4}}{62\frac{1}{2} \cdot 5 \cdot 6\frac{1}{2}} = \frac{11}{13}$.

10. Impact Wheel. Borda Turbine.—A jet moving in the direction OC (Fig. 216), with a velocity $v_1 (= OC)$ impinges upon a flat vane, driving it in the direction OE with a velocity $u (= OE)$. Join CE .

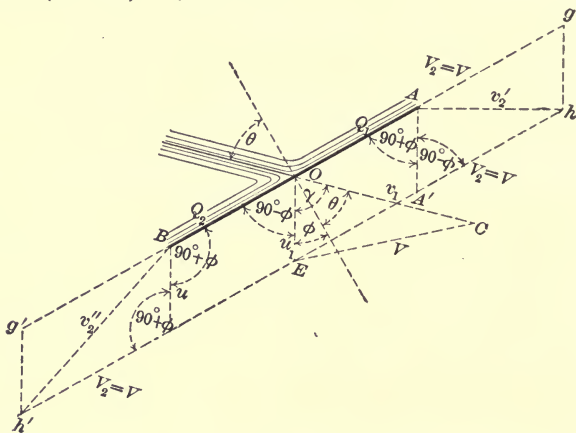


FIG. 216.

Let Q_1 be the quantity of flow towards A , and m_1 its mass.

" Q_2 " " " " " " B , " m_2 "

Disregarding the effect of gravity, which is equivalent to the assumption that the movement of the water on the vane is sensibly in a horizontal plane, and also disregarding friction, the water leaves the vane at A and B with a relative velocity $V = Ag = Bg' = CE$, coincident in direction with AB produced.

Draw AA' and BB' parallel and equal to $OE = u$.

Complete the parallelograms $A'g$ and $B'g'$. Then

$Ah, = v_2'$, represents in direction and magnitude the absolute velocity with which the water leaves the vane at A , and

$Bh', = v_2''$, represents in direction and magnitude the absolute velocity with which the water leaves the vane at B .

From the triangle OCE ,

$$V^2 = v_1^2 + u^2 - 2v_1u \cos \gamma.$$

From the triangle $AA'h$,

$$v_2'^2 = V^2 + u^2 - 2Vu \sin \phi.$$

From the triangle $BB'h'$,

$$v_2''^2 = V^2 + u^2 + 2Vu \sin \phi.$$

Hence

$$\frac{v_1^2 - v_2'^2}{2} = u(v_1 \cos \gamma - u + V \sin \phi)$$

and

$$\frac{v_1^2 - v_2''^2}{2} = u(v_1 \cos \gamma - u - V \sin \phi).$$

Also,

$$m_1 = \frac{w}{g} \frac{Q_1}{v_1} \left(v_1 - u \frac{\cos \phi}{\cos \theta} \right)$$

and

$$m_2 = \frac{w}{g} \frac{Q_2}{v_2} \left(v_1 - u \frac{\cos \phi}{\cos \theta} \right).$$

Therefore the useful work

$$\begin{aligned} &= m_1 \frac{v_1^2 - v_2'^2}{2} + m_2 \frac{v_1^2 - v_2''^2}{2} \\ &= \frac{w}{g} \frac{u}{v_1} \left(v_1 - u \frac{\cos \phi}{\cos \theta} \right) \{ Q(v_1 \cos \gamma - u) + (Q_1 - Q_2)V \sin \phi \}, \end{aligned}$$

where

$$Q = Q_1 + Q_2.$$

If the directions of motion of the vane and of the impinging jet coincide,

$$\gamma = \theta + \phi = 0 \quad \text{and} \quad V = v_1 - u,$$

and therefore the useful energy imparted to the vane

$$= \frac{w}{g} \frac{u}{v_1} (v_1 - u)^2 (Q + \overline{Q_1 - Q_2} \sin \phi).$$

For a maximum effect $v_1 = 3u$.

Series of Vanes.—If a number of vanes are successively introduced at the same point in the path of the jet, then

$$m_1 = \frac{w}{g} Q_1 \quad \text{and} \quad m_2 = \frac{w}{g} Q_2.$$

Thus the useful energy becomes

$$\frac{w}{g} u \{ Q(v_1 \cos \gamma - u) + (Q_1 - Q_2) V \sin \phi \};$$

and if the directions of motion of the vane and the impinging jet coincide,

$$\gamma = \theta + \phi = 0, \quad V = v_1 - u,$$

and the useful energy

$$= \frac{w}{g} u (v_1 - u) (Q + \overline{Q_1 - Q_2} \sin \phi).$$

For a maximum effect $v_1 = 2u$.

Flow in One Direction.—If the whole of the water flows away in the direction OA so that $Q_2 = 0$ and $Q_1 = Q$, the useful energy for a *single* vane

$$= \frac{wQ}{g} \frac{u}{v_1} \left(v_1 - u \frac{\cos \phi}{\cos \theta} \right) (v_1 \cos \gamma - u + V \sin \phi)$$

and the useful energy for a *series* of vanes

$$= \frac{wQ}{g} u(v_1 \cos \gamma - u + V \sin \phi).$$

For a given value of ϕ this last is greatest when $\gamma (= \theta + \phi) = 0$, and therefore $V = v_1 - u$. Then

$$\text{maximum useful energy} = \frac{wQ}{g} u(v_1 - u)(1 + \sin \phi),$$

which increases with ϕ or as the angle of exit OAA' ($= 90^\circ - \phi$) diminishes, indicating that it is advantageous to curve the outlet lip of the vane.

Denote the exit angle by e , Fig. 217. Then

$$V^2 = u^2 + v_1^2 - 2uv_1 \cos \gamma$$

and

$$v_2^2 = u^2 + V^2 - 2uV \cos e.$$

Thus the useful energy imparted to the vane

$$\begin{aligned} &= \frac{wQ}{g} \frac{v_1^2 - v_2^2}{2} \\ &= \frac{wQ}{g} u(v_1 \cos \gamma - u + V \cos e). \end{aligned}$$

If e is so small that $\cos e = 1$, approximately, then the useful energy

$$\begin{aligned} &= \frac{wQ}{g} u(v_1 \cos \gamma - u + V) \\ &= \frac{wQ}{g} u(v_1 \cos \gamma - u \\ &\quad + \sqrt{u^2 + v_1^2 - 2uv_1 \cos \gamma}). \end{aligned}$$

This is greatest and $= \frac{wQ}{g} \frac{v_1^2}{2}$ when $u = \frac{1}{2}v_1 \sec \gamma$, which is the best speed of the wheel. In such case the whole of the jet's energy is transformed into useful work.

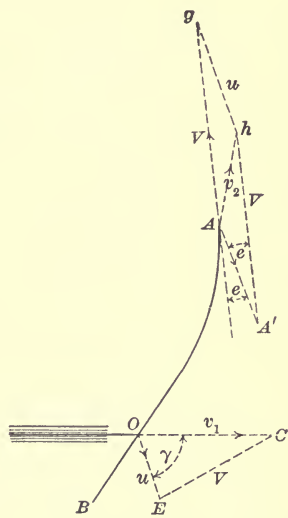


FIG. 217.

In the simplest kind of impact wheel the jet strikes the vane more or less perpendicularly and spreads over the surface in all directions. Wheels of 5 ft. diameter are used for falls of from 10 to 20 ft. The vanes are 15 ins. \times 8 ins. to 10 ins. measured radially, and are inclined at from 50° to 70° to the horizon. The water strikes the vane in a direction making an angle of from 10° to 20° with the horizon, i.e., nearly at right angles.

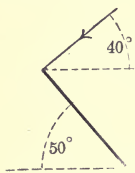


FIG. 218.

In a Borda turbine (Fig. 219) revolving about a vertical axis OO , the vanes are curved and the water, as it flows over them, acts principally by pressure. The vanes are set between two concentric drums which should be of considerable depth

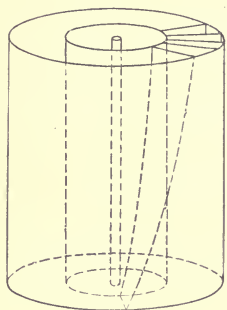


FIG. 219.

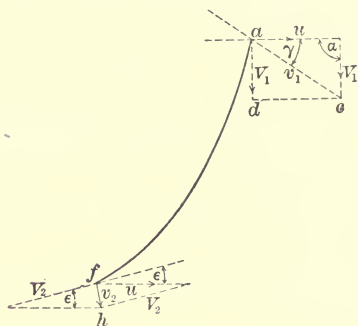


FIG. 220.

and should have a large mean diameter. Borda found that with such a turbine an efficiency of 75 per cent could be obtained under favorable conditions. As the water passes through the turbine the fluid particles move wholly in cylindrical surfaces, and there is little if any change in the distance of a particle from the axis. Thus the effect of centrifugal force may be disregarded.

Let Fig. 220 be a section of the vane at a distance from the axis equal to the mean radius.

The water strikes the vane at a in the direction ac , falls

upon the vane through a vertical distance h_2 , and is discharged at f in the direction fh with a velocity v_2 .

Let V_1 , V_2 be the relative velocities ad at a and fg at f , respectively. Then

$$V_2^2 = V_1^2 + 2gh_2.$$

If, again, the angle of exit e at f is so small that $\cos e = 1$, approximately,

$$v_2 = V_2 - u.$$

Suppose that the water leaves the turbine without energy, i.e., so that $v_2 = 0 = V_2 - u$, then

$$\begin{aligned} u^2 &= V_2^2 = V_1^2 + 2gh_2 \\ &= u^2 + v_1^2 - 2uv_1 \cos \gamma + 2gh_2 \end{aligned}$$

and

$$\begin{aligned} 2uv_1 \cos \gamma &= v_1^2 + 2gh_2 \\ &= 2g(H + h_2) \\ &= 2gH_1, \end{aligned}$$

or

$$uv_1 \cos \gamma = gH_1,$$

an equation giving the best speed of the turbine.

H is the head required to give the velocity v_1 at entrance.

H_1 is the total head under which the turbine works.

There should be no loss in shock at entrance, and to insure this ad ($= V_1$), the relative velocity, must be tangential to the lip at a .

The lip angle α is then given by

$$\frac{u}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha} = \cos \gamma + \cot \alpha \sin \gamma,$$

or

$$\cot \alpha = \frac{u}{v_1} \operatorname{cosec} \gamma - \cot \gamma.$$

Since $u = V_2$, the triangle fgh is isosceles, and

$$v_2 = 2u_2 \sin \frac{e}{2}.$$

$$\text{The useful work} = \frac{wQ}{g} \eta \left(1 - \frac{v_2^2}{2g} \right),$$

η being the efficiency.

Let R be the mean radius.

“ t “ “ water thickness, measured radially.

Then,

$$2\pi R t V_2 \sin e = Q.$$

Allowance may be made for the principal hydraulic resistances (friction, etc.) by taking

$f_2 \frac{v_1^2}{2g}$ to represent the loss of head up to the inlet, and

$f_4 \frac{V_2^2}{2g}$ “ “ “ “ “ “ in the wheel-passages.

Then

$$(1 + f_2) \frac{v_1^2}{2g} = H$$

and

$$(1 + f_4) \frac{V_2^2}{2g} = \frac{V_1^2}{2g} + h_2,$$

f_2 and f_4 being coefficients to be determined by experiment.

Usually f_2 varies from .025 to .2 and upwards, an average value being .125, and f_4 varies from .1 to .2.

The normal distance, Fig. 221, between two consecutive vanes should be $>$ the stream's normal thickness between the

vanes, i.e., $> \frac{v_1}{V} \times$ the normal thickness of the stream before impact.

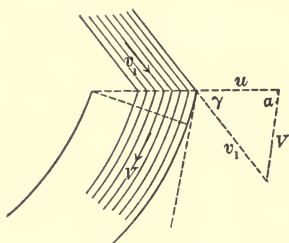


FIG. 221.

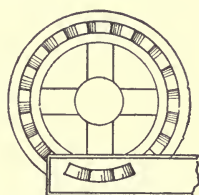


FIG. 222.

Burdin's (Fig. 222) is among the best of impact wheels, differing only from the simple Borda in receiving the water at several points simultaneously and in distributing the outlet openings in three concentric circles.

Ex. A 5-H.P. Borda turbine, of 4 ft. mean diameter and 4 ft. depth, works under a total head of 20 ft. The direction of the jet before impact is inclined at $33^\circ 33'$ (γ) to the horizon, and the angle of exit (e) is $19^\circ 8'$. The jet delivers 3 cu. ft. of water per second. Find (a) the best speed of the turbine; (b) the lip angle α ; (c) the velocity, v_2 , of the water as it leaves the turbine; (d) the hydraulic efficiency; (e) the practical efficiency.

$$(a) \quad 20 - 4 = 16 = \text{head required to produce } v_1 = \frac{v_1^2}{64}.$$

Therefore $v_1 = 32$ ft. per second.

The best speed is then given by

$$uv_1 \cos \gamma = gH_1,$$

$$\text{or} \quad u \cdot 32 \cdot \cos 33^\circ 33' = 32.20,$$

$$\text{or} \quad u \cdot 32 = 32.20 \cdot \frac{6}{5} \quad \text{and} \quad u = 24 \text{ ft. per second.}$$

$$\text{The number of revolutions per minute} = \frac{60 \times 24}{\frac{\pi}{2} \times 4} = 114\frac{6}{11}.$$

$$(b) \quad \frac{\sin(\alpha + \gamma)}{\sin \alpha} = \frac{u}{v_1} = \frac{24}{32} = \frac{3}{4} = \cos \gamma + \cot \alpha \sin \gamma,$$

$$\text{or} \quad \cot(180^\circ - \alpha) = \cot 33^\circ 33' - \frac{3}{4} \operatorname{cosec} 33^\circ 33' = .1509,$$

$$\text{and} \quad \alpha = 98^\circ 35'.$$

(c) Assuming $V_2 = u$, then

$$v_2 = 2u \sin \frac{\epsilon}{2} = 48 \sin 9^\circ 34' = 48 \times \frac{1}{6} = 8 \text{ ft. per second.}$$

(d) The hydraulic efficiency $= 1 - \frac{8^2}{64 \cdot 20} = \frac{19}{20} = .95$.

(e) If η is the practical efficiency,

$$\eta \cdot 62\frac{1}{2} \cdot 3 \left(20 - \frac{8^2}{2 \times 32} \right) = 5.550$$

and $\eta = .772$.

Danaïdes.—These are wheels capable of revolving about a vertical axis and consist of two casings which are more or less in the form of inverted truncated cones (Fig. 223) and which enclose a space divided into a number of water-passages by vanes which may be flat, spiral, or screw-shaped. In the wheels described by Belidor the inner casing with the vanes

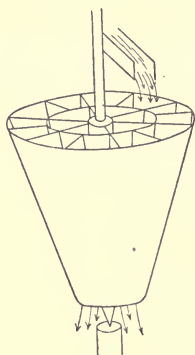


FIG. 223.

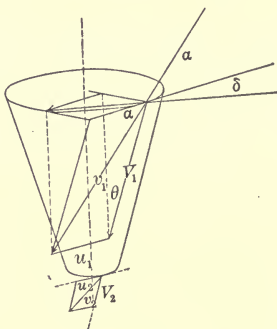


FIG. 224.

attached is made to closely fit the outer conical casing, which is fixed. In another form of Danaïde the vessel is divided into two equal parts by a vertical partition. Thus in wheels of this type the water approaches the axis in its descent, developing a centrifugal force which must be taken into account.

Consider the case of a Danaïde with double conical casing and flat vertical vanes, Fig. 224.

The relative velocities V_1 , V_2 are evidently at right angles to the corresponding peripheral velocities at inlet and exit. Therefore

$$v_1^2 = V_1^2 + u_1^2 \quad \text{and} \quad v_2^2 = V_2^2 + u_2^2.$$

Also, if h_2 is the depth of the wheel,

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + h_2 - \frac{u_1^2 - u_2^2}{2g},$$

the term $\frac{u_2^2 - u_1^2}{2g}$ being due to the effect of centrifugal force.

Hence

$$\frac{v_1^2 - v_2^2}{2g} = \frac{u_1^2 - u_2^2}{g} - h_2,$$

and the mechanical effect

$$= wQ \left(\frac{v_1^2}{2g} + h_2 - \frac{v_2^2}{2g} \right) = \frac{wQ}{g} (u_1^2 - u_2^2).$$

Tub-wheel.—This form of impact wheel, Fig. 225, consists of a number of floats fixed to a vertical shaft. The wheel is either fitted into a well, a small clearance being allowed, or it is given a larger diameter and is placed just below the well. The water is brought along a properly designed race, enters the well tangentially with considerable velocity and acquires a rotary motion. Thus it acts upon the floats both by impact and by pressure. The efficiency of the wheel is small, as a large portion of the water escapes without producing its full effect. Practical experience indicates that the best speed of the middle of the floats is about *one third* of the velocity of the current, and that the efficiency varies from 15 to 40 per cent, but rarely exceeds 30 per cent.

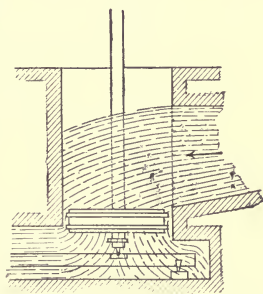


FIG. 225.

Draw DF parallel and equal to AC ($= u$).

Complete the parallelogram EF .

Then the diagonal DG evidently represents in direction and magnitude the absolute velocity v_2 with which the water leaves the vane.

Draw AK equal and parallel to DG ($= v_2$).

Join BK . Then BK represents the total change of velocity between A and D in direction and magnitude.

Thus if R is the resultant pressure on the vane, then $R = m \cdot BK$.

Let ML be the projection of BK upon AC .

Then ML represents the total change of velocity in the direction of the vane's motion.

Let P be the pressure upon the vane in this direction.

Then

$$P = m \cdot LM. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{The useful work} = Pu = mu \cdot LM = m \frac{v_1^2 - v_2^2}{2}. \quad . \quad (4)$$

$$\text{The total available work} = \frac{w}{g} A \frac{v_1^3}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{The efficiency} \frac{mu \cdot LM}{\frac{w}{g} A \frac{v_1^3}{2}} = 2mg \frac{v_1^2 - v_2^2}{w A v_1^3}. \quad . \quad . \quad . \quad . \quad (6)$$

Again, join CK .

Then, since AC is equal and parallel to DF , and AK to DG , the line CK is equal and parallel to DE , and is therefore equal to CB .

Thus in the isosceles triangle CBK , CB is equal and parallel to the relative velocity V at A , CK is equal and parallel to the relative velocity V at D , and the base BK represents the total change of motion.

Let δ be the angle through which the direction of the water is deviated, i.e., the angle between AB and AK . Then

$$\begin{aligned} V^2 &= CK^2 = AK^2 + AC^2 - 2AK \cdot AC \cos (A + \delta) \\ &= v_2^2 + u^2 - 2v_2u \cos (A + \delta), \quad . \quad . \quad . \quad . \quad . \quad (7) \end{aligned}$$

and also

$$\begin{aligned} V^2 &= CK^2 = CB^2 = AB^2 + AC^2 - 2AB \cdot AC \cos A \\ &= v_1^2 + u^2 - 2v_1u \cos A. \quad . \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

Hence

$$\frac{v_1^2 - v_2^2}{2} = u \{v_1 \cos A - v_2 \cos (A + \delta)\}. \quad . \quad . \quad (9)$$

If BH is drawn parallel to the tangent at D , BK evidently bisects the angle between BC and BH , and this angle is equal to the angle between the tangents to the vane at A and D .

Let α be the angle between the normals at A and D . Then the angle $KCB = \alpha$, and

$$\text{the angle } CBK = \frac{1}{2}(180^\circ - \alpha) = 90^\circ - \frac{\alpha}{2}.$$

Therefore

$$BK = 2CB \left(\cos 90^\circ - \frac{\alpha}{2} \right) = 2V \sin \frac{\alpha}{2}.$$

Hence

$$R = m \cdot BK = 2mV \sin \frac{\alpha}{2}. \quad . \quad . \quad . \quad (10)$$

Let X , Y be the components of R in the direction of the normal at A and at right angles to this direction. Then

$$X = R \cos \frac{\alpha}{2} = mV \sin \alpha, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$Y = R \sin \frac{\alpha}{2} = 2mV \sin^2 \frac{\alpha}{2} = mV(1 - \cos \alpha). \quad . \quad (12)$$

The efficiency is a maximum when

$$\frac{d(Pu)}{du} = 0 = u \frac{dP}{du} + P. \quad \dots \quad (13)$$

The efficiency is nil when

$$Pu = 0, \quad \text{i.e., when } u = 0 \text{ or } P = 0. \quad \dots \quad (14)$$

In the latter case, since $P = m \cdot LM$, the projection LM must be nil, and therefore BK must be at right angles to AC , as in Fig. 227.

FIG. 227.

227.

The angle ACB is now

$$= 180^\circ - \frac{\alpha}{2}. \quad \text{Therefore}$$

$$\begin{aligned} \frac{u}{v_1} &= \frac{\sin ABC}{\sin ACB} \\ &= \frac{\sin \left(\frac{\alpha}{2} - A \right)}{\sin \frac{\alpha}{2}}. \end{aligned}$$

If BK is parallel to AC , Fig. 228, then the angle

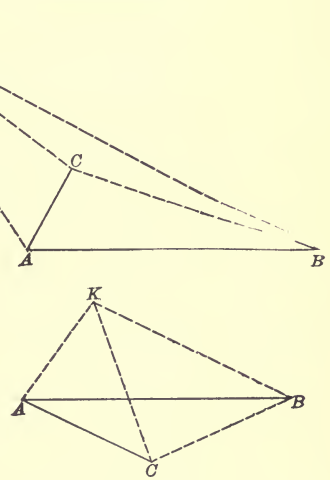


FIG. 228.

$$\begin{aligned} ACB &= \frac{1}{2}(180^\circ - \alpha) + \alpha \\ &= 90^\circ + \frac{\alpha}{2}, \end{aligned}$$

and therefore

$$\frac{u}{v_1} = \frac{\sin ABC}{\sin ACB} = \frac{\cos \left(\frac{\alpha}{2} + A \right)}{\cos \alpha}.$$

Let the direction of the impinging jet be tangential to the vane at A , Fig. 226, and let the jet and vane move in the same direction. Then

$$V = v_1 - u, \quad m = \frac{w}{g} A(v_1 - u);$$

$$P = Y = \frac{w}{g} A(v_1 - u)^2 (1 - \cos \alpha) = 2 \frac{w}{g} A(v_1 - u)^2 \sin^2 \frac{\alpha}{2};$$

$$\text{useful work} = Pu = 2 \frac{w}{g} Au(v_1 - u)^2 \sin^2 \frac{\alpha}{2};$$

$$\text{efficiency} = 4 \frac{u(v_1 - u)^2}{v_1^3} \sin^2 \frac{\alpha}{2}.$$

This is a maximum and equal to $\frac{16}{27} \sin^2 \frac{\alpha}{2}$ when $v_1 = 3u$.

These results are identical with those for a concave cup when $\alpha = 180^\circ$.

Instead of one vane let a series of vanes be successively introduced at short intervals at the same point in the path of the jet. Then

$$m = \frac{w}{g} Av_1,$$

and hence the pressure P , useful work, and efficiency respectively become

$$\frac{w}{g} Av_1 \cdot LM; \quad \frac{w}{g} Av_1 \frac{v_1^2 - v_2^2}{2}; \quad \frac{v_1^2 - v_2^2}{v_1^2}.$$

N.B. Frictional resistance may be taken into account by assuming that it absorbs a fractional portion of the head corresponding to the velocity of the jet relatively to the surface over which it spreads. Thus the loss of head $= f \frac{V^2}{2g}$, and the corresponding loss of energy $= wQf \frac{V^2}{2g}$.

Ex. A curved vane in the form of the quadrant of a circle receives *without shock*, at an edge, a stream of water flowing at the rate of 12 ft. per second, which drives the vane with a velocity of 4 ft. per second in a direction making an angle of 60° with the receiving edge.

At the receiving edge the triangle abc is a triangle of velocities in which the angle $abc = 120^\circ$, $ac = 12$ ft., $ab = 4$ ft., and $bc = V$, the relative velocity at a which must be parallel to the tangent at a , as there is to be no loss in shock. Then

$$12^2 = 4^2 + V^2 - 2 \cdot 4 \cdot V \cos 120^\circ,$$

or

$$128 = V^2 + 4V,$$

and

$$V = 9.4891 \text{ ft. per second.}$$

Also, if γ is the angle between ac and the receiving edge, then the angle $cab = 60^\circ - \gamma$, and

$$\frac{9.4891}{4} = \frac{bc}{ab} = \frac{\sin (60^\circ - \gamma)}{\sin \gamma} = \sin 60^\circ \cot \gamma - \cos 60^\circ,$$

or

$$\cot \gamma = 3.3166 \quad \text{and} \quad \gamma = 16^\circ 47'.$$

At the discharging edge $fgkh$ is the parallelogram of velocities in which fg , parallel to ab , $= 4$ ft., fk , tangential at f , $= 9.4891$ ft., the relative velocity, and fh is the absolute velocity in direction and magnitude with which the water leaves the vane. Let the angle $hfk = \delta$. Then

$$v_2^2 = 4^2 + (9.4891)^2 - 2 \times 4 \times 9.4891 \cos 30^\circ = 40.2993,$$

and

$$v_2 = 6.3481 \text{ ft. per sec.}$$

$$\text{Again, } \frac{9.4891}{4} = \frac{fk}{hk} = \frac{\sin (\delta + 30^\circ)}{\sin \delta} = \cos 30^\circ + \sin 30^\circ \cot \delta,$$

and

$$\cot \delta = 3.0126, \quad \text{or} \quad \delta = 18^\circ 22'.$$

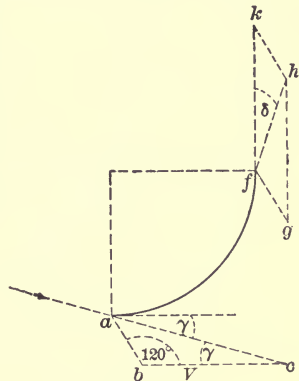


FIG. 229.

12. Tangential or Centrifugal Turbines.—Suppose that the vane AD is constrained to revolve about a vertical axis O with a constant angular velocity ω . If OP , OQ are consecutive radii and if PN is drawn at right angles to OQ , then the

work of the centrifugal force as a mass m of fluid moves from P to the consecutive position Q

$$= m\omega^2 r \cdot QN$$

$$= m\omega^2 r \cdot dr,$$

where $OP = r$.

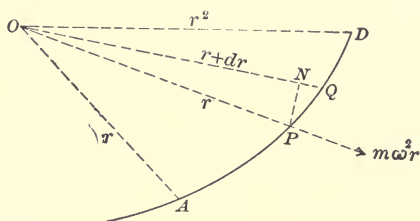


FIG. 230.

The total work in the movement from A to D

$$= \int_{r_1}^{r_2} m\omega^2 r \, dr$$

$$= \frac{m\omega^2}{2} (r_2^2 - r_1^2)$$

$$= \frac{m}{2} (u_2^2 - u_1^2),$$

u_1 , u_2 being the linear velocities at A and D respectively.

If the flow is from A to D , $\frac{u_2^2 - u_1^2}{2g}$ is evidently a gain of head, while it is a loss of head if the flow is from D to A .

In tangential or centrifugal flow turbines a number of vanes are encased and have concentric inlet and outlet surfaces. The flow, which is more or less radial, is *towards* the axis in the inward-flow and *from* the axis in the outward-flow turbine. Since the axis of rotation is vertical, the effect of gravity may be disregarded.

If v_r' , v_r'' are the radial components of v_1 and v_2 respectively,

$$v_r' = v_1 \sin \gamma \quad \text{and} \quad v_r'' = V_2 \sin \beta.$$

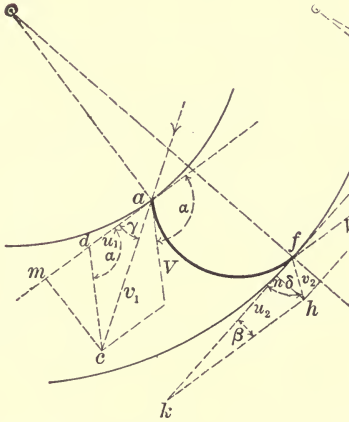


FIG. 231.

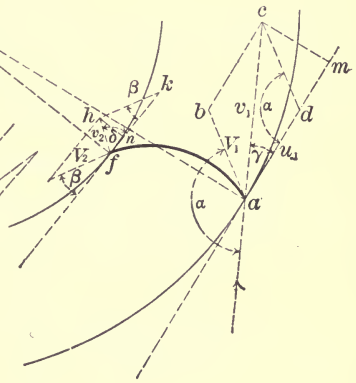


FIG. 232.

Then, by the condition of continuity of flow, and disregarding the thickness of the vanes,

$$2\pi r_1 d_1 v_r' = 2\pi r_1 d_1 v_1 \sin \gamma = Q = 2\pi r_2 d_2 v_r'' = 2\pi r_2 d_2 V_2 \sin \beta,$$

and

$$r_1 d_1 v_1 \sin \gamma = \frac{Q}{2\pi} = r_2 d_2 V_2 \sin \beta, \quad . \quad . \quad . \quad (1)$$

d_1 and d_2 being the inlet and outlet depths of the wheel.

First. Disregard hydraulic resistances. Then

$$\begin{aligned} V_2^2 &= V_1^2 + u_2^2 - u_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \\ &= u_1^2 + v_1^2 - 2u_1 v_1 \cos \gamma + u_2^2 - u_1^2 \\ &= v_1^2 - 2u_1 v_1 \cos \gamma + u_2^2, \end{aligned}$$

or

$$v_1^2 = V_2^2 - u_2^2 + 2u_1 v_1 \cos \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also, from the triangle fkh ,

$$v_2^2 = V_2^2 + u_2^2 - 2V_2u_2 \cos \beta. \quad . \quad . \quad . \quad (4)$$

Hence

$$\begin{aligned} \text{the useful work} &= \frac{wQ}{g} \frac{v_1^2 - v_2^2}{2} \\ &= \frac{wQ}{g} (u_1v_1 \cos \gamma + u_2V_2 \cos \beta - u_2^2). \end{aligned} \quad (5)$$

The energy carried away by the water on leaving the turbine should, of course, be as small as possible. Two assumptions are usually made in practice, viz.,

$$\text{either } u_2 = V_2,$$

and then also, by eq. (2), $u_1 = V_1$, so that the triangles fkh and acd are isosceles;

$$\text{or } \delta = 90^\circ,$$

so that the flow at outlet is radial, or $v_2 = v_r''$, and therefore the tangential component of v_2 , or, as it is called, the outlet velocity of whirl, is nil.

Adopting the assumption $u_2 = V_2$, in which case the triangles fkh and acd are isosceles, then

$$u_1 = V_1 = \frac{v_1}{2} \sec \gamma = \frac{r_1}{r_2} u_2 = \frac{r_1}{r_2} V_2. \quad . \quad . \quad (6)$$

Hence eq. (1) becomes

$$r_1 d_1 v_1 \sin \gamma = \frac{Q}{2\pi} = r_2 d_2 u_2 \sin \beta = \frac{r_2^2 d_2 v_1 \sin \beta}{r_1^2 2 \cos \gamma},$$

$$\text{or} \quad r_1^2 d_1 \sin 2\gamma = r_2^2 d_2 \sin \beta, \quad . \quad . \quad . \quad (7)$$

and, by eqs. (5) and (6),

$$\begin{aligned} \text{the useful work} &= \frac{wQ}{g} \left(\frac{v_1^2}{2} + u_2^2 \cos \beta - u_2^2 \right) \\ &= \frac{wQ}{g} \frac{v_1^2}{2} \left(1 - \frac{r_2^2 \sin^2 \frac{\beta}{2}}{r_1^2 \cos^2 \gamma} \right). \end{aligned} \quad (8)$$

The corresponding efficiency

$$= 1 - \frac{r_2^2 \sin^2 \frac{\beta}{2}}{r_1^2 \cos^2 \gamma}. \quad \dots \dots \dots (9)$$

Adopting the assumption $\delta = 90^\circ$, then

$$v_r' \cot \beta = v_2 \cot \beta = u_2 = V_2 \cos \beta. \quad \dots (10)$$

Eq. (1) now becomes

$$r_1 d_1 v_1 \sin \gamma = \frac{Q}{2\pi} = r_2 d_2 u_2 \tan \beta = \frac{r_2^2}{r_1} d_2 u_1 \tan \beta, \quad (11)$$

and, by eqs. (1), (5), and (10),

$$\text{the useful work} = \frac{wQ}{g} u_1 v_1 \cos \gamma = \frac{wQ}{g} \frac{v_1^2}{2} \frac{r_1^2 d_1 \sin 2\gamma}{r_2^2 d_2 \tan \beta}. \quad (12)$$

The corresponding efficiency

$$= \frac{r_1^2 d_1 \sin 2\gamma}{r_2^2 d_2 \tan \beta}. \quad \dots \dots \dots (13)$$

Second. The principal hydraulic resistances may be taken into account by taking the loss of head up to inlet $= f_2 \frac{v_1^2}{2g}$, and the loss of head in the wheel-passages $= f_4 \frac{V_2^2}{2g}$, so that the total loss due to the resistances in question

$$= f_2 \frac{v_1^2}{2g} + f_4 \frac{V_2^2}{2g}. \quad \dots \dots \dots (14)$$

Eq. (2) now becomes

$$(1 + f_4) \frac{V_2^2}{2g} = V_1^2 + u_2^2 - u_1^2; \quad \dots \dots (15)$$

and if H is the head over the inlet,

$$(1 + f_2) \frac{v_1^2}{2g} = H. \quad \dots \dots \dots (16)$$

Ex. 1. A centrifugal inward-flow turbine, with equal inlet and outlet depths and working under the head of 200 ft., passes 1 cu. ft. of water per second. The angle γ is 15° ; $4r_1 = 5r_2$; and it is assumed that $u_2 = V_2$. Find (a) the peripheral speed; (b) the lip angle at outlet; (c) the energy carried away by the water; (d) the energy lost in hydraulic resistance; (e) the useful work; (f) the efficiency. (Disregard the thickness of the vanes.)

$$\sin 15^\circ = .259, \quad \cos 15^\circ = .966, \quad \text{and let } f_2 = f_1 = .125.$$

$$(a) \quad \left(1 + \frac{1}{8}\right) \frac{v_1^2}{64} = 200. \quad \text{Therefore } v_1 = 106\frac{2}{3} \text{ ft. per sec.}$$

$$\text{By eq. (14),} \quad V_1^2 + u_2^2 - u_1^2 = \frac{9}{8} V_2^2 = \frac{9}{8} u_2^2,$$

$$\text{or} \quad v_1^2 - 2v_1u_1 \cos 15^\circ = \frac{1}{8} u_2^2 = \frac{1}{8} \left(\frac{4}{5}\right)^2 u_1^2 = .08u_1^2,$$

$$\text{or} \quad v_1 = 1.972u_1 = 106\frac{2}{3},$$

$$\text{so that} \quad u_1 = 54.09 \text{ ft. per sec.}$$

$$\text{Also,} \quad \frac{1}{1.972} = \frac{u_1}{v_1} = \frac{\sin(\alpha + 15^\circ)}{\sin \alpha} = \cos 15^\circ + \cot \alpha \sin 15^\circ.$$

$$\text{Therefore} \quad \cot(180^\circ - \alpha) = \cot 15^\circ - \frac{\operatorname{cosec} 15^\circ}{1.972} = 1.77277,$$

$$\text{and} \quad \alpha = 145^\circ 40''.$$

$$(b) \text{ By eq. (1),} \quad r_1 v_1 \sin \gamma = r_2 u_2 \sin \beta = \frac{r_2^2}{r_1} \sin \beta,$$

$$\text{or} \quad \sin \beta = \left(\frac{5}{4}\right)^2 \times 1.972 \times .259 = .79804,$$

$$\text{and} \quad \beta = 52^\circ 56'.$$

(c) The energy carried away by the water

$$= 62\frac{1}{2} \cdot 1 \cdot \frac{v_2^2}{64} = \frac{125}{128} 4u_2^2 \sin^2 \frac{\beta}{2} = \frac{25}{4} u_1^2 (1 - \cos \beta)$$

$$= \frac{5}{4} (54.09)^2 \times .397 = 1451.8 \text{ ft.-lbs.}$$

$$= 2.64 \text{ H.P.}$$

(d) By eq. (14), the loss in hydraulic resistance

$$\begin{aligned} &= \frac{1}{8} \cdot \frac{62\frac{1}{2}}{64} \cdot 1 \left(106\frac{2}{3}^2 + \frac{16}{25} (54.09)^2 \right) = 1617.5 \text{ ft.-lbs.} \\ &= 2.94 \text{ H.P.} \end{aligned}$$

(e) The total possible work = $62\frac{1}{2} \cdot 1 \cdot 200 = 12,500$ ft.-lbs.

The useful work = $12500 - 1451.8 - 1617.5 = 9430.7$ ft.-lbs.
= 17.146 H.P.

(f) The efficiency = $\frac{9430.7}{12500} = .754$.

EX. 2. A centrifugal outward-flow turbine with an efficiency of 80 per cent and working under the head of 200 ft. over the inlet passes 1 cu. ft. of water per second. The angle $\gamma = 15^\circ$; $5r_1 = 4r_2$; and the velocity at outlet is radial, i.e., $\delta = 90^\circ$. Find (a) the peripheral speed; (b) the lip angle at inlet; (c) the ratio of the inlet to the outlet depth; (d) the lip angle at outlet; (e) the energy carried away by the water; (f) the useful work. (Disregard the blade thickness and the hydraulic resistance.)

(a) $\frac{v_1^2}{64} = 200$, and therefore $v_1 = 80\sqrt{2} = 113.17$ ft. per sec.

But $.8 = \text{the efficiency} = \frac{u_1 v_1 \cos 15^\circ}{32 \times 200} = \frac{u_1 \cos 15^\circ}{40\sqrt{2}}$,

or $u_1 = 32\sqrt{2} \sec 15^\circ = 46.851$ ft. per sec.

(b) $\frac{32\sqrt{2} \sec 15^\circ}{80\sqrt{2}} = \frac{2}{5} \sec 15^\circ = \frac{u_1}{v_1} = \frac{\sin(\alpha + 15^\circ)}{\sin \alpha}$
= $\cos 15^\circ + \cot \alpha \sin 15^\circ$,

and $\cot(180^\circ - \alpha) = \cot 15^\circ - \frac{2}{5} \sec 15^\circ \operatorname{cosec} 15^\circ$
= $3.7320508 - 1.6 = 2.1320508$,

and $\alpha = 154^\circ 52'$.

(c) By eqs. (10) and (12),

$$u_2^2(1 + \tan^2 \beta) = V_2^2 = V_1^2 + u_2^2 - u_1^2 = u_2^2 + v_1^2 - 2u_1v_1 \cos \gamma,$$

or $u_2^2 \tan^2 \beta = \left(\frac{4}{5}\right)^2 \left(\frac{d_1}{d_2}\right)^2 v_1^2 \sin^2 15^\circ = v_1^2 - 2u_1v_1 \cos 15^\circ$,

or $u_2^2 \tan^2 \beta = 8192 \sin^2 15^\circ \left(\frac{d_1}{d_2}\right)^3 = 80\sqrt{2}(80\sqrt{2} - 64\sqrt{2}) = 2560$,

which gives $\left(\frac{d_1}{d_2}\right)^3 = 4.665$,

and therefore $\frac{d_1}{d_2} = 2.16$.

(d) By (c),

$$\tan^2 \beta = \frac{2560}{u_2^2} = \frac{2560}{\left(\frac{5}{4}\right)^2 u_1^2} = \frac{2560}{25 \cdot (32\sqrt{2} \sec 15^\circ)^2} = \frac{4}{5} \cos^2 15^\circ = .7464.$$

Therefore $\tan \beta = .864$,
and $\beta = 40^\circ 50'$.

(e) The energy carried away by the water

$$\begin{aligned}
 &= 62\frac{1}{2} \cdot 1 \cdot \frac{v_2^2}{64} = \frac{125}{128} u_2^2 \tan^2 \beta = \frac{125}{128} \times 2560 \\
 &= 2500 \text{ ft.-lbs.} \\
 &= 4\frac{6}{11} \text{ H.P.}
 \end{aligned}$$

(f) The total possible work

$$\begin{aligned}
 &= 1.62\frac{1}{2} \cdot 200 \\
 &= 12,500 \text{ ft.-lbs.}
 \end{aligned}$$

Hence the useful work

$$\begin{aligned}
 &= \text{total possible work} - 2500 \\
 &= 12500 - 2500 = 10,000 \text{ ft.-lbs.} \\
 &= 18\frac{8}{11} \text{ H.P.}
 \end{aligned}$$

13. Jet Turbine.—In the jet turbine the water passes along the axis and is distributed radially in all directions so that the angle $\gamma = 90^\circ$. It is no longer possible to have $u_1 = V_1$, and it cannot therefore be assumed that $u_2 = V_2$. A fair efficiency may, however, be secured by making $u_2 = v_1$.

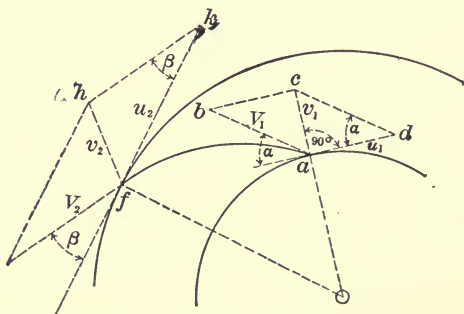


FIG. 233.

First, disregard hydraulic resistances. Then, from the triangle adc ,

$$\begin{aligned}
 u_1^2 + v_1^2 &= V_1^2 = V_2^2 - u_2^2 + u_1^2 \\
 &= V_2^2 - v_1^2 + u_1^2,
 \end{aligned}$$

and $2v_1^2 = V_2^2$.

From the triangle fkh ,

$$\begin{aligned} v_2^2 &= u_2^2 + V_2^2 - 2u_2V_2 \cos \beta \\ &= v_1^2(3 - 2\sqrt{2} \cos \beta). \end{aligned}$$

Hence

$$\begin{aligned} \text{the useful work} &= \frac{wQ}{g} \frac{v_1^2 - v_2^2}{2} \\ &= wQ \frac{v_1^2}{2g} (2\sqrt{2} \cos \beta - 2), \end{aligned}$$

and $\text{the efficiency} = 2\sqrt{2} \cos \beta - 2$.

Hence, too, $\cos \beta > \frac{1}{\sqrt{2}}$, i.e., β must not exceed 45° .

Second, taking the hydraulic resistances into account,

$$\begin{aligned} u_1^2 + v_1^2 = V_1^2 &= (1 + f_4)V_2^2 - u_2^2 + u_1^2 \\ &= (1 + f_4)V_2^2 - v_1^2 + u_1^2, \end{aligned}$$

or $2v_1^2 = (1 + f_4)V_2^2$.

Also, the loss of head up to inlet $= f_2 \frac{v_1^2}{2g}$.

“ “ “ “ in wheel-passages $= f_4 \frac{V_2^2}{2g} = \frac{f_4}{1 + f_4} \cdot \frac{2v_1^2}{2g}$,

and the total loss of head due to the *principal* hydraulic resistances

$$= \frac{v_1^2}{2g} \left(f_2 + \frac{2f_4}{1 + f_4} \right).$$

If H is the head over the inlet,

$$(1 + f_2) \frac{v_1^2}{2g} = H.$$

NOTE.—Impact, centrifugal, and jet turbines will work with the axis inclined at any angle to the vertical.

14. Resistance to the Motion of Solids in a Fluid Mass.

—The preceding results indicate that the pressure due to the impact of a jet upon a surface may be expressed in the form

$$P = K w A \frac{V^2}{2g},$$

A being the sectional area of the jet, V the velocity of the jet relatively to the surface, and K a coefficient depending on the position and form of the surface.

Again, the normal pressure (N) on each side of a thin plate, completely submerged in an indefinitely large mass of still water, is the same. If the plate is made to move horizontally with a velocity V , a forward momentum is developed in the water immediately in front of the plate, while the plate tends to leave behind the water at the back. A portion of the water carried on by the plate escapes laterally at the edges and is absorbed in the neighboring mass, while the region it originally occupied is filled up with other particles of water. Thus the normal pressure N , in front of the plate, is increased by an amount n , while at the back eddies and vortices are produced, and the normal pressure N at the back is diminished by an amount n' . The total resultant normal pressure, or the normal resistance to motion, is $n + n'$, and this increases with the speed. In fact, as the speed increases, n' approximates more and more closely to N , and in the limit the pressure at the back would be nil, so that a vacuum might be maintained.

Confining the attention to a plate moving in a direction normal to its surface, the resistance is of the same character as if the plate is imagined to be at rest and the fluid moving in the opposite direction with a velocity V . So, if both the water and the plate are in motion, imagine that a velocity equal and opposite to that of the water is impressed upon every particle of the plate and of the water. The resistance is then of the same character as that of a plate moving in still water, the velocity of the plate being the velocity relatively to the

water. Thus, in general, the resistance to the motion of such a plane moving in the direction of the normal to its surface, with a velocity V relatively to the water, may be expressed in the form

$$R = KwA \frac{V^2}{2g},$$

A being the area of the plate, and K a coefficient depending upon the form of the plate and also upon the relative sectional areas of the plate and of the water in which it is submerged.

According to the experiments of Dubuat, Morin, Piobert, Didion, Mariotte, and Thibault, the value of K may be taken at 1.3 for a plate moving in still water, and at 1.8 for a current moving on a fixed plate. Unwin points out the unlikelihood of such a difference between the two values, and suggests that it might possibly be due to errors of measurement.

Again, reasoning from analogy, the resistance to the motion of a solid body in a mass of water, whether the body is wholly or only partially immersed, has been expressed by the formula

$$R = KwA \frac{V^2}{2g},$$

V being the relative velocity of the body and water, A the greatest sectional area of the immersed portion of the body at right angles to the direction of motion, and K a coefficient depending upon the form of the body, its position, the relative sectional areas of the body and of the mass of water in which it is immersed, and also upon the surface wave-motion.

The following values have been given for K :

$K = 1.1$ for a prism with plane ends and a length from three to six times the least transverse dimension;

$K = 1.0$ for a prism, plane in front, but tapering towards the stern, the curvature of the surface changing gradually

so that the stream-lines can flow past without any production of eddy motion, etc.;

$K = .5$ for a prism with tapering stern and a cut-water or semicircular prow;

$K = .33$ for a prism with a tapering stern and a prow with a plane front inclined at 30° to the horizon;

$K = .16$ for a well-formed ship.

Froude's experiments, however, show that the resistance to the motion of a ship, or of a body tapering in front and in the rear, so that there is no abrupt change of curvature leading to the production of an eddy motion, is almost entirely due to skin-friction (see Art. 1, Chap. II).

15. Pressure of a Steady Stream in a Uniform Pipe against a Thin Plate AB Normal to the Direction of Motion.

—The stream-lines in front of the plate are deviated and a contraction is formed at C_2C_2 . They then converge, leaving a mass of eddies behind the plate.

Consider the mass bounded by the transverse planes C_1C_1 , C_3C_3 , where the stream-lines are again parallel.

At C_1C_1 let p_1 , A_1 , v_1 , z_1 be the mean intensity of the pressure, the sectional area of the waterway, the velocity of flow, and the elevation of the C. of G. of the section above datum.

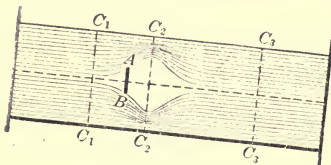


FIG. 234.

Let p_2 , A_2 , v_2 , z_2 be corresponding symbols at C_2C_2 .

Let p_3 , A_3 , v_3 , z_3 be corresponding symbols at C_3C_3 .

Let a be the area of the plate.

Let c_c be the coefficient of contraction.

Neglect the skin and fluid friction between C_1C_1 and C_3C_3 .

Then, by Bernoulli's theorem,

$$z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g} = z_3 + \frac{p_3}{w} + \frac{v_3^2}{2g} + \frac{(v_2 - v_3)^2}{2g},$$

the term $\frac{(v_2 - v_1)^2}{2g}$ representing the loss of head due to the bending of the stream-lines between C_2C_2 and C_3C_3 .

Hence

$$z_1 - z_3 + \frac{p_1 - p_3}{w} = \frac{(v_2 - v_1)^2}{2g}.$$

Again, let R be the total pressure on the plane. Then

$$p_1 A_1 - p_3 A_1 = (p_1 - p_3) A_1 = \left\{ \begin{array}{l} \text{fluid pressure in the direction} \\ \text{of the axis,} \end{array} \right.$$

$$w A_1 C_1 C_3 \frac{z_1 - z_3}{C_1 C_3} = w A_1 (z_1 - z_3)$$

= component of the weight in the direction of the axis.

Thus

$$(p_1 - p_3) A_1 + w A_1 (z_1 - z_3) - R = \text{change of motion in direction of axis}$$

$$= 0,$$

since the motion is steady.

Hence

$$\frac{R}{w A_1} = \frac{p_1 - p_3}{w} + z_1 - z_3 = \frac{(v_2 - v_1)^2}{2g}.$$

But $A_1 v_1 = A_2 v_2 = c_c (A_1 - a) v_2$. Therefore

$$\begin{aligned} R &= w A_1 \frac{v_1^2}{2g} \left\{ \frac{A_1}{c_c (A_1 - a)} - 1 \right\} \\ &= w a \frac{v_1^2}{2g} m \left\{ \frac{m}{c_c (m - 1)} - 1 \right\}^2, \end{aligned}$$

where $m = \frac{A_1}{a}$, or

$$R = Kwa \frac{v_1^2}{2g},$$

where $K = m \left\{ \frac{m}{c_c(m-1)} - 1 \right\}^2$.

16. Pressure of a Steady Stream in a Uniform Pipe on a Cylindrical Body about Three Diameters in Length.—The stream-lines in front of the body are deviated and a contraction is formed at C_2C_2 . They then converge, flow in parallel lines, and converge a second time at C_3C_3 , leaving a mass of eddies behind the body.

Consider the mass bounded by the planes C_1C_1 , C_4C_4 .

As in the previous article, let

p_1 , A_1 , v_1 , z_1 be the intensity of pressure, sectional area of the waterway, velocity of flow, and elevation of C. of G. above datum at

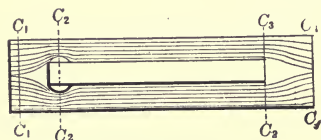


FIG. 235.

C_1C_1 ,
 p_2 , A_2 , v_2 , z_2 be similar symbols
 for C_2C_2 .
 p_3 , A_3 , v_3 , z_3 be similar symbols
 for C_3C_3 .

p_4 , A_4 , v_4 , z_4 be similar symbols for C_4C_4 .

Neglect the skin and fluid friction between C_1C_1 and C_4C_4 .

Then, by Bernoulli's theorem,

$$\begin{aligned} z_1 + \frac{p_1}{w} + \frac{v_1^2}{2g} &= z_2 + \frac{p_2}{w} + \frac{v_2^2}{2g} = z_3 + \frac{p_3}{w} + \frac{v_3^2}{2g} + \frac{(v_2 - v_3)^2}{2g} \\ &= z_4 + \frac{p_4}{w} + \frac{v_1^2}{2g} + \frac{(v_3 - v_1)^2}{2g} + \frac{(v_2 - v_3)^2}{2g}, \end{aligned}$$

$\frac{(v_2 - v_3)^2}{2g}$ being the loss of head between C_2C_2 and C_3C_3 and

$\frac{(v_3 - v_1)^2}{2g}$ being the loss of head between C_3C_3 and C_4C_4 .

Hence

$$z_1 - z_4 + \frac{p_1 - p_4}{w} = \frac{(v_3 - v_1)^2}{2g} + \frac{(v_2 - v_3)^2}{2g}.$$

But $A_1 v_1 = A_2 v_2 = A_3 v_3, \quad c_c(A_1 - a) = A_2,$

and $A_3 = A_1 - a.$

Therefore

$$\begin{aligned} z_1 - z_4 + \frac{p_1 - p_4}{w} &= \frac{v_1^2}{2g} \left[\left(\frac{A_1}{A_1 - a} - 1 \right)^2 + \left\{ \frac{A_1}{c_c(A_1 - a)} - \frac{A_1}{A_1 - a} \right\}^2 \right] \\ &= \frac{v_1^2}{2g} \left[\left(\frac{m}{m - 1} - 1 \right)^2 + \left\{ \frac{m}{c_c(m - 1)} - \frac{m}{m - 1} \right\}^2 \right], \end{aligned}$$

where $m = \frac{A_1}{a}.$

Also, as in the preceding article,

$$(p_1 - p_4)A_1 + wA_1(z_1 - z_4) - R = 0.$$

Hence

$$\begin{aligned} R &= wa \frac{v_1^2}{2g} m \left\{ \frac{1}{(m - 1)^2} + \frac{m^2}{(m - 1)^2} \left(\frac{1}{c_c} - 1 \right)^2 \right\} \\ &= Kwa \frac{v_1^2}{2g}, \end{aligned}$$

where $m = \frac{A_1}{a},$ and

$$K = m \left\{ \frac{1}{(m - 1)^2} + \frac{m^2}{(m - 1)^2} \left(\frac{1}{c_c} - 1 \right)^2 \right\}.$$

This value of K is always less than the value of K for the plate in the preceding article for the same values of $m, a,$ and $c_c.$

Hence the pressure on the cylinder is also less than the corresponding pressure on the plate.

In every case K should be determined by experiment.

EXAMPLES.

1. A stream with a transverse section of 24 sq. ins. delivers 10 cu. ft. of water per second against a flat vane in a normal direction. Find the pressure on the vane.

Ans. 1171 $\frac{1}{2}$ lbs.

2. If the vane in example 1 moves in the same direction as the impinging jet with a velocity of 24 ft. per second, find (a) the pressure on the vane; (b) the useful work done; (c) the efficiency.

Ans. (a) 421 $\frac{1}{2}$ lbs.; (b) 10,125 ft.-lbs.; (c) .288.

3. What must be the speed of the vane in example 2, so that the efficiency of the arrangement may be a maximum? Find the maximum efficiency.

Ans. 20 ft. per sec.; $\frac{8}{27}$.

4. Find (a) the pressure, (b) the useful work done, (c) the efficiency, when, instead of the single vane in example 2, a series of vanes are introduced at the same point in the path of the jet at short intervals.

Ans. (a) 703 $\frac{1}{2}$ lbs.; (b) 16,875 ft.-lbs.; (c) .48.

What must be the speed of the vane to give a maximum efficiency? What will be the maximum efficiency?

Ans. 30 ft. per sec.; .5.

5. A stream of water delivers 7500 gallons per minute at a velocity of 15 ft. per second and strikes an indefinite plane. Find the normal pressure on the vane when the stream strikes the vane (a) normally; (b) at an angle of 60° to the normal.

Ans. (a) 585.9 lbs.; 292.9 lbs.

6. A railway truck, full of water, moving at the rate of 10 miles an hour, is retarded by a jet flowing freely from an orifice 2 ins. square in the front, 2 ft. below the surface. Find the retarding force.

Ans. 7.97 lbs.

7. A jet of water of 48 sq. ins. sectional area delivers 100 gallons per second against an indefinite plane inclined at 30° to the direction of the jet; find the total pressure on the plane, neglecting friction. How will the result be affected by friction?

Ans. 750 lbs.

8. If the plane in example 7 move at the rate of 24 ft. per second in a direction inclined at 60° to the normal to the plane, find the useful work done and the efficiency.

Ans. 2250 ft.-lbs.; $\frac{1}{16}$.

At what angle should the jet strike the plane so that the efficiency might be a maximum? Find the maximum efficiency.

Ans. $\sin^{-1} \frac{3}{4}$; $\frac{1}{6}$.

9. A stream of 32 sq. ins. sectional area delivers 7 $\frac{1}{2}$ cu. ft. of water per second. At short intervals a series of flat vanes are introduced at the same point in the path of the stream. At the instant of impact the direction of the jet is at right angles to the vane, and the vane itself moves in a direction inclined at 45° to the normal to the vane. Find

the speed of the vane which will make the efficiency a maximum. Also find the maximum efficiency and the useful work done.

Ans. 15.08 ft. per. sec.; $\frac{8}{27}$; 2106 $\frac{2}{3}$ $\frac{4}{3}$ ft.-lbs.

10. A stream of water of $\frac{1}{8}$ sq. ft. sectional area delivers 16 cu. ft. per second normally against a flat vane. Find the pressure on the vane.

If the vane moves in the same direction as the impinging jet, with a velocity of 32 ft. per second, find (a) the pressure on the vane; (b) the useful work done; (c) the efficiency.

How would the results be affected if the vane were inclined at 45° to the jet, and moved in the direction of its normal with a velocity of 24 ft. per second?

Ans. 4000 lbs.; 2250 lbs., 72,000 ft.-lbs., $\frac{9}{32}$; 1802.8 lbs., 43,268 ft.-lbs., .169.

11. Two cubic feet of water are discharged per second under a pressure of 100 lbs. per sq. in. through a thin-lipped orifice in the vertical side of a vessel, and strike against a vertical plate. Find the pressure on the plate and the reaction on the vessel.

Ans. 475.82 lbs.

12. A stream moving with a velocity of 16 ft. per second in the direction *ABC* strikes obliquely against a flat vane and drives it with a velocity of 8 ft. per second in the direction *BD*, the angle *CBD* being 30° . Find (a) the angle between *ABC* and the normal to the plane for which the efficiency is a maximum; (b) the maximum efficiency; (c) the velocity with which the water leaves the vane; (d) the useful work per cubic foot of water.

Ans. (a) $21^\circ 44'$; (b) .25664; (c) 12.6 ft. per sec.; (d) 256.64 ft.-lbs.

13. At 8 knots an hour the resistance of the Water-witch was 5500 lbs.; the two orifices of her jet propeller were each 18 ins. by 24 ins. Find (a) the velocity of efflux; (b) the delivery of the centrifugal pump; (c) the useful work done; (d) the efficiency; (e) the propelling H.P., assuming the efficiency of the pump and engine to be .4.

Ans. (a) 29.4 ft. per sec.; (b) 1104.6 gallons per sec.; (c) 74.393 ft.-lbs.; (d) .63; (e) 536.7.

14. If feathering-paddles are substituted for the jet propeller in question 15, what would be the area of stream driven back for a slip of 25%? Find the efficiency and the water acted on in gallons per minute.

Ans. 34.63 sq. ft.; .75; 234.206.

15. A jet issues horizontally, under a head of 20 ft., from a $\frac{1}{2}$ -in. orifice in the vertical face of a tank and strikes normally the centre of a vane at a distance of 48 ins., measured horizontally, from the tank's face. By measurement the vertical distance of the point of impact, below the axis of the orifice, was found to be 2.582 inches. Find the coefficient of velocity (c_v), the inclination of the vane's axis to the horizontal, and the coefficient of impact (c_i) in the following cases:

(a) A flat 12-in. circular vane, the balancing weight (*W*) being 3.015 lbs.

(b) A hemispherical vane of 12 ins. diameter, W being 3.556 lbs.

(c) A hemispherical vane of 3 ins. diameter, W being 5.776 lbs.

(d) A parabolic vane with a base of 12 ins. in diameter and 6 ins. in height, W being 3.535 lbs.

(e) An elliptic vane, 6 ins. in height and having a base of 12 ins. diameter, W being 3.56 lbs. The vane edge is inclined at 20° to the axis.

Ans. .96411; $6^\circ 8'$; (a) .9834; (b) .5799; (c) .9419; (d) .6086;

(e) .5986.

16. Find the angle of blade at entrance, the useful effect, and the efficiency of a Borda turbine from the following data: the jet at entrance makes an angle of 60° with the horizontal; the depth of the turbine is 3 ft.; the total fall to the point of discharge is 19 ft.; the mean diameter of the turbine is 4 ft.; the quantity of water passing through the turbine is 4 cu. ft. per second; the angle of blade at exit is 30° . (Disregard hydraulic resistance.)

Ans. $51^\circ 33'$; 7.256 H.P.; .84.

17. What advantages are gained by increasing the depth and diameter of a Borda turbine and by curving the outlet lips of the buckets?

18. A Borda turbine of 3.5 ft. mean diameter has a head of 12.96 ft. over the inlet, a practical efficiency of .75, a theoretic efficiency (i.e., disregarding hydraulic resistances) of .9265 and delivers 3 horse-power. The radial width of the water-passages is 3 ins., and the depth of the turbine is 2.04 ft. If there is to be no shock at entrance, find (a) the inlet and outlet lip angles; (b) the velocity (v_2) of discharge; (c) the quantity of water used by the turbine.

Ans. (a) $111^\circ 25'$, $25^\circ 12'$; (b) 8.4 ft. per sec.; (c) 2.532 cu. ft. per sec.

19. In a railway truck, full of water, an opening 2 ins. in diameter is made in one of the ends of the truck, 9 ft. below the surface of the water. Find the reaction (a) when the truck is standing; (b) when the truck is moving at the rate of 10 ft. per second in the same direction as the jet; (c) when the truck is moving at the rate of 10 ft. per second in a direction opposite to that of the jet. If this movement of the truck is produced by the reaction of the jet, find the efficiency.

Ans. (a) 24.55 lbs. per sq. in.; (b) 34.78 lbs. per sq. in.;

(c) 14.3 lbs. per sq. in.; .588.

20. From a ship moving forward at 6 miles an hour a jet of water is sent astern with a velocity relative to the ship of 30 ft. per second from a nozzle having an area of 16 sq. ins.; find the propelling force and the efficiency of the jet as a propeller without reference to the manner in which the supply of water may be obtained. *Ans.* $138\frac{1}{8}$ lbs.; .4535.

21. A reaction wheel is inverted and worked as a pump. Find the speed of maximum efficiency and the maximum efficiency, the coefficient of hydraulic resistance referred to the orifices being .125.

Ans. Speed = twice that due to lift; .758.

22. A reaction wheel with orifices 2 ins. in diameter makes 80 revolutions per minute under a head of 5 ft. The distance between the centre of an orifice and the axis of rotation is 12 inches. Find the H.P. and the efficiency. *Ans.* .146; .596.

23. In a reaction wheel the speed of maximum efficiency is that due to the head. In what ratio must the resistance be diminished to work at $\frac{4}{3}$ this speed, and what will then be the efficiency? Obtain similar results when the speed is diminished to three fourths of its original amount. *Ans.* .94; .8896; 1.067; .75

24. In a reaction wheel, determine the per cent of available effect lost (1) if $V^2 = 2gH$; (2) if $V^2 = 4gH$; (3) if $V^2 = 8gH$.

What conclusion may be drawn from the results?

Efficiencies are respectively .828, .9, .945.

25. A stream of 64 sq. ins. section strikes with a 40-ft. velocity against a fixed cone having an angle of convergence = 100° ; find the hydraulic pressure. *Ans.* 492.1 lbs.

26. A jet of 9 sq. ins. sectional area, moving at the rate of 48 ft. per second, impinges upon the convex surface of a paraboloid in the direction of the axis and drives it in the same direction at the rate of 16 ft. per second. Find the force in the direction of motion, the useful work done, and the efficiency. The base of the paraboloid is 2 ft. in diameter and its length is 8 ins. *Ans.* 25 lbs.; 400 ft.-lbs.; $\frac{8}{133}$.

27. A stream of water of 16 sq. ins. sectional area delivers 12 cu. ft. of water per second against a vane in the form of a surface of revolution, and drives in the same direction, which is that of the axis of the vane. The water is turned through an angle of 60° from its original direction before it leaves the vane. Neglecting friction, find the speed of vane which will give a maximum effect. Also find impulse on vane, the work on vane, and the velocity with which the water leaves the vane.

Ans. 36 ft. per sec.; $562\frac{1}{2}$ lbs.; 20,250 ft.-lbs.; 95.24 ft. per. sec.

28. A surface of revolution is driven in the direction of its axis with a velocity of 16 ft. per second by means of a jet of water of 18 sq. ins. sectional area, which moves in the direction of the axis with a velocity of 80 ft. per second, and impinges upon the convex side of the surface. The tangent at the edge of the surface makes an angle of 30° with the vertical. Find the pressure on the surface and the efficiency.

Ans. 500 lbs.; .128.

29. A jet of water under a head of 20 ft., issuing from a vertical thin-lipped orifice 1 in. in diameter, impinges upon the centre of a vane 3 ft. from the orifice. Determine the position of the vane and the force of the impact (a) when the vane is a plane surface; (b) when the vane is 6 ins. in diameter and in the form of a portion of a sphere of 6 ins. radius.

Ans. (a) 13,679 lbs.; (b) 20,518 lbs. or 6.839 lbs. according as vane is concave or convex.

30. A stream of water 1 in. thick and 8 ins. wide, moving with a velocity of 18 ft. per second, strikes without shock a circular vane, of a length subtending an angle of 90° at the centre. The vane is driven in the direction of the stream with a velocity of 6 ft. per second. Find the pressure on the vane, the work done, and the efficiency.

Ans. $22\frac{3}{8}$ lbs.; $93\frac{3}{4}$ ft.-lbs.; $\frac{8}{27}$.

31. A Pelton wheel of 2 ft. diameter makes 822 revolutions per minute under a pressure-head of 200 lbs. per square inch, the delivery of water being 100 cu. ft. per minute. Find the total H.P., assuming that the buckets are so formed that the water is returned parallel to its original direction, and leaves without energy.

If the actual H.P. is 70.3, what is the efficiency?

Ans. 87.22; .805.

32. A vane moves in the direction ABC with a velocity of 10 ft. per second, and a jet of water impinges upon it at B in the direction BD with a velocity of 20 ft. per second; the angle between BC and BD is 30° . Determine the direction of the receiving-lip of the vane, so that there may be no shock.

Ans. The angle between lip and $BC = 23^\circ 47'$.

33. A jet moves in a direction ABC with a velocity v and impinges upon a vane which it drives in the direction BD with a velocity $\frac{1}{2}v$. The angle ABD is 165° . Determine the direction of the lip of the vane at B , so that there may be no shock at entrance.

Ans. The angle between lip and direction of stream $= 14^\circ 3'$.

34. The lip angle of a given bucket is 30° , the relative velocity (V) is one half the velocity (v) with which the water reaches the lip. If there is to be no "loss in shock," find the speed (u) of the bucket, the direction (γ) of the entering water, and show that if the speed is to be increased 10 per cent, the lip angle must also be increased by 55.6 per cent.

Ans. $\gamma = 15^\circ 31'$.

35. A stream moving with a velocity v impinges without shock upon a curved vane and drives it in a direction inclined at an angle to the direction of the stream. The angle between the lip of the vane and the direction of the stream is x , and V is the relative velocity of the water with respect to the vane. If the speed of the vane is changed by a small amount, say n per cent, show that the corresponding change in the direction of the lip, in order that the water might still strike the vane without shock, is $\frac{n}{100} \frac{v}{V} \sin x$.

36. A jet issues through a thin-lipped orifice 1 sq. in. in sectional area in the vertical side of a vessel under a pressure equivalent to a head of 900 ft. and impinges on a curved vane, driving it in the direction of the axis of the jet. The water enters without shock and turns through an angle of 60° before it leaves the vane. Find (a) the speed of the vane which will give a maximum effect; (b) the pressure on the

vane; (c) the work done; (d) the absolute velocity with which the water leaves the vane; (e) the reaction on the vessel, disregarding contraction.

Ans. (a) 80 ft. per sec.; (b) 320.9 lbs.; (c) 46.68 H.P.; (d) 184 ft. per sec.; (e) 781.25 lbs.

37. A stream of thickness t and moving with the velocity v impinges without shock upon the concave surface of a cylindrical vane of a length subtending an angle 2α at the centre. Determine the total pressure upon the vane (a) if it is fixed; (b) if it is moving in the same direction as the stream with the velocity u . In case (b) also find (c) the work done on the vane.

Ans. (a) $2\frac{w}{g}btv^2 \sin \alpha$; (b) $2\frac{w}{g}bt(v-u)^2 \sin \alpha$; (c) $2\frac{w}{g}btu(v-u)^2 \sin^2 \alpha$.

38. A stream of water, 2 sq. ins. in sectional area, delivers 1 cu. ft. per second against the concave side of a hemispherical cup, which moves with a velocity of 20 ft. per second in the direction of the jet.

Find the impulse, the work done, and the efficiency.

39. A curved vane subtends an angle of 90° at the point of intersection of the normals at the two edges, and receives without shock a stream of water 2 ft. wide and $\frac{1}{2}$ in. thick, moving with a velocity of 20 ft. per second and driving the vane in the same direction. The actual direction of the water is turned through an angle of 45° . Find (a) the speed of the vane; (b) the velocity with which the water leaves the vane; (c) the total pressure on the vane; (d) the efficiency.

Ans. (a) 10 ft. per sec.; (b) 14.14 ft. per sec.; (c) 23,017 lbs.; (d) .25.

40. A vane is in the form of the segment AB of a circle subtending an angle of 120° at the centre O . A stream of water, moving with a velocity v_1 , strikes the vane tangentially at A and drives it in the same direction with a velocity u . Find the velocity (v_2) with which the water leaves the vane, and show that it leaves in the direction OB if $v_1 = 1\frac{1}{2}u$, and that the direction has turned through 90° if $v_1 = 3u$. Find the efficiency in the two cases, and show that $v_1 = 3u$ corresponds to maximum efficiency.

Ans. $v_2^2 = v_1^2 - 3v_1u + u^2$; $\frac{2}{3}$; $\frac{4}{9}$. If $v_1 = 2u$, $v_2 = u$, the direction turns through 60° and $\eta = \frac{2}{3}$.

41. A stream of water of 36 sq. ins. section moves in a direction ABC and delivers 4 cu. ft. of water per second upon a vane moving in a direction BD with a velocity of 8 ft. per second, the angle between BC and BD being 30° . Find (a) the best form to give to the vane; (b) the velocity of the water as it leaves the vane; (c) the mechanical effect of the impinging jet; and (d) the efficiency, the angle turned through by the jet being 90° .

Ans. (a) The angle between lip and $BC = 23^\circ 48'$; (b) 3.088 ft. per second; (c) 962.8 ft. per second; (d) .963.

42. In an I. F. tangential turbine find (a) the loss due to hydraulic resistances, (b) the useful effect, (c) the efficiency, (d) the lip angles from

the following data: $Q = 1$ cu. ft. per second; $h = 100$ ft.; $f_2 = f_4 = \frac{1}{8}$; $\gamma = 15^\circ$; $5r_1 = 6r_2$; $d_1 = d_2$; and $u_2 = V_2$.

Ans. (a) 736.72 ft.-lbs.; (b) 4866.9 ft.-lbs.; (c) .7787; $\alpha = 150^\circ 33'$, $\beta = 47^\circ 16'$.

43. In an O. F. turbine of the tangential type find the lip angles, the losses of head due to the velocity (v_2) of the effluent water, and to hydraulic resistances, and also find the efficiency from the following data: $\gamma = 30^\circ$; $2r_1 = r_2$; $H = 30$ ft.; $d_1 = d_2$; $u_2 = V_2$; $f_2 = 2f_4 = .125$.

Ans. $\alpha = 123^\circ 27'$; $\beta = 13^\circ 30'$; 1.688 ft.; 5.243 ft.; .769.

44. An I. F. tangential turbine, with parallel faces ($d_1 = d_2$) and an inlet surface of 6 ft. diameter, delivers 10.76 H.P. under a head of 150 ft. The direction of the inflowing stream, which is 5 ins. wide, makes an angle (γ) of 10° with the turbine's periphery, and the diameters of the inlet and outlet surfaces are in the ratio of 4 to 3. If $f_2 = f_4 = .1$, and if also it is assumed that $u_2 = V_2$, find (a) the inlet and outlet lip angles; (b) the loss of H.P. due to hydraulic resistances; (c) the loss of H.P. corresponding to the velocity with which the water leaves the turbine; (d) the efficiency; (e) the quantity of water passing through the turbine; (f) the thickness of the inflowing stream; (g) the speed in revolutions per minute.

Ans. (a) $161^\circ 13'$, $40^\circ 36'$; (b) 1.309; (c) .708; (d) .842; (e) $\frac{3}{4}$ cu. ft. per second; (f) .231 in.; (g) 141.

45. In the preceding example if, instead of making $u_2 = V_2$, it is assumed that the flow at outlet is radial, find the inlet and outlet lip angles so that the efficiency may remain the same. Also find the losses in H.P. due to hydraulic resistance and to the energy carried away by the effluent water, and determine the speed in revolutions per minute. ($c_v^2 = \frac{1}{11}$.)

Ans. $161^\circ 21'$, $33^\circ 17'$; 1.369; .623; 139.8.

46. A stream 4 ins. wide and supplying $\frac{3}{4}$ cu. ft. of water per second drives an O. F. turbine of the tangential type, in which the diameters of the inlet and outlet surfaces are in the ratio of 3 to 4. The turbine faces are parallel, and the inflowing stream makes an angle of 20° with the periphery. The head is 100 ft. *First* assuming that $u_2 = V_2$, and *second* that the outlet flow is radial, the efficiency being the same, determine (a) the inlet and outlet lip angles; (b) the useful work; (c) the efficiency; (d) the thickness by the stream; (e) the speed in revolutions per minute, the outer diameter being 5 ft. (Disregard hydraulic resistances.)

Ans. First. (a) 140° , $21^\circ 12'$; (b) 4368.16 ft.-lbs.; (c) .932; (d) .3375 in.; (e) 216.7. *Second.* (a) $142^\circ 19'$, $21^\circ 10'$; (b) 4394.7 ft.-lbs.; (c) .937; (d) .3375 in.; (e) 202.45.

47. Solve the preceding example when hydraulic resistances are taken into account, assuming $f_2 = f_4 = .1$ and $c_v^2 = .9$.

Ans. First. (a) 140° , $21^\circ 12'$; (b) 3766 ft.-lbs.; (c) .8034;

(*d*) .356 in.; (*e*) 154.2. *Second.* (*a*) $141^{\circ} 29', 20^{\circ} 40'$; (*b*) 3766 ft.-lbs.; (*c*) .8034; (*d*) .356 in.; (*e*) 147.79.

48. A jet turbine, of $5\frac{1}{11}$ ft. exterior diameter and with equal inlet and outlet depths, passes 1890 cu. ft. of water per hour under a head of 100 ft.; the diameter of the outlet surface is twice that of the inlet, and the velocity of the outlet periphery (u_2) is equal to that of the inflowing stream (v_1), which is radial in direction. Find (*a*) the useful effect in horse-power; (*b*) the efficiency; (*c*) the speed in revolutions per minute, *first* disregarding hydraulic resistances and *second* taking these resistances into account. ($f_4 = 2f_2 = .2$ and $c_v^2 = \frac{10}{11}$.)

Ans. First. (*a*) 3.85 H.P.; (*b*) .645; (*c*) 300. *Second.* (*a*) 2.063 H.P.; (*b*) .3458; (*c*) 286.04.

49. In the preceding example how much water must the turbine pass, when hydraulic resistances are taken into account, to give the delivery of 4 H.P.? *Ans.* 1.017 cu. ft. per second.

50. A centrifugal outward-flow turbine with equal inlet and outlet depths and working under the head of 200 ft. over the inlet passes 1 cu. ft. of water per second. The angle $\gamma = 15^{\circ}$; $5r_1 = 4r_2$; and the velocity at outlet is radial, i.e., $\delta = 90^{\circ}$. Find (*a*) the peripheral speed; (*b*) the lip angle at inlet; (*c*) the lip angle at outlet; (*d*) the areas at inlet and outlet; (*e*) the efficiency, taking $f_2 = f_4 = .125$.

Ans. (*a*) 55.215 ft. per second; (*b*) $157^{\circ} 18'$; (*c*) $18^{\circ} 40'$; (*d*) .3623 sq. ft., .0428 sq. ft.; (*e*) .778.

51. A centrifugal inward-flow turbine with an efficiency of 80 per cent and working under the head of 200 ft. passes 1 cu. ft. of water per second. The angle $\gamma = 15^{\circ}$; $4r_1 = 5r_2$; and $u_2 = V_2$. Find (*a*) the peripheral speed; (*b*) the lip angles at inlet and outlet; (*c*) the inlet and outlet areas; (*d*) the useful work, taking $f_2 = f_4 = .125$.

Ans. (*a*) 55.215 ft. per second; (*b*) $157^{\circ} 18', 32^{\circ} 28'$; (*c*) .03623 sq. ft., .0369 sq. ft.; (*d*) $8888\frac{8}{9}$ ft.-lbs.

CHAPTER VI.

VERTICAL WATER-WHEELS.

1. Classification of Water-wheels.—Water-wheels are large vertical wheels which are made to turn on a horizontal axis by water falling from a higher to a lower level. These wheels may be divided into three classes:

(a) *Undershot Wheels*, in which the water is received near the bottom and acts *by impulse*.

(b) *Breast Wheels*, in which the water is received a little below the axis of rotation and acts *partly by impulse and partly by its weight*.

(c) *Overshot Wheels*, in which the water is delivered nearly at the top and acts *chiefly by its weight*.

2. Undershot Wheels.—Wheels of this class, with plane floats or buckets, are simple in construction, are easily kept in repair, and were in much greater use formerly than they are now. They are still found in remote districts where there is an abundance of water-power, and are also employed to work floating mills, for which purpose they are suspended in an open current by means of piles or suitably moored barges. They are made from 10 to 25 ft. in diameter, and the floats, which are from 24 to 28 ins. deep, are fixed either normally to the periphery of the wheel, or with a slight slope towards the supply-sluice, the angle between the float and radius being from 15° to 30° . The depth of a float is from one fourth to one fifth of the radius and should not be less than from 12 to 14 ins. They are from 14 to 16 ins. apart, and generally from

one half to one third of the total depth of float is acted upon by the water.

Let Fig. 236 represent a wheel with plane floats working in an open current.

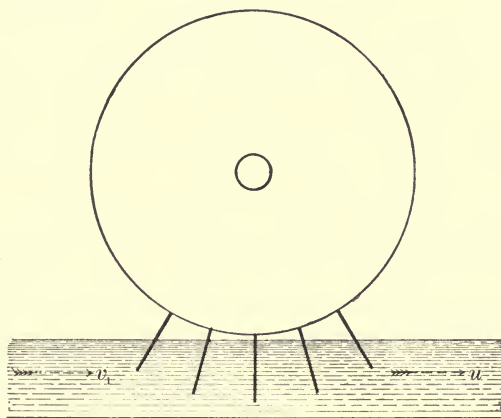


FIG. 236.

Let v_1 be the velocity of the current.

Let u be the velocity of the wheel's periphery.

Let Q be the delivery of water in cubic feet per second.

The water impinging upon a float, is reduced to relative rest, and is carried along with the velocity u . Thus

$$\text{the impulse} = \frac{wQ}{g}(v_1 - u),$$

and

$$\text{the useful work per second} = \frac{wQ}{g}u(v_1 - u).$$

Hence

$$\text{the efficiency} = \frac{\frac{wQ}{g}u(v_1 - u)}{\frac{wQ}{g} \frac{v_1^2}{2}} = \frac{2u(v_1 - u)}{v_1^2},$$

which is a maximum and equal to $\frac{1}{2}$ when $u = \frac{1}{2}v_1$.

Theoretically, therefore, the wheel works to the best advantage when the velocity of its periphery is one half of the current velocity. Even then its maximum theoretic effect is only 50 per cent, and in practice this is greatly reduced by frictional and other losses, so that the useful effect rarely exceeds 30 per cent. Undershot wheels with plane floats are cumbrous, have little efficiency, and should not be used for falls of more than 5 ft.

Again, let A be the water-area of a float, and w be the specific weight of the water.

wQ is somewhat less than wAv_1 , as there will be an escape of water on both sides of the float.

Let $wQ = kwAv_1$, k being some coefficient (< 1) to be determined by experiment. Then

$$\text{the useful work per second} = kAw\frac{v_1^2 u}{g}(v_1 - u),$$

$$\text{and its maximum value} = \frac{kA}{4g}v_1^3 w.$$

According to Bossut's and Poncelet's experiments a mean value of k is $\frac{4}{5}$, and the best effect is obtained when $u = \frac{2}{5}v_1$,

the corresponding useful work being $\frac{24}{125} \frac{wAv_1^3}{g}$ and the efficiency $\frac{48}{125}$.

3. Wheels in Straight Race.—Generally the water is let on to the wheel through a channel made for the purpose, and closely fitting the wheel, so as to prevent the water escaping without doing work. For this reason, also, the space between the ends of the floats in their lowest positions and the channel is made as small as is practicable and should not exceed 2 ins. Hence k , and therefore also the efficiency, will be increased.

Assume the channel to be of a uniform rectangular section and to have a bed of so slight a slope that it may be regarded as horizontal without sensible error.

The wheel is usually from 24 to 48 ft. in diameter, with 24 to 48 floats, either radial or inclined. The floats are 12 to 20 ins. deep, or about $2\frac{1}{2}$ to 3 times the depth of the approaching stream. The fall should not exceed 4 ft. Let the floats be radial, Fig. 237.

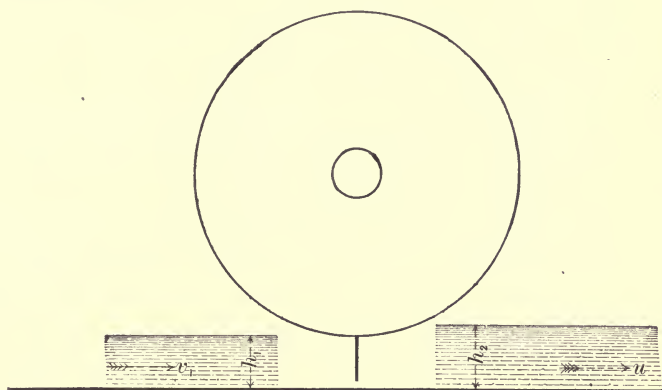


FIG. 237.

Let h_1 be the depth of the water on the *up-stream* side of the wheel.

Let h_2 be the depth of the water on the *down-stream* side of the wheel.

Let b be the width of the race. Then

$$bh_1v_1 = Q = bh_2u.$$

The *impulse* = impulse due to change of velocity
+ impulse due to change of pressure

$$= \frac{wQ}{g}(v_1 - u) + \frac{wb}{2}(h_1^2 - h_2^2)$$

$$= \frac{wQ}{g}(v_1 - u) + \frac{wQ}{2}\left(\frac{h_1}{v_1} - \frac{h_2}{u}\right),$$

and the *useful work per second*

$$= \text{impulse} \times u = \frac{wQ}{g}u(v_1 - u) + \frac{wQ}{2}\left(\frac{h_1}{v_1} - \frac{h_2}{u}\right)u.$$

$$\text{The total available work} = \frac{wQ}{g} \frac{v_1^2}{2}.$$

$$\text{Hence the efficiency} = \frac{2u}{v_1^2}(v_1 - u) + \frac{gu}{v_1^2}\left(\frac{h_1}{v_1} - \frac{h_2}{u}\right).$$

The second term is negative, since $h_2 > h_1$, and the maximum theoretic efficiency may be easily shown to be $< .5$.

Ex. An undershot wheel with straight floats and weighing 15,000 lbs. works in a rectangular channel with horizontal bed and of the same width as the wheel, viz., 4 ft.; the stream delivers 28 cu. ft. of water per second, and the efficiency of the wheel is $\frac{1}{3}$. Find the relation between the up-stream (v_1) and down-stream (u) velocities.

If the up-stream velocity is 20 ft. per second, find the down-stream velocity. If the diameters of the wheel and bearings are 20 ft. and 4 ins., respectively, and if the coefficient of friction is .008, determine the mechanical effect.

$$28 = 4h_1v_1 = 4h_2u,$$

$$\text{or} \quad h_1 = \frac{7}{v_1} \quad \text{and} \quad h_2 = \frac{7}{u}.$$

$$\begin{aligned} \text{Therefore the efficiency} &= \frac{2u}{v_1^2}(v_1 - u) + \frac{32u}{v_1^2}\left(\frac{7}{v_1^2} - \frac{7}{u^2}\right) = \frac{1}{3}, \\ &\frac{2u}{v_1^2}(v_1 - u)\left(1 - 112\frac{v_1 + u}{v_1^2u^2}\right) = \frac{1}{3}. \end{aligned}$$

If $v_1 = 20$ ft. per second, then

$$\frac{u}{200}(20 - u)\left(1 - \frac{7}{25} \frac{20 + u}{u^2}\right) = \frac{1}{3}.$$

It is found by trial that u lies between 5.9 and 6 and is very approximately 5.97 ft. per second.

$$\text{The total available power} = \frac{62\frac{1}{2}}{32} \cdot 28 \cdot \frac{20^2}{2} = 10937.5 \text{ ft.-lbs. per. sec.}$$

$$\begin{aligned} \text{Therefore the actual mechanical effect} &= \frac{1}{3}(10937.5) \\ &= 3645.83 \text{ ft.-lbs. per sec.} \end{aligned}$$

$$\begin{aligned} \text{The work absorbed by bearing friction} &= .008 \times 15000 \times 5.97 \times \frac{1}{10} \\ &= 11.94 \text{ ft.-lbs. per sec.} \end{aligned}$$

$$\text{The net delivery in ft.-lbs.} = 3645.83 - 11.94 = 3633.89.$$

Losses.—Four principal losses may be considered, viz.:

(1) The loss of Q_1 cubic feet of the deeper fluid elements which do not impinge upon some of the foremost floats.

According to Gerstner,

$$Q_1 = \frac{cQ}{n_1^2} \left(\frac{r_1}{r_1 - u} \right)^2,$$

n_1 being the number of the floats immersed, and c being $\frac{1}{8}$ or $\frac{2}{8}$ according as the bottom of the race is straight or falls abruptly at the lowest point of the wheel.

(2) The loss of Q_2 cubic feet of water which escapes between the wheel and the race-bottom.

Approximately, the play at the bottom may be said to vary from a minimum, $s_1 = BC$, when a float AB is in its lowest position, Fig. 238, to a maximum, $B_1C_1 = CD = B_2C_2$, when

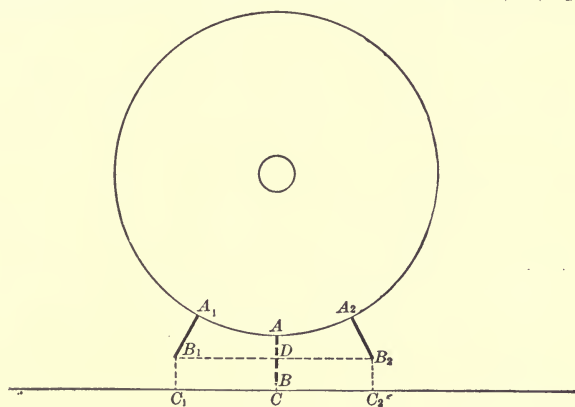


FIG. 238.

two floats A_1B_1 , A_2B_2 are equidistant from the lowest position, Fig. 238. Thus the mean clearance

$$\begin{aligned} &= \frac{1}{2}(BC + B_1C_1) = \frac{1}{2}(s_1 + CD) \\ &= \frac{1}{2}(2s_1 + BD) = s_1 + \frac{1}{4} \frac{(B_1D)^2}{r_1}, \text{ nearly,} \end{aligned}$$

r_1 being the wheel's radius.

But $\frac{2\pi r_1}{n}$ = distance between two consecutive floats

$$= 2 \cdot B_1 D, \text{ very nearly,}$$

n being the total number of floats. Hence

$$B_1 D = \frac{\pi r_1}{n},$$

and therefore the mean clearance $= s_1 + \frac{1}{4} \frac{\pi^2 r_1}{n^2}$.

Again, the difference of head on the up-stream and down-stream sides

$$= h_2 - h_1 = h_1 \left(\frac{v_1}{u} - 1 \right),$$

and the velocity of discharge, v_d , through the clearance is given by the equation

$$v_d^2 = v_1^2 - 2g(h_2 - h_1) = v_1^2 - 2gh_1 \left(\frac{v_1}{u} - 1 \right).$$

Hence

$$Q_2 = b_1 \left(s_1 + \frac{1}{4} \frac{\pi^2 r_1}{n^2} \right) v_d.$$

Introducing .7 as a coefficient of hydraulic resistance,

$$Q_2 = .7 b_1 \left(s_1 + \frac{1}{4} \frac{\pi^2 r_1}{n^2} \right) v_d.$$

If the depth of the stream is the same on both sides of the wheel, i.e., if $h_1 = h_2$, then

$$v_d = v_1.$$

(3) The loss of Q_3 cubic feet of water which escapes between the wheel and the race-sides.

Let s_2 be the clearance on each side. Then

$$Q_3 = .7 \times 2h_1 s_2 v_d = 1.4 h_1 s_2 v_d,$$

.7 being a coefficient of hydraulic resistance.

(4) Finally, if W lbs. is the weight on the wheel-journals, the loss due to journal friction

$$= \mu W \frac{\rho}{r_1} u,$$

μ being the journal coefficient of friction, and ρ the journal radius.

Actual Delivery.—Thus the actual delivery of the wheel in foot-pounds

$$= \left\{ \frac{u(v_1 - u)}{g} + \frac{u}{2} \left(\frac{h_1}{v} - \frac{h_2}{u} \right) \right\} \left\{ w(Q - Q_1 - Q_2 - Q_3) \right\} - \mu W \frac{\rho}{r_1} u.$$

Remarks.—These wheels are most defective in principle, as they utilize only about one third of the total available energy. They may be made to work to somewhat better advantage by introducing the following modifications:

(a) The supply may be so regulated by means of a sluice-board that the mean thickness of the impinging stream is about 6 or 8 ins. If the thickness is too small, the relative loss of water along the channel will be very great. If the thickness is too great, the floats, as they emerge, will have to raise a heavy weight of water. The sluice-board is inclined at an angle of 30° to 40° to the vertical, so that the sluice-opening may be as near the wheel as possible, thus diminishing the loss of head due to channel friction, and is rounded at the bottom to prevent a contraction of the issuing fluid. Neglecting frictional losses, etc.,

$$\begin{aligned} \text{the useful effect} &= wQ \left(H + \frac{v_1^2}{2g} - \frac{u^2}{2g} \right) - \left\{ \begin{array}{l} \text{loss of energy} \\ \text{due to shock} \end{array} \right\} \\ &= wQ \left(H + \frac{v_1^2}{2g} - \frac{u^2}{2g} \right) - \frac{wQ}{g} \frac{(v_1 - u)^2}{2} \\ &= wQ \left\{ H + \frac{u}{g} (v_1 - u) \right\}, \end{aligned}$$

H being the difference of level between the point at which the water enters the wheel and the surface of the water in the tail-

race, i.e., the fall. H is usually very small and may be negative.

If the vanes are inclined, the resistance to emergence is not so great, and the frictional bed resistance between the sluice and float is practically reduced to *nil*. With a straight bed and small slope (1 in 10) the minimum convenient diameter of wheel is about 14 ft.

(b) The bed of the channel for a distance at least equal to the interval between two consecutive vanes may be curved to the form of a circular arc concentric with the wheel, with the view of preventing the escape of the water until it has exerted its full effect upon the wheel. When the bed is curved, the minimum convenient diameter of wheel is about 10 ft.

An undershot wheel with a curb is in reality a low breast wheel, and its theory is the same.

(c) The down-stream channel may be deepened so that the velocity of the water as it flows away becomes $> v_1$. The *impulse due to pressure* is then positive, which increases the useful work and therefore also the efficiency.

(d) The down-stream channel may be widened and a slight counter-inclination given to the bed. What is known as a *standing-wave* is then produced, in virtue of which there is a sudden rise of surface-level on the down-stream side above that on the up-stream side. This allows of the wheel being lowered by an amount equal to the difference of level between the surfaces of the standing-wave and of the water-layer as it leaves the wheel, thus giving a corresponding gain of head.

(e) The introduction of a sudden fall has been advocated in order to free the wheel from back-water, but it must be borne in mind that all such falls diminish the available head.

4. Poncelet Wheel (Figs. 239, 240).—Thus undershot wheels with flat buckets have a small efficiency because of the loss of energy in shock at entrance and because of the loss of energy carried away by the water on leaving the wheel. These losses have been considerably modified in

Poncelet's wheel, which is often the best motor to adopt when the fall does not exceed $6\frac{1}{2}$ ft., and which, in its design, is

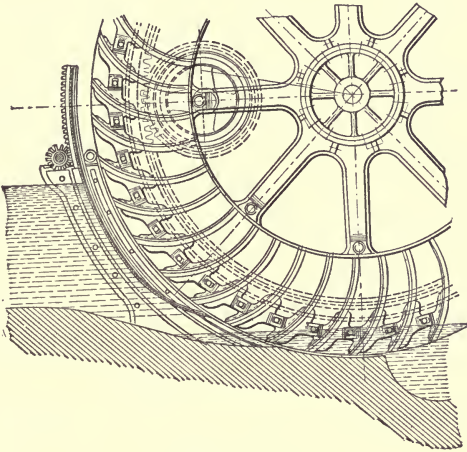


FIG. 239.

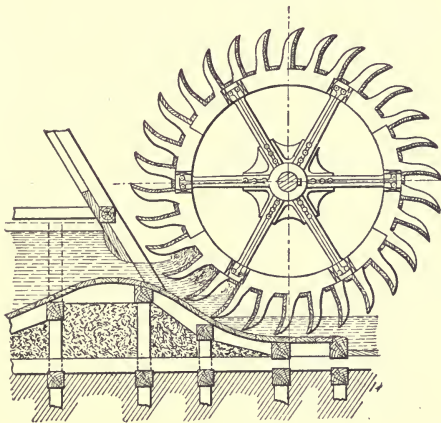


FIG. 240.

governed by two principles that should govern every perfect water-motor, viz. :

(1) *That the loss of energy in shock at entrance should be a minimum.*

(2) That the velocity of the water as it leaves the wheel should be a minimum.

The vanes are curved and are comprised between two crowns, at a slightly greater distance apart than the vane-width; the inner ends of the vanes are radial, and the water acts in nearly the same manner as in an impulse turbine.

A Poncelet wheel of from 10 to 13 ft. in diameter has 36 floats, while for wheels of from 20 to 23 ft. in diameter the number of floats is about 48. The wheels are usually from 10 to 20 ft. in diameter and have from 32 to 48 floats which may be of plate-iron or wood.

First. Assume that the outer end of a vane is tangential to the wheel's periphery, that the impinging layer is infinitely thin, and that it strikes a float tangentially.

Let af (Fig. 24I) be a float, and aq the tangent at a .

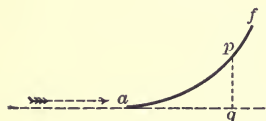


FIG. 24I.

The velocity of the water relatively to the float $= v_1 - u$.

The water, in virtue of this velocity, ascends on the bucket to a height

$$pq = \frac{(v_1 - u)^2}{2g}, \text{ then falls back and leaves}$$

the float with the *relative* velocity $v_1 - u$ and with an *absolute* velocity $v_1 - 2u$. This absolute velocity is nil when the speed of the wheel is such that $u = \frac{1}{2}v_1$, and the theoretical height of a float is $pq = \frac{1}{4} \frac{v_1^2}{2g}$. The total available head is thus

changed into useful work, and the efficiency is *unity*, or perfect.

Taking R as the mean radius of the crown and u_m as the corresponding linear velocity, the mean centrifugal force on each unit of fluid mass is $\frac{u_m^2}{R}$ and acts very nearly in the direction of gravity, so that the height pq of a float may be approximately expressed in the form

$$pq = \frac{V^2}{2\left(g + \frac{u_m^2}{R}\right)},$$

V being the velocity with which the water commences to rise on the float.

Practically, however, the float is *not* tangential to the periphery at a , as the water could not then enter the wheel. Also, the impinging water is of sensible thickness, strikes the periphery at some appreciable angle, and in rising and falling on the floats loses energy in shocks, eddies, etc.

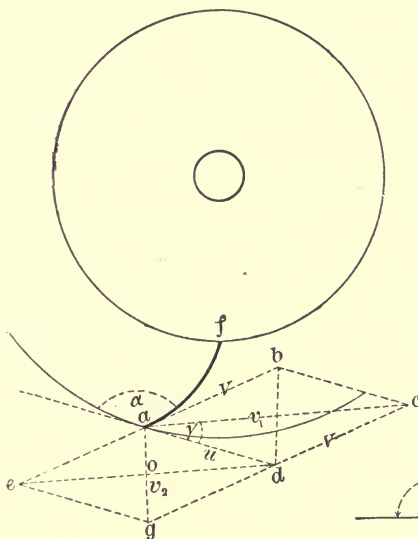


FIG. 242.

Let the water impinge in the direction ac , Fig. 242, and take $ac = v_1$.

Take ad in the direction of and equal to u , the velocity of the wheel's periphery.

Complete the parallelogram bd .

Then $cd = ab = V$ is the velocity of the water relatively to the float.

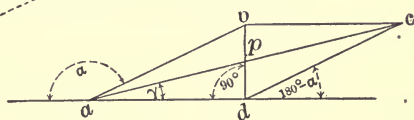


FIG. 243.

That there may be no shock at entrance, ab must be a tangent to the vane at a .

Again, the water leaves the vane in the direction of ba produced, and with a relative velocity $ac = ab = V$.

Complete the parallelogram de . Then $ag (= v_2)$ is the *absolute* velocity of the water leaving the wheel.

Evidently cdg is a straight line.

Let the angle $cad = \gamma$, and the angle $bad = \pi - \alpha$.

From the triangle adc ,

$$V^2 = v_1^2 + u^2 - 2v_1u \cos \gamma; \quad . \quad . \quad . \quad (1)$$



$$v_1^2 = V^2 + u^2 - 2Vu \cos \alpha; \quad . \quad . \quad . \quad (2)$$

$$\frac{V}{v_1} = \frac{\sin \gamma}{\sin \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From the triangle adg ,

$$v_2^2 = V^2 + u^2 + 2Vu \cos \alpha. \quad . \quad . \quad . \quad (4)$$

By equations (1), (2), and (4),

$$\frac{v_1^2}{2} - \frac{v_2^2}{2} = -2Vu \cos \alpha = v_1^2 - V^2 - u^2 = 2u(v_1 \cos \gamma - u).$$

Therefore the useful work per second

$$= \frac{wQ}{g} 2u(v_1 \cos \gamma - u). \quad . \quad . \quad . \quad (5)$$

This is a maximum and equal to $\frac{wQ}{g} \frac{v_1^2 \cos^2 \gamma}{2}$ when $u = \frac{v_1 \cos \gamma}{2}$, and the maximum efficiency is $\cos^2 \gamma$. Hence, too, the angle $adb = 90^\circ$, and, by Fig. 243,

$$\tan (\pi - \alpha) = \frac{bd}{ad} = \frac{2pd}{ad} = 2 \tan \gamma. \quad . \quad . \quad (6)$$

Also,

$$\frac{V}{u} = \frac{ab}{ad} = \sec (\pi - \alpha). \quad . \quad . \quad . \quad (7)$$

The efficiency is perfect if γ is nil, and therefore $\alpha = 180^\circ$. Practically this is an impossible value, but the preceding calculations indicate that γ should not be too large (usually $< 30^\circ$), and that the speed of the wheel should be a little less than one half of the velocity of the inflowing stream.

Take $\gamma = 15^\circ$ as a mean value. Then

$$u = v_1 \times .484, \text{ and the efficiency} = .993.$$

The best practice indicates the relation $11v_1 = 20u$. It must be borne in mind that the theory applies to one elementary layer only, say the mean layer, and that all the other layers enter the wheel at angles differing from 15° , thus giving rise to "losses of energy in shock." The losses of energy in frictional resistance, eddy motion, etc., in the vane-passages have also been disregarded. Tangential entrance is not possible in practice and the efficiency does not exceed .65 for falls up to 4 ft., is .60 for falls of from 4 to 5.5 ft., and is from .55 to .50 for falls of from 5.5 to 6.5 ft. The greater efficiency of the Poncelet wheel, as compared with wheels having flat buckets, very clearly shows the importance of bringing the water on to the wheel in such a manner as to avoid loss of energy in shock and in the production of eddies. The layers of water, flowing to the wheel under an adjustable sluice and with a velocity very nearly equal to that due to the total head, may be all made to enter at angles approximately equal to 15° , and the corresponding losses in shock reduced to a minimum by forming the *course* as follows:

The first part of the course FG , Fig. 244, is curved in such

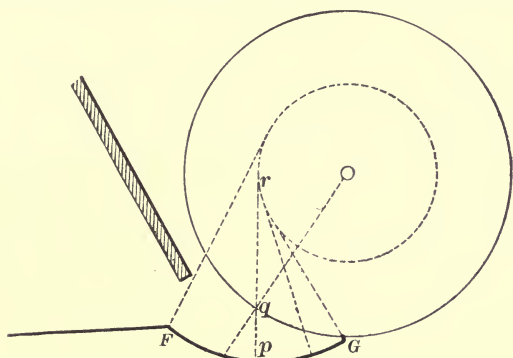


FIG. 244.

a manner that the normal pqr at any point p makes an angle of 15° with the radius oq . The water moves sensibly parallel to the bottom FG , and therefore in a direction at right angles

to pr . Hence at q the direction of motion makes an angle of 15° with the tangent to the wheel's periphery. If or is drawn perpendicular to pr , then $or = oq \sin 15^\circ = \text{a constant}$.

Thus the normal pqr touches at r a circle concentric with the wheel and of a certain constant diameter.

The initial point F of the profile FG is the point in which the tangent to this circle, passing through the upper edge of the sluice-opening, cuts the bed of the supply-channel.

Let d be the depth of the *crown* or *shrouding*, i.e., the normal distance between the outer and inner peripheries of the wheel.

Let b be the width and t the thickness of the sheet of water entering the wheel.

Then, disregarding the thickness of the floats, the capacity of the portion of the wheel passing in front of the entering stream per second is approximately bdu_m . Practically, the whole of this space cannot be occupied by the water and

$$mbdu_m = Q = btv_1,$$

m being a coefficient varying from $\frac{1}{2}$ to $\frac{2}{5}$.

$$\begin{aligned} \text{Thus } t, \text{ the thickness of the stream,} &= md \frac{u_m}{v_1} \\ &= md \frac{R}{r_1} \frac{u}{v_1}. \end{aligned}$$

If the efficiency is a maximum, $v_1 \cos \gamma = 2u$, and then

$$t = \frac{m}{2} d \frac{R}{r_1} \cos \gamma.$$

The head over the mean water layer at the point of entrance

$$= H - \frac{t}{2},$$

H being the available fall. Hence

$$v_1 = c_v \sqrt{2g \left(H - \frac{t}{2} \right)},$$

an average value of c_v being .9, and if, as according to Grashof, $H = 16t$,

$$v_1 = c_v \sqrt{2gH \cdot \frac{31}{32}}.$$

Morin makes the radius (r_1) of the wheel from *two* to *three* times the depth (d) of the crown, and Poncelet considers that this depth should be about $\frac{H}{3}$ and not less than $\frac{H}{4}$. In order, indeed, to prevent the water from rising over the top of the floats, d should be from $\frac{H}{2}$ to $\frac{2}{3}H$, and therefore r_1 from H to $2H$, the latter being often adopted in practice.

The area of the sluice-opening usually varies from $1.25bt$ to $1.3bt$.

The inside width of the wheel is about $(b + \frac{1}{3})$ ft.

If λ is the angle subtended at the centre O of the wheel by the water-arc between the point of entrance a and the lowest point C , Fig. 245, of the wheel, and if Aq' is drawn horizontally, then Aq' is approximately the height of the float, and the theoretic depth d of the crown is given by

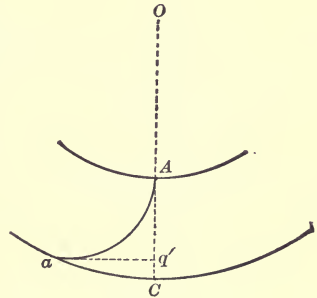


FIG. 245.

$$\begin{aligned} d &= AC = Aq' + Cq' \\ &= Aq' + OC - Oq' \\ &= \frac{V^2}{2\left(g + \frac{u_m^2}{R}\right)} + r_1(1 - \cos \lambda). \end{aligned}$$

In practice it is usual to increase this depth by t , the thickness of the impinging water-layer, and therefore

$$d = \frac{2}{3} \frac{V^2}{g + \frac{u_m^2}{R}} + r_1(1 - \cos \lambda) + t.$$

The buckets are usually placed about 1 ft. apart, measured along the circumference, but the number of the buckets is not a matter of great importance. There are generally 36 buckets in wheels of 10 to 14 ft. diameter, and 48 buckets in wheels of 20 to 23 ft. diameter.

It may be assumed that the water-arc is equally divided by the lowest point C of the wheel, so that

$$\text{the length of the water-arc} = 2\lambda r_1 = 2uT,$$

T being the time of the ascent or descent of the water in the bucket.

In the middle position, the upper end of the bucket should be vertical, and if the float is in the form of a circular arc, its radius $r' = d \sec(\pi - \alpha)$, α being the angle between the bucket's lip and the wheel's periphery.

The time of ascent or descent is also given by

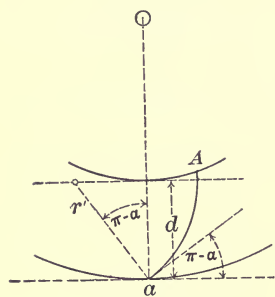


FIG. 246.

$$T = \frac{9\psi + \sin \psi}{16} \sqrt{\frac{r'}{g + \frac{u_1^2}{R}}}$$

where $\sin \frac{\psi}{2} = \sqrt{\cos(\pi - \alpha)}$.

5. Efficiency corresponding to a Minimum Velocity of Discharge (v_2).—From Fig. 242,

$$\frac{\sin \gamma}{\sin aod} = \frac{ao (= \frac{1}{2}ag)}{ad} = \frac{\frac{1}{2}(v_2)}{u}.$$

Hence for any given values of u and γ , v_2 is a minimum when $\sin aod$ is greatest, that is, when $aod = 90^\circ$, or ag is at

right angles to de . Then also $ad = ae = ab$, or $u = V$, and ac bisects the angle bad . Thus

$$v_1 = 2u \cos \gamma \quad \text{and} \quad v_2 = 2u \sin \gamma.$$

The useful work

$$= \frac{W}{g} \cdot \frac{v_1^2 - v_2^2}{2} = \frac{W}{g} 2u^2 \cos 2\gamma = \frac{W}{g} \frac{v_1^2}{2} \frac{\cos 2\gamma}{\cos^2 \gamma}.$$

The total available work

$$= \frac{W}{g} \frac{v_1^2}{2}.$$

$$\text{Therefore the efficiency, } \eta, = \frac{\cos 2\gamma}{\cos^2 \gamma},$$

$$\text{and the H.P. of the wheel} = \eta \frac{62\frac{1}{2}QH}{550}.$$

Experience indicates that the most favorable value for u lies between $.5v_1$ and $.6v_1$, and that the average value of the efficiency is about 60 per cent.

Although, under normal conditions of working, the efficiency of a Poncelet wheel is a little less than that of the best turbines, the advantage is with the former when working with a reduced supply.

Ex. To design a Poncelet wheel for a fall of $4\frac{1}{2}$ ft. and a water-supply of 24 cu. ft. per second, taking, as a first approximation, $\gamma^\circ = \lambda^\circ = 20^\circ$.

Mean velocity (v_1) at point of admission:

$$v_1 = .9\sqrt{2g \cdot 4\frac{1}{2} \cdot \frac{31}{32}} = 15.0329 \text{ ft. per sec.}$$

Best speed of periphery:

$$u = \frac{1}{2}v_1 \cos 20^\circ = 7.06318 \text{ ft. per sec.}$$

Lip angle α :

$$\tan (\pi - \alpha) = 2 \tan 20^\circ = .728,$$

$$\text{and} \quad \pi - \alpha = 36^\circ 3', \quad \text{or} \quad \alpha = 143^\circ 57'.$$

Value of ψ :

$$\sin \frac{\psi}{2} = \sqrt{\cos 36^\circ 3'} = .89917,$$

$$\text{and } \psi^\circ = 128^\circ .6' = 128^\circ .1.$$

Relative velocity (V) at admission:

$$V = u \sec 36^\circ 3' = 8.7361 \text{ ft. per sec.}$$

Value of r_1 . Taking, as a first approximation,

$$R = r_1 = 3d, \quad u_1 = u, \quad \text{and } \lambda^\circ = 20^\circ, \quad \text{then}$$

$$r' = d \sec 36^\circ 3' = r_1 \times .4123, \quad \text{and}$$

$$\pi \frac{20}{180} r_1 = 7.06318 \frac{9\pi \frac{128.1}{180} + \sin 128^\circ 6'}{16} \sqrt{\frac{r_1 \times .4123}{32 + \frac{(7.06318)^2}{r_1}}}$$

which gives $r_1 = 7.445 \text{ ft.}$, or, say, $7\frac{1}{2} \text{ ft.}$

Depth (d) of crown. Taking, as an approximation, $u_1 = u$ and $R = r_1$,

$$d = \frac{2}{3} \frac{(8.7361)^2}{32 + \frac{7\frac{1}{2}}{(7.06318)^2}} + 7\frac{1}{2}(1 - \cos 20^\circ) + t$$

$$= 1.7755 + t = 1.8 \text{ ft., suppose.}$$

More correct radius of float:

$$r' = 1.8 \sec 36^\circ 3' = 2.226 \text{ ft.}$$

Values of R and u_1 :

$$R = 7.5 - \frac{1}{2}(1.8) = 6.6 \text{ ft.}$$

$$u_1 = \frac{R}{r_1} u = \frac{6.6}{7.5} 8.73618 = 6.2156 \text{ ft. per sec.}$$

More correct value of λ :

$$\lambda \cdot 7\frac{1}{2} = 7.06318 \frac{9\pi \frac{128.1}{180} + \sin 128^\circ 6'}{16} \sqrt{\frac{2.226}{32 + \frac{(6.2156)^2}{6.6}}}$$

$$\text{or } \lambda = .298479,$$

$$\text{and } \lambda^\circ = 17^\circ.1.$$

Thickness (t) of stream:

$$t = \frac{1}{2} \frac{1.8}{2} \cos 20^\circ \cdot \frac{6.6}{7.5} = .372 \text{ ft.}$$

Width (b) of wheel:

$$b = \frac{24}{.372 \times 15.0329} = 4.29 \text{ ft.}$$

Time (T) of ascent or descent of water on float:

$$T = \lambda \frac{r_1}{u} = .298479 \frac{7\frac{1}{2}}{7.06318} = .317 \text{ secs.}$$

Number of floats (N). If spaced 1 ft. apart,

$$N = 2\pi \cdot 7\frac{1}{2} = 47\frac{1}{2}, \text{ or, say, } 48.$$

Theoretical maximum power of wheel

$$= \frac{62\frac{1}{2}}{32} 24(7.06318)^2 = 4679.6 \text{ ft.-lbs. per sec.}$$

Total available power

$$= 62\frac{1}{2} \cdot 24 \cdot 4\frac{1}{2} = 6750 \text{ ft.-lbs. per sec.}$$

$$\text{Efficiency} = \frac{4679.6}{6750} = .693.$$

6. Form of Bucket.—The form of the bucket is arbitrary, and may be assumed to be a circular arc. In practice there are various methods of tracing its form.

METHOD I (Fig. 247). The tangent am to the bucket at a makes a given angle α with the tangent at a to the wheel's outer periphery. The radius of is also a tangent to the bucket at f . If the angle aof is known, the position of f on the inner periphery is at once fixed, and the form of the bucket can be easily traced.

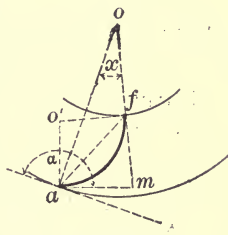


FIG. 247.

Let the angle $aof = x$. Join af and let the tangents to the bucket at a and f meet in m . Then

the angle $oam = \alpha - 90^\circ$,

“ “ $oma = 180^\circ - oam - aom = 270^\circ - \alpha - x$,

“ “ $mfa = \text{the angle } maf = \frac{1}{2}(180^\circ - fma)$

$$= \frac{\alpha + x}{2} - 45^\circ.$$

Let r_1, r_2 be the radii of the outer and inner peripheries of the wheel. Then

$$\frac{r_1}{r_2} = \frac{oa}{of} = \frac{\sin ofa}{\sin oaf} = \frac{\sin mfa}{\sin oaf} = \frac{\sin \left(\frac{\alpha + x}{2} - 45^\circ \right)}{\sin \left(\frac{\alpha - x}{2} - 45^\circ \right)},$$

since the angle $oaf = oam - maf = \frac{\alpha - x}{2} - 45^\circ$.

METHOD III (Fig. 249). Let the bed with a slope of, say, 1 in 10 extend to the point c , and then be made concentric with the wheel for a distance cc subtending an angle of 30° at the centre of the wheel. Let the mean layer, half way between the sloping bed and the surface of the advancing water, strike the outer periphery at the point f . Draw fk making an angle of 23° with of , and take fk equal to *one half* or seven tenths of the available fall. k is the centre of the circular arc required, and kf is its radius.

7. Sluices.—The water is rarely admitted to the wheel without some sluice arrangement, which may take the form of an overfall sluice (Fig. 250), an underflow sluice (Fig. 251), or a bucket or pipe sluice (Fig. 252).

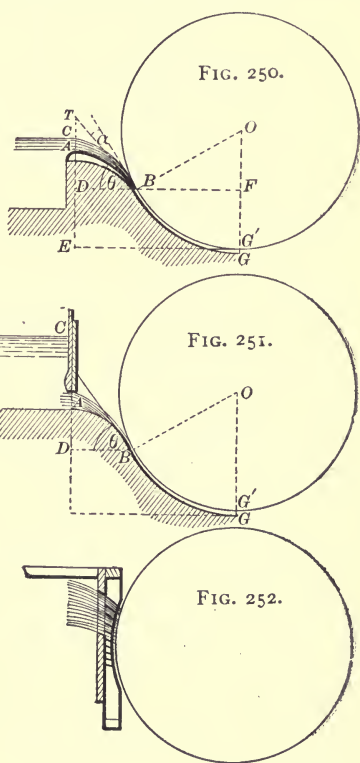
The pipe sluice is especially adapted for a varying supply, being provided, for a certain vertical distance, with a series of short tubes, so inclined as to insure that the water enters the wheel in the right direction. Taking .85 as the mean coefficient of hydraulic resistance for these tubes, the head h_1 required to produce the velocity of entrance v_1 is

$$h_1 = \left(\frac{1}{.85}\right)^2 \frac{v_1^2}{2g};$$

and if H is the total available fall,

$$H - h_1 = H - \frac{1}{(.85)^2} \frac{v_1^2}{2g}$$

= remainder of fall available for pressure-work.



The profile AB in an overfall and an underflow sluice should coincide with the parabolic path of the lowest streamlines of the jet. The crest of the overfall should be properly curved, and the inner edges of the underflow opening should be carefully rounded so as to eliminate losses due to contraction.

The underflow sluice-opening should also be normal to the axis of the jet.

Let h_0 be the head above the crest of an overfall sluice. Then

$$Q = \frac{2}{3}cb_1\sqrt{2gh_0^{\frac{3}{2}}},$$

b_1 being the width of the crest, and c the coefficient of discharge. The width b_1 is usually 3 or 4 ins. less than the width b of the wheel.

From this equation

$$h_0 = \left(\frac{3Q}{2cb_1\sqrt{2g}} \right)^{\frac{2}{3}},$$

and the depth of water over the crest or lip is usually about 9 ins.

Again, the head $h_1 (= CD)$ required to produce the velocity v_1 at the point of entrance B is

$$CD = h_1 = \frac{11}{10} \frac{v_1^2}{2g},$$

10 per cent being allowed for loss due to friction.

Thus the height of the crest A above B , the point of entrance,

$$\begin{aligned} &= AD = CD - CA = h_1 - h_0 \\ &= \frac{11}{10} \frac{v_1^2}{2g} - \left(\frac{3Q}{2cb_1\sqrt{2g}} \right)^{\frac{2}{3}}. \end{aligned}$$

But BA is a parabola with its vertex at A , and therefore, if θ is the angle between the horizontal BD and the tangent BT to the parabola at B ,

$$\frac{v_1^2 \sin^2 \theta}{2g} = AD = \frac{11}{10} \frac{v_1^2}{2g} - \left(\frac{3Q}{2cb_1 \sqrt{2g}} \right)^{\frac{2}{3}}.$$

Also,

$$BD = \frac{v_1 \sin 2\theta}{2g}.$$

The head available for pressure-work

$$= DE = FG = H - h_1.$$

Let α be the angle between BT and the tangent to the wheel's periphery at B . Then

$$\alpha + \theta = \text{the angle } BOF,$$

BO being the radius to the centre of the wheel and OFG' vertical.

If the lowest point G' of the wheel just clears the tail-race, the head available for pressure-work

$$\begin{aligned} &= H - h_1 = FG' = OG' - OF \\ &= r_1(1 - \cos BOF) = 2r_1 \sin^2 \frac{BOF}{2}, \end{aligned}$$

r_1 being the radius to the outer periphery of the wheel.

If, again, the water enters the wheel tangentially,

$$\alpha = 0, \text{ and the angle } BOF = \theta,$$

so that

$$H - h_1 = 2r_1 \sin^2 \frac{\theta}{2}.$$

If the sluice-opening is not at the vertex of the parabola, the axis of the opening should be tangential to the parabola.

8. Breast Wheels.—These wheels are usually adopted for falls of from 5 to 15 ft., and for a delivery of from 5 to 80 cu. ft. per second.

The diameter should be at least 11 ft. 6 ins., and rarely exceeds 24 ft. The velocity (u) of the wheel's periphery is generally from $3\frac{1}{2}$ ft. to 5 ft. per second, the most useful average velocity being about $4\frac{1}{4}$ ft. per second.

The width of the wheel should not exceed from 8 to 10 ft.

It is of great importance to retain the water in the wheel as long as possible, and this is effected either by introducing the water at the inner periphery, Fig. 253, or by surrounding the

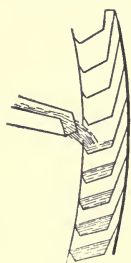


FIG. 253.

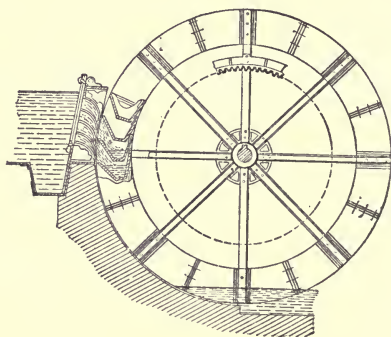


FIG. 254.

water-arc with an apron, or a curb, or a *breast*, Fig. 254, which may be constructed of timber, iron, or stone. In this case the buckets may be plane floats, as the curb retains the water, but they should be set at an angle to the periphery of the wheel, so as to rise out of the water with the least resistance.

Wheels with curbs are designated as *high breast*, *breast*, or *low breast* according as the water reaches the wheel near the summit, middle, or bottom, while if there is no curb they are termed *overshot*, *middle-shot*, and *undershot*, respectively.

The depth of a float should not be less than 2.3 ft., and

the space between two consecutive floats should be filled to at least one half, and even to two thirds, of its capacity. The head (measured from still water) over the sill or lip should be about 9 ins.

The play between the outer edge of the floats and the curb varies from $\frac{1}{2}$ in., in the best constructed wheels, to 2 ins.

The distances between the floats is from $1\frac{1}{3}$ to $1\frac{2}{3}$ times the head over the sill for slow wheels, and a little more for quick wheels.

Breast wheels are among the best of hydraulic motors, having an efficiency which may be as great as 80 per cent. The efficiency is usually about 70 per cent for a fall of about 8 ft., and 50 per cent for a fall of 4 ft.

9. Speed of Wheel.—The water leaves the buckets and flows away in the race with a velocity not sensibly different

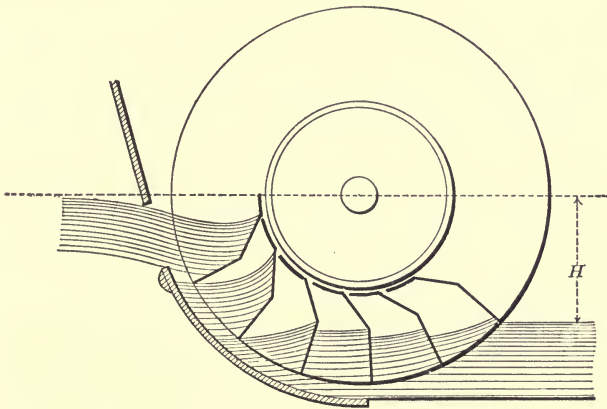


FIG. 255.

from the velocity u of the wheel, which, in practice, is usually about *one half* of the velocity $\left(\frac{v_1}{2}\right)$ with which the water enters the wheel.

Let b be the width of the wheel.

Let x be depth of the water in the lowest bucket.

Allowing for the thickness of the buckets, the play between the wheel and curb, etc.,

$$Q = cbxu,$$

c being an empirical coefficient whose average value is about .9. Hence

$$u = \frac{10}{9} \frac{Q}{bx}.$$

In practice b is often taken to be $\frac{Q}{15}$ to $\frac{Q}{25}$. It is important that b should be as small as possible and hence x should be as large as possible, its value being usually $1\frac{1}{2}$ ft. to 2 ft.

It must be borne in mind, however, that any increase in the value of x will cause an increase in the weight of water lifted by the buckets as they emerge from the race, and will therefore tend to diminish the efficiency.

10. Mechanical Effect.—Theoretically, the total mechanical effect

$$= wQ\left(H - \frac{v_2^2}{2g}\right) = wQ\left(H - \frac{u^2}{2g}\right),$$

H being the fall from the surface of still water in the supply-channel to the surface of the water in the tail-race.

This, however, is reduced by the following losses:

(a) Owing to frictional resistance, etc., there is a loss of head in the supply-channel which may be measured by $\nu \frac{v_1^2}{2g}$, ν being approximately $\frac{1}{20}$ to $\frac{1}{10}$.

The head required to produce the velocity v_1 at entrance

$$= (1 + \nu) \frac{v_1^2}{2g}.$$

(b) Let af , Fig. 256, represent in direction and magnitude v_1 the velocity of the water entering the bucket.

Let ad , in the direction of the tangent to the wheel's periphery, represent the velocity u of the periphery in direction and magnitude.

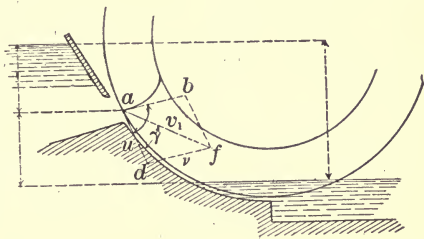


FIG. 256.

Complete the parallelogram bd . Then ab evidently represents the velocity V of the water relatively to the wheel. This velocity V is rapidly destroyed, the corresponding loss of head being

$$\frac{V^2}{2g} = \frac{u^2 + v_1^2 - 2uv_1 \cos \gamma}{2g}, \dots \dots \dots (1)$$

γ being the angle daf .

Assuming that the water enters the race with a velocity equal to u , the speed of the wheel, the theoretical useful work per pound of water per second due to impact

$$= \frac{v_1^2}{2g} - \frac{V^2}{2g} - \frac{u^2}{2g} = \frac{u}{g}(v_1 \cos \gamma - u),$$

which is a maximum and $= \frac{v_1^2 \cos^2 \gamma}{4g} = \frac{u^2}{g}$,

when $v_1 \cos \gamma = 2u$.

In practice γ is usually about 30° , and

$$\text{the maximum useful work} = \frac{3}{8} \frac{v_1^2}{2g},$$

corresponding to the relation $4u = \sqrt{3}v_1$, or $u = .433v_1$.

To diminish as much as possible the loss in shock at entrance due to the dissipation of the energy $\frac{V^2}{2g}$ in eddy motion, the direction ab of the relative velocity V should be parallel to

the arm or tangential to the lip of the bucket and should therefore be approximately at right angles to the wheel's periphery.

If, at the point of entrance, the inlet lip is the lowest point of the bucket, the water flows upwards, and the relative velocity V , instead of *being* wholly destroyed in eddy motion, is *partially* destroyed by gravity. This latter is again restored to the water on its return, and increases the wheel's efficiency.

For a given speed (u) of the wheel, the velocity (v_1) with which the water should reach the wheel in order to make the loss of $\frac{V^2}{2g}$ a minimum is found by making $dV = 0$ in eq. (1), and then

$$0 = v_1 \cdot dv_1 - u \cos \gamma \cdot d\dot{v}_1,$$

or

$$v_1 = u \cos \gamma.$$

This is an impossible relation, as it makes $v_1 < u$ and the useful work negative. In fact the angle $afd (= baf)$ in such case would be 90° , and the direction af of v_1 would be practically tangential, so that no water would enter the wheel.

Again, for a given velocity v_1 of the water as it reaches the wheel, the speed of wheel which would make the loss of $\frac{V^2}{2g}$ a minimum is given by

$$0 = u \cdot du - v_1 \cos \gamma \cdot du,$$

or

$$u = v_1 \cos \gamma.$$

This is also an impossible relation, as it makes the useful work *nil*.

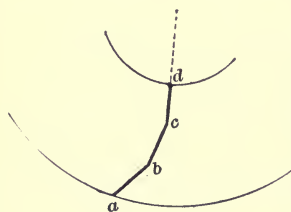


FIG. 257.

It will be found advantageous to use curved or polygonal buckets and not plane floats. A bucket, for example, may consist of three straight portions, ab , bc , cd , Fig. 257. Of these the inner portion, cd , should be radial; the outer portion, ab , is nearly

normal to the periphery of the wheel, and the central portion, bc , may make angles of about 135° with ab and cd .

Disregarding all other losses, the theoretical delivery of the wheel in foot-pounds

$$= wQ \left\{ \frac{u(v_1 \cos \gamma - u)}{g} + h_2 \right\},$$

where h_2 = total fall — fall (h_1) required to produce the velocity v_1 .

If η be the efficiency, then, according to the results of Morin's experiments,

$$\eta = .40 \text{ to } .45 \text{ if } h_1 = \frac{1}{4}H;$$

$$\eta = .42 \text{ to } .49 \text{ if } h_1 = \frac{2}{5}H;$$

$$\eta = .47 \quad \text{if } h_1 = \frac{2}{3}H;$$

$$\eta = .55 \quad \text{if } h_1 = \frac{3}{4}H.$$

(c) There is a loss of head due to frictional resistance along the channel in which the wheel works.

Let l = length of the channel (or curb).

Let t = thickness of water-layer leaving the wheel.

Let b = breadth of wheel.

The mean velocity of flow in this curb channel is approximately $\frac{4}{3}u$, and the loss of head due to channel friction

$$= f \frac{b + 2t}{bt} \frac{\left(\frac{4}{3}u\right)^2}{2g} l = \frac{4}{3} f \frac{b + 2t}{bt} \frac{v_1^2}{2g} l,$$

where f = coefficient of friction, $b + 2t$ = wetted perimeter, bt = water area, and γ being 30° .

(d) There is a loss of head due to the escape of water over the ends and sides of the buckets.

Let s_1 be the play between the ends of the buckets and the channel.

Let s_2 be the play at the sides. ($s_1 = s_2$, approximately.)

Let $z_1, z_2, \dots z_n$ be the depths of water in a bucket corresponding to n successive positions in its descent from the receiving to the lowest point.

Let $l_1, l_2, \dots l_n$ be the corresponding water-arcs measured along the wheel's periphery.

The orifice of discharge at end of a bucket $= bs_1$.

The mean amount of water escaping from a bucket over its end

$$= cb s_1 \sqrt{2g} \frac{\sqrt{z_1} + \sqrt{z_2} + \dots \sqrt{z_n}}{n},$$

c being the coefficient of discharge.

The water escapes at the sides as over a series of weirs, and the mean amount of water escaping from a bucket over the sides

$$= 2 \times \frac{2}{3} c s_2 \sqrt{2g} \frac{l_1 \sqrt{z_1} + l_2 \sqrt{z_2} + \dots l_n \sqrt{z_n}}{n}.$$

Hence the total loss of effect from escape of water

$$c \sqrt{2g} \frac{wh}{n} \left\{ bs_1 (\sqrt{z_1} + \sqrt{z_2} + \dots \sqrt{z_n}) + \frac{4}{3} s_2 (l_1 \sqrt{z_1} + l_2 \sqrt{z_2} + \dots l_n \sqrt{z_n}) \right\}$$

per sec., h being the vertical distance between the point of entrance and the surface of the water in the tail-race

$$= H - (1 + v) \frac{v_1^2}{2g}.$$

(e) There is a loss of head due to journal friction.

Let W = weight of wheel.

Let w_1 = weight of water on the wheel.

Let r_1 = radius of wheel's outer periphery.

Let r' = radius of axle.

Loss per second of mechanical effect due to journal friction

$$= \mu(W + w_1) \frac{r'}{r_1} u,$$

r being the coefficient of journal friction.

There is a loss of mechanical effect due to the resistance of the air to the motion of the floats (buckets), but this is practically very small, and may be disregarded without sensible error. A deepening of the tail-race produces a further loss of effect, and should only be adopted when back-water is feared.

Hence the total actual mechanical effect, putting

$$Z = bs_1(\sqrt{z_1} + z_2 + \dots \sqrt{z_n}) + \frac{4}{3}s_2(l_1\sqrt{z_1} + l_2\sqrt{z_2} + \dots + l_n\sqrt{z_n}),$$

$$\begin{aligned} \text{is} &= wQ\left(H - \frac{u^2}{2g}\right) - wQ\left(v \frac{v_1^2}{2g} + \frac{u^2 + v_1^2 - 2uv_1 \cos \gamma}{2g}\right) \\ &\quad - wQf \frac{b + 2t}{bt} \frac{4}{3} \frac{v_1^3}{2g} l - c \sqrt{2g} \frac{wh}{n} Z - \mu(W + w_1) \frac{r'}{r_1} u \end{aligned}$$

$$\begin{aligned} &= wQ \left\{ H - (1 + v) \frac{v_1^2}{2g} \right\} + \frac{wQ}{g} u(v_1 \cos \gamma - u) \\ &\quad - wQf \frac{b + 2t}{bt} \frac{4}{3} \frac{v_1^3}{2g} l - c \sqrt{2g} \frac{wh}{n} Z - \mu(W + w_1) \frac{r'}{r_1} u \end{aligned}$$

$$\begin{aligned} &= \left(wQ - c \sqrt{2g} \frac{wZ}{n} \right) \left(H - \frac{v_1^2}{1 + v} \frac{1}{2g} \right) + \frac{wQ}{g} u(v_1 \cos \gamma - u) \\ &\quad - wQf \frac{b + 2t}{bt} \frac{4}{3} \frac{v_1^3}{2g} l - \mu(W + w_1) \frac{r'}{r_1} u. \end{aligned}$$

Hence for a given value of v_1 the mechanical effect (omitting the last term) is a maximum when

$$u = \frac{v_1 \cos \gamma}{2} (= .433 \times v_1, \text{ if } \gamma = 30^\circ).$$

In practice the speed of the wheel is made about *one half* of the velocity with which the water enters the wheel.

For a given speed of wheel, and disregarding the loss of

effect due to curb friction, which is always small, the mechanical effect is a maximum for a value of v_1 given by

$$-\left(wQ - c\sqrt{2g}\frac{wZ}{n}\right)\frac{1+r}{g}v_1 + \frac{wQ}{g}u \cos \gamma = 0,$$

or

$$v_1 = \frac{u \cos \gamma}{(1+r)\left(1 - \frac{c\sqrt{2g}Z}{nQ}\right)}.$$

The loss by escape of water, viz., $c\sqrt{2g}\frac{Z}{n}$, varies, on an average, from 10 to 15 per cent of the whole supply, so that $c\sqrt{2g}\frac{Z}{n}$ varies from $\frac{Q}{10}$ to $\frac{3Q}{20}$.

Ex. The buckets of a low breast wheel, of 24 ft. diameter, are half filled with water which flows from a flume through a vertical rectangular sluice-opening at the rate of 15 cu. ft. per second. The linear speed of the wheel's periphery is 5 ft. per second. At the point of admission the inflowing jet has a velocity of 10 ft. per second and makes an angle of

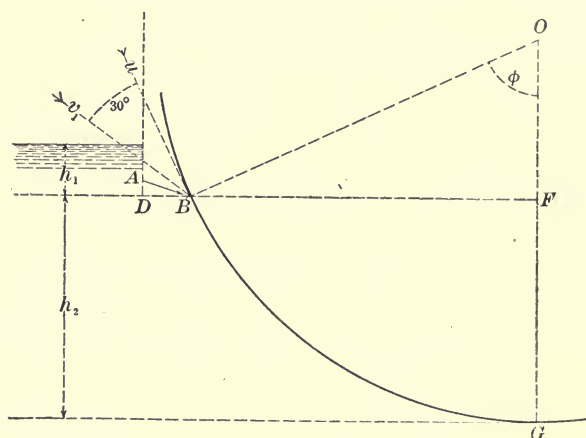


FIG. 258.

30° with the rim. The total available fall is $8\frac{1}{2}$ ft. Find (a) the position of the point of admission; (b) the work done by impact and weight; (c) the position and dimensions of the sluice-opening, the depth of the shrouding being 12 ins.

(a) Let OB be the radius to the point of admission B , and let ϕ be its inclination to the vertical.

Draw the vertical OG and the horizontal BF .

Theoretically, h_1 , the head required to develop a velocity of 10 ft. per second,

$$= \frac{10^2}{64} = 1\frac{9}{16} \text{ ft.}$$

Then $8\frac{1}{2} - 1\frac{9}{16} = 6\frac{1}{16}$ ft. = head available for work by weight
= the vertical fall on the wheel
= FG .

$$\text{Therefore } \cos \phi = \frac{OF}{OB} = \frac{12 - 6\frac{1}{16}}{12} = .421875,$$

and $\phi = 65^\circ 3'$, defining the position of B .

(b) The useful theoretical work done by impact

$$= \frac{62\frac{1}{2}}{32} 15 \cdot 5 (10 \cos 30^\circ - 5) = 536.133 \text{ ft.-lbs.}$$

The useful theoretical work done by weight

$$= 62\frac{1}{2} \cdot 15 \cdot 6\frac{1}{16} = 6503.906 \text{ ft.-lbs.,}$$

and the combined useful work = 7040.039 ft.-lbs.

(c) Let AD , BD be the vertical and horizontal distances of the lift A from B .

The angle between the direction of v_1 at B and the horizontal

$$= \phi - 30^\circ = 35^\circ 3'$$

$$\text{Therefore } AD = \frac{10^2}{64} \sin^2 35^\circ 3' = .51533 \text{ ft.}$$

$$\text{and } BD = \frac{10^2}{64} \sin 70^\circ 6' = 1.4692 \text{ ft.}$$

$$\text{Again, the width of the wheel} = \frac{15}{\frac{1}{2} \cdot 1 \cdot 5} = 6 \text{ ft.,}$$

and the width of the sluice may be taken to be about 3 ins. less than this, or $5\frac{3}{4}$ ft. The head over the lip = $1\frac{9}{16} - .51533 = 1.0472$;

the average velocity of flow through the sluice = $.9 \sqrt{64 \times 1.0472} = 7.3656$ ft. per second, and the depth of the sluice-opening = $\frac{15}{5\frac{3}{4} \times 7.3656} = .354$ ft.

11. Sagebien Wheels, Fig. 259, have plane floats inclined to the radius at from 40° to 45° in the direction of the wheel's rotation. The floats are near together and sink slowly into the fluid mass. The level of the water in the float-passages gradually varies, and the volume discharged in a given time may be very greatly changed. The efficiency of these wheels

is over 80 per cent, and has reached even 90 per cent. The action is almost the same as if the water were transferred from

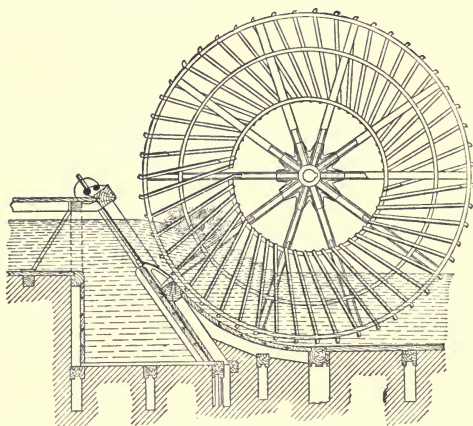


FIG. 259.

the upper to the lower race, without agitation, frictional resistance, etc., flowing away without obstruction into the tail-race.

12. Overshot Wheels.—Since the introduction and development of the turbine these wheels have become almost obsolete. They have been considered among the best of hydraulic motors for falls of 8 to 70 ft. and for a delivery of 3 to 25 cu. ft. per second, and have proved especially useful for falls of 12 to 20 ft. The efficiency of overshot wheels of the best construction is from .70 to .85.

The thickness of the sheet of water passing through the sluice on to the wheel rarely exceeds 4 or 5 ins., and is often less than 2 ins.

If the level of the head-water is liable to a greater variation than 2 ft., it is most advantageous to employ a pitch-back or high breast wheel, which receives the water on the same side as the channel of approach.

13. Wheel Velocity.—This evidently depends upon the work to be done, and upon the velocity with which the water arrives on the wheel. Overshot wheels should have a low cir-

cumferential speed, varying from 10 ft. per second for large wheels to 3 ft. per second for small wheels, and should not be less than $2\frac{1}{2}$ ft. per second. At a higher speed than 6 ft. per second, if the buckets are more than *two thirds* full, the efficiency does not exceed 60 per cent.

In order that the water may enter the buckets easily, its velocity should be greater than the peripheral velocity of the wheel.

14. Effect of Centrifugal Force.—Consider a molecule of weight w in the “unknown” surface of the water in a bucket (Fig. 260). At each moment there is a dynamical equilibrium between the “forces” acting on m , viz.: (1) its weight w ; (2) the centrifugal force $\frac{w}{g}\omega^2 r$; (3) the resultant T of the neighboring reactions.

Take $MF = w$, $MG = \frac{w}{g}\omega^2 r$, and complete the parallelogram FG . Then $MH = T$. The direction of T is, of course, normal to the surface of the water in the bucket.

Let HM produced meet the vertical through the axis O of the wheel in E . Then

$$\frac{MG}{MF} = \frac{\frac{w}{g}\omega^2 r}{w} = \frac{FH}{MF} = \frac{OM}{OE} = \frac{r}{OE},$$

and therefore

$$OE = \frac{g}{\omega^2} = \frac{2915}{n^2} \text{ ft.},$$

taking $g = 32$ ft. and n being the number of revolutions per minute.

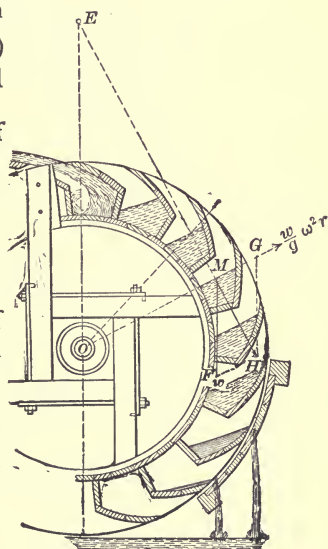


FIG. 260.

Thus the position of E is independent of r and of the position of the bucket, so that all the normals to the water-surface in a bucket meet in E , and the surface is the arc of a circle having its centre at E , or, rather, a cylindrical surface with axis through E parallel to the axis of rotation.

15. Weight of Water on Wheel and Arc of Discharge.—

Let Q = volume supplied per sec., and N = number of buckets.

Then $\frac{N\omega}{2\pi}$ = number of buckets fed per second,

and $\frac{2\pi Q}{N\omega}$ = volume of water received by each bucket per sec.

Hence the area occupied by the water until spilling commences = $\frac{2\pi Q}{bN\omega}$, b being the bucket's width (= width of wheel between the shroudings).

The water flows on to the wheel through a channel (Fig. 261), usually of the same width b as the wheel, and the supply is regulated by means of an adjustable sluice, which may be either vertical, inclined, or horizontal.

When the water springs clear from the sluice, as in Fig. 261, the axis of the sluice should be tangential to the axis of the jet, and the inner edges of the sluice-opening should be rounded so as to eliminate contraction.

Let y , z be the horizontal and vertical distances between the sluice and the point of entrance.

Let T be the time of flow between the sluice and entrance.

Let v_0 , v_1 be the velocities of flow on leaving the sluice and on entering the bucket.

Then

$$\begin{aligned} v_1 \cos (\gamma + \delta) T &= y, \\ v_1 \sin (\gamma + \delta) T - \frac{1}{2} g T^2 &= z, \end{aligned}$$

and

$$v_1^2 = v_0^2 + 2gz,$$

δ being the angular deviation of the point of entrance from the

summit, and γ the angle between the direction of motion of the water and the wheel at the point of entrance.

If the bed of the channel is horizontal, and if also the sluice is vertical, opening upwards from the bed, and is of the same width b as the wheel, then

$$Q = bt \sqrt{2gh_1},$$

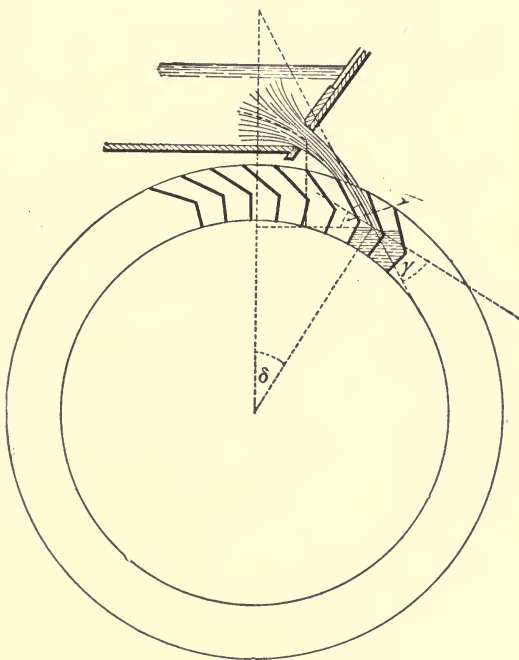


FIG. 261.

t being the depth of sluice-opening and h_1 the effective head over the sluice. This effective head is about $\frac{9}{10}$ of the actual head.

Thus, taking $g = 32$, $\frac{Q}{b} = 8th_1^{\frac{1}{2}}$ gives the delivery per foot width of wheel.

Taking .6 ft. and 3.6 ft. as the extreme limits between which h_1 should lie, and .2 ft. and .33 ft. as the extreme limits

between which t should lie, then $\frac{Q}{b}$ must lie between the limits 1.24 and 5, and an average value of $\frac{Q}{b}$ is 3. Thus the width of the wheel should be on the average $\geq \frac{Q}{3}$.

Again, disregarding the thickness of the buckets, the capacity of the portion of the wheel passing in front of the water-supply per second

$$= b\omega \left\{ \frac{r_1^2}{2} - \frac{(r_1 - d)^2}{2} \right\} = bd\omega \left(r_1 - \frac{d}{2} \right) = bdr_1\omega, \text{ approximately,}$$

$$= bdu = bd \frac{\pi r_1 n}{30},$$

r_1 being the radius and u the velocity of the outer circumference of the wheel, d the depth of the shrouding, or crown, and n the number of revolutions per minute.

Only a portion, however, of the space can be occupied by the water, so that the capacity of a bucket is $mubd$, m being a fraction less than unity and usually $\frac{1}{3}$ or $\frac{1}{4}$. For very high wheels m may be $\frac{1}{5}$. Hence

$$mbdu = Q.$$

Therefore
$$mdu = \frac{Q}{b}.$$

The delivery $\left(\frac{Q}{b} \right)$ per foot of width must not exceed a certain limit, otherwise either d or u will be too great. In the former case the water would acquire too great a velocity on entering the buckets, which would lead to an excessive loss in eddy motion and a corresponding loss of efficiency; while if the speed u of the wheel is too great, the efficiency is again diminished and might fall even below 40 per cent.

The depth of a bucket or of the shrouding varies from 10 to 16 ins., being usually from 10 to 12 ins., and the buckets

are spread along the outer circumference at intervals of 12 to 14 ins. The number of the buckets is approximately $5r_1$ or $6r_1$, r_1 being in feet.

The efficiency of the wheel necessarily increases with the number of the buckets, but the number is limited by certain considerations, viz.: (*a*) the bucket thickness must not take up too much of the wheel's periphery; (*b*) the number of the

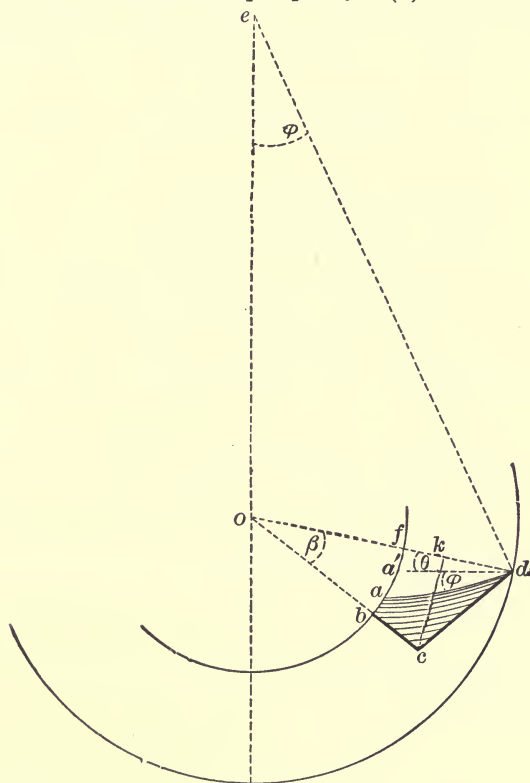


FIG. 262.

buckets must not be so great as to obstruct the free entrance of the water; (c) the form of the bucket essentially affects the number.

Let the bucket, Fig. 262, consist of two portions, an inner

portion bc , which is radial, and an outer portion, cd ; c being a point on what is called the division circle. The length bc is usually one half or two thirds of the depth d of the shrouding.

Take $bc = \frac{1}{2}d$.

It may also be assumed without much error that the water-surface ad is approximately perpendicular to the line ed , so that the angle eda is approximately a right angle.

The spilling evidently commences when the cylindrical surface, having its axis at e and cutting off from the bucket a water-area equal to $\frac{2\pi Q}{bN\omega}$, passes through the outer edge d of the bucket.

Let β be the bucket angle cOd .

Let θ be the inclination of Od to the horizon.

Let ϕ be the inclination of ad to the horizon.

Let r_1 be the radius of the outer periphery.

Let R be the radius of the division circle.

Let r_2 be the radius of the inner periphery.

Then

$$\frac{g}{r_1\omega^2} = \frac{Oe}{Od} = \frac{\cos(\theta \pm \phi)}{\sin \phi}, \quad \dots \quad (1)$$

the sign being plus or minus according as the bucket is below or above the horizontal, and in the latter case, if $\theta = \phi$, then $r_1\omega^2 = g \sin \phi$.

Again,

$$af = fd \tan(\theta + \phi), \text{ approximately.}$$

Therefore

$$\text{the area } dfa = \frac{fd^2}{2} \tan(\theta + \phi) = \frac{d^2}{2} \tan(\theta + \phi),$$

where $d = r_1 - r_2$. Hence

$$\begin{aligned} \text{the area } abcd &= \text{area } cod - \text{area } bof - \text{area } dfa \\ &= \frac{r_1 R}{2} \sin \beta - \frac{r_2^2}{2} \beta - \frac{d^2}{2} \tan(\theta + \phi) = \frac{2\pi Q}{bN\omega}. \quad (2) \end{aligned}$$

Equations (1) and (2) give θ and ϕ , and therefore the position of the bucket when spilling commences.

The bucket will be completely emptied when it has reached a position in which cd is perpendicular to a line from e to middle point of cd , or, approximately, when edc is a right angle.

Let θ_1 , ϕ_1 be the corresponding values of θ and ϕ , and let γ_1 be the angle between cd and the tangent at d to the wheel's periphery. Then

$$\gamma_1 = 90^\circ - (\theta_1 + \phi_1),$$

and

$$\frac{\sin \gamma_1}{\sin \phi_1} = \frac{g}{r_1 \omega^2},$$

two equations giving ϕ_1 and θ_1 .

Also, if ck is drawn perpendicular to od ,

$$\tan \gamma_1 = \cot cdk = \frac{dk}{ck} = \frac{r_1 - R \cos \beta}{R \sin \beta} = \frac{r_1}{R} \operatorname{cosec} \beta - \cot \beta.$$

The vertical distance between the points where spilling begins and ends, viz., $r_1(\sin \theta_1 - \sin \theta)$ can now be determined.

The pitch-angle ($= \psi$) is the angle between two consecutive buckets so that $\psi = \frac{360^\circ}{N}$. In order to obtain a small angle ($= \gamma_1$) between the lip of the bucket and the wheel's periphery, it is usual to make the bucket angle β greater than ψ .

For example,

$$\beta = \frac{5}{4}\psi = \frac{5}{4} \frac{360^\circ}{N} = \frac{450^\circ}{N}.$$

The interval between the buckets should be at least sufficient to prevent any bucket dipping into the one below at the moment the latter begins to spill.

Let coc' , Fig. 263, be the division angle, and t the thickness of the bucket.

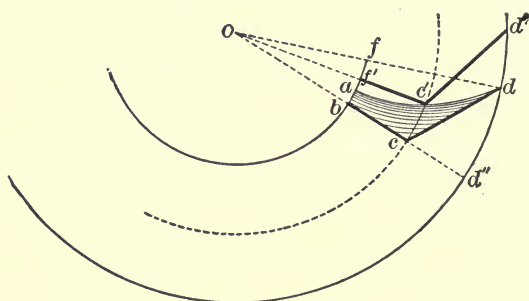


FIG. 263.

Then

$$ff' = \frac{fa}{2} = \frac{d}{2} \tan (\theta + \phi) = \frac{d}{2} \tan \theta,$$

approximately, and therefore

$$N \left(r_2 \beta + t - \frac{d}{2} \tan \theta \right) = 2\pi r_2. \quad . \quad . \quad (3)$$

Also, by equation (2),

$$\frac{r_1 R}{2} \sin \beta - \frac{r_2^2}{2} \beta = \frac{d^2}{2} \tan \theta + \frac{2\pi Q}{bN\omega}. \quad . \quad . \quad (4)$$

These last two equations give N and θ .

The number of buckets may also be approximately found from the formula

$$N = \frac{2\pi r_1}{d}.$$

15. Form and Capacity of Bucket.—In practice the bucket may be delineated as follows:

In Fig. 263 let dd' = distance between two buckets.

Take $dd'' = \frac{5}{4}dd'$ to $\frac{6}{5}dd'$; also take $bc = \frac{d}{2}$, and join dc .

This gives the form of a suitable wooden bucket.

If the bucket is of iron, circular arc is substituted for the portions bc , cd .

Again, let pm , Fig. 264, be the thickness of the stream just before entering the bucket.

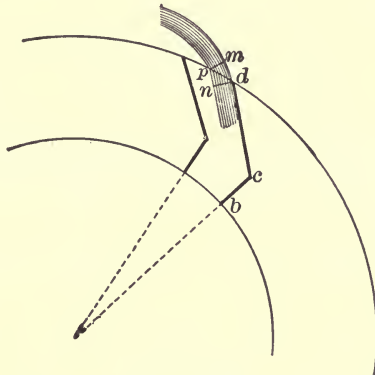


FIG. 264.

Let dn be the thickness of the stream just after entering the bucket.

Let γ_1 be the angle between the bucket's lip and the wheel's periphery.

Then

$$\begin{aligned} mbdu_1 &= \text{capacity of bucket} = bv_1 \cdot pm = bV \cdot dn \\ &= bv_1 dp \sin \gamma = bV \cdot dp \cdot \sin \gamma_1, \end{aligned}$$

and therefore

$$dp = \frac{mbdu_1}{v_1 \sin \gamma} = \frac{mbdu_1}{V \sin \gamma_1}.$$

Now overshot wheels cannot be ventilated, and it is consequently necessary to leave ample space above the entering

stream for the free exit of air. Thus, neglecting float thickness,

$$\begin{aligned}\frac{2\pi r_1}{N} &= \text{the distance between consecutive floats} \\ &= dd' \text{ (Fig. 263)} > dp > \frac{mdu_1}{V \sin \gamma_1},\end{aligned}$$

and N , the number of buckets,

$$< \frac{2\pi r_1 V \sin \gamma_1}{mdu_1}.$$

For efficient action the number of the buckets is much less than the limit given by this relation, often not exceeding one half of such limit.

If γ_1 is very small, $V = v_1 - u_1$, approximately, and therefore

$$N < \frac{2\pi r_1 \sin \gamma_1}{md} \left(\frac{v_1}{u_1} - 1 \right).$$

The capacity of a bucket depends upon its form; and the bucket must be so designed that the water can enter freely and without shock, is retained to the lowest possible point, and is finally discharged without let or hindrance. Hence flat buckets, Fig. 265, are not so efficient as the curved iron bucket in Fig. 268 and as the compound bucket made of three or two pieces in Figs. 266, 267, and 269. The resistance to entrance is least in the curved bucket, as there are no abrupt changes of direction due to angles. The capacity of a compound bucket may be increased, without diminishing the ease of entrance, by making the inner portion strike the inner periphery at an acute angle, Fig. 269. The objection to this construction, especially if the relative velocity V is large, is that the water tends to return in the opposite direction and escape from the bucket.

FIG. 265.

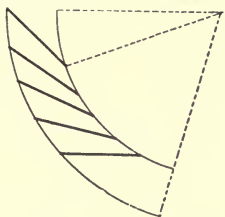


FIG. 266.

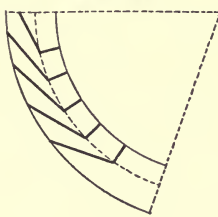


FIG. 267.

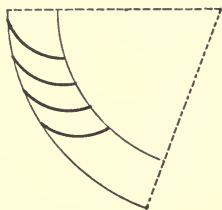
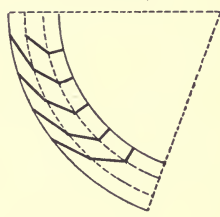


FIG. 268.

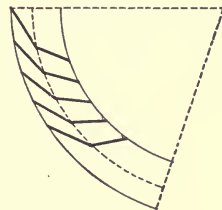


FIG. 269.

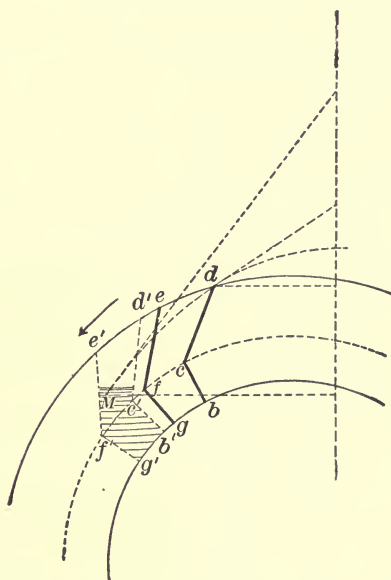


FIG. 270.

Let bcd , efg , Fig. 270, represent two consecutive buckets of an overshot wheel turning in the direction shown by the arrow.

Water will cease to enter the bucket-space between bcd and efg , and impact will therefore cease, when the upper parabolic boundary of the supply-stream intersects the edge d . The last fluid elements will then strike the water already in the bucket at a point M , whose vertical distance below d may be designated by z . The velocity v_1' with which the entering particles reach M is given by the equation

$$v_1' = \sqrt{v_1^2 + 2gz}. \quad \dots \quad (1)$$

Again, while the fluid particles move from d to M let the buckets move into the positions $d'c'b'$, $e'f'g'$.

Let arc $dd' = s_1 = ec'$.

Let arc $dM = s_2$.

Let T be the time of movement from d to d' (or d to M).

Then

$$s_1 = uT$$

and

$$s_2 = \frac{v_1 + v_1'}{2}T,$$

assuming that the mean velocity from d to M is an arithmetic mean between the initial and final velocity of entrance. Thus

$$\frac{2s_2}{v_1 + v_1'} = T = \frac{s_1}{u}. \quad \dots \quad (2)$$

Also, since the angle between dM and the wheel's periphery is small, it may be assumed that

the arc $dM = de + ef + ee'$, approximately,

$$= \frac{2\pi r_1}{N} + \frac{2\pi r_1}{N} \cdot \frac{v_1 - u}{u} + s_1.$$

(NOTE.— $ef = ed \frac{V}{u} = ed \frac{v_1 - u}{u} = \frac{2\pi r_1}{N} \cdot \frac{v_1 - u}{u}$, nearly.)

Thus

$$s_2 = \frac{2\pi r_1 v_1}{N u} + s; \quad . \quad . \quad . \quad . \quad (3)$$

and by equations (2) and (3),

$$s_1 \left(\frac{v_1 + v_1' - 2u}{2u} \right) = \frac{2\pi r_1 v_1}{N u},$$

an equation giving approximately the distance s_1 passed through by a float during impact. The buckets can now be plotted in the positions they occupy at the end of the impact. The amount of water in each bucket being also known, the water-surface can be delineated, and hence the vertical distance z can be at once found.

EX. 1. Find the angular depression of the water-surface below the horizontal (*a*) when the bucket lip is $37^\circ 14'$ above the centre, and (*b*) when the bucket lip is on a level with the centre; also find (*c*) the position of the bucket below the centre when a horizontal through the lip bisects the angle between the water-surface and the radius to the lip. The wheel has a diameter of 32 ft. and makes $10\frac{1}{2}$ revolutions per minute.

The angular velocity $\omega = \frac{44}{7} \frac{10\frac{1}{2}}{60} = \frac{11}{10}$. Then

$$(a) \quad \frac{16 \left(\frac{11}{10} \right)^2}{32} = \frac{\sin \phi}{\cos (37^\circ 14' - \phi)} = \frac{121}{200}.$$

$$\text{Therefore} \quad \frac{200}{121} = \frac{\cos (37^\circ 14' - \phi)}{\sin \phi} = \cos 37^\circ 14' \cot \phi + \sin 37^\circ 14',$$

$$\text{or} \quad \cot \phi = 1.316 \quad \text{and} \quad \phi = 37^\circ 14'.$$

$$(b) \quad \frac{121}{200} = \frac{\sin \phi}{\cos (\theta + \phi)} = \tan \phi = .605,$$

$$\text{and} \quad \phi = 31^\circ 10'.$$

$$(c) \quad \frac{121}{200} = \frac{\sin \phi}{\cos (\theta + \phi)} = \frac{\sin \phi}{\cos 2\phi} = \frac{\sin \phi}{1 - 2 \sin^2 \phi},$$

$$\text{or} \quad \sin^2 \phi + \frac{100}{121} \sin \phi = \frac{1}{2},$$

$$\text{and} \quad \sin \phi = .4058,$$

$$\text{or} \quad \phi = 23^\circ 56' = \theta.$$

EX. 2. An overshoot wheel has a diameter of 32 ft., a 12-in. crown, and its peripheral speed is 4 ft. per second. The lip of the bucket is $1\frac{1}{2}$

ins. thick. Water enters the wheel in a direction inclined at 60° to the vertical at a point $12^\circ 30'$ from the summit and with a velocity of 16 ft. per second. Spilling commences at 120° from the summit. Find (a) the relative velocity (V) at admission; (b) the angle between the horizontal and the water-surface at $1^\circ 47' 33''$ above and at 30° below the centre; (c) the angle (γ) between the bucket lip and the rim; (d) the point where the bucket is emptied; (e) the bucket angle; (f) the elbow angle; (g) the number of buckets; (h) the bucket water area.

At the point of admission d let dgh be the triangle of velocities so that $dg = 16$ ft., $dh = 4$ ft., and the angle $gdh = 30^\circ - 12^\circ 30' = 17^\circ 30'$.

Assuming also that the water enters *without shock*, the relative velocity $V = (dg)$ is parallel to the bucket arm cd , and the angle $cdk = \gamma_1 = \text{angle } ghk$.

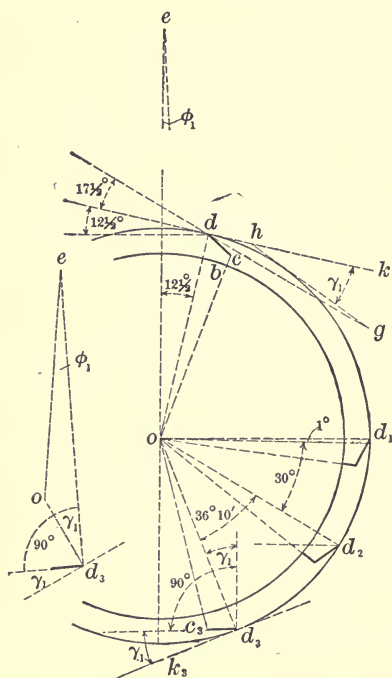


FIG. 270.

Then

$$\begin{aligned} (a) \quad V^2 &= hg^2 = dg^2 + dh^2 - 2 \cdot gd \cdot dh \cos 17^\circ 30' \\ &= 16^2 + 4^2 - 2 \cdot 16 \cdot 4 \cos 17^\circ 30' \\ &= 149.9242, \end{aligned}$$

and $V = 12.2443$ ft. per sec.

(b) When $\theta = 1^\circ 47' 33''$,

$$\frac{32}{16(\frac{4}{16})^2} = \frac{\cos(1^\circ 47' 33'' - \phi)}{\sin \phi} = 32,$$

or

$$\cot \phi = 32 \sec 1^\circ 47' 33'' - \tan 1^\circ 47' 33'',$$

and

$$\phi = 1^\circ 47' 33''.$$

$$\text{When } \theta = 30^\circ, \quad \frac{\cos(30^\circ + \phi)}{\sin \phi} \approx 32,$$

or $\cot \phi = .32 \sec 30^\circ + \tan 30^\circ = 37.5277,$

and $\phi = 1^\circ 31' 24''.$

(c) $\frac{\sin 17^\circ 30'}{\sin \gamma_1} = \frac{12.2443}{10},$

or $\operatorname{cosec} \gamma_1 = \frac{12.2443}{16} \operatorname{cosec} 17^\circ 30' = 2.545,$

and $\gamma_1 = 23^\circ 8'.$

(d) At the point d_3 , where the spilling is completed, Od_3k , is a right angle and the angle $Od_3e = \text{angle } c_3d_3k_3 = \gamma_1$. Then

$$\frac{\sin \phi_1}{\sin \gamma_1} = \frac{ed_3}{oe} = \frac{r_1}{\frac{g}{\omega^2}} = \frac{16 \left(\frac{4}{16} \right)^2}{32} = \frac{1}{32},$$

or $\sin \phi_1 = \frac{\sin 23^\circ 8'}{32} = .0122772,$

and $\phi_1 = 0^\circ 42'.$

Therefore $\theta_1 = 90^\circ - (\gamma_1 + \phi_1) = 66^\circ 10',$
and the bucket is emptied at $90^\circ + 66^\circ 10' = 156^\circ 10'$ from the summit.

(e) $r_1 = 16'$; $R = 15\frac{1}{2}'$. Therefore

$$\tan 23^\circ 8' = \tan \gamma_1 = \frac{16 - 15\frac{1}{2} \cos \beta}{15\frac{1}{2} \sin \beta} = .4272.$$

This last equation is easily reduced to the form

$$\cos^2 \beta - 1.7458 \cos \beta = -.74674,$$

and $\cos \beta = .9962,$

or $\beta = 5^\circ 8',$

$$= \frac{121}{1350} \text{ in circular measure.}$$

(f) The elbow angle $Ocd = 180^\circ - \beta - Odc = 18^\circ - \beta - (90^\circ - \gamma_1)$
 $= 90^\circ - 5^\circ 8' + 23^\circ 8'$
 $= 108^\circ.$

(g) $N \left(15 \times \frac{121}{1350} + \frac{1}{8} - \frac{1}{2} \tan 30^\circ \right) = \frac{44}{7} \cdot 15,$

or $N = 79.8, \text{ say } 80.$

An empirical approximate rule makes

$$N = \frac{2\pi r_1}{d} = \frac{44}{7} \cdot \frac{16}{1} = 100\frac{4}{7}.$$

$$(h) \frac{16 \times 15\frac{1}{2}}{2} \sin 5^\circ 8' - \frac{15^2}{2} \times \frac{121}{1350} = \frac{1}{2} \tan 30 + \text{water area of bucket.}$$

Therefore

$$\begin{aligned} \text{the water area} &= 11.09475 - 10.08333 = .28867 \\ &= .72275 \text{ sq. ft.} \\ &= 104 \text{ sq. ins.} \end{aligned}$$

EX. 3. An overshot water-wheel, of 40 ft. diameter, is 12 in. wide and has a 9.6-in. shrouding. The pitch-angle is 4° and the thickness of the bucket lip is 1 in. At the point where spilling commences the bucket water area is $24\frac{1}{2}$ sq. ins. Find the number of buckets, the point where spilling commences, and the angle between the rim and the bucket lip.

$$r_1 = 20 \text{ ft.}; \quad R = 20 - \frac{1}{2} \frac{9.6}{12} = 19.6 \text{ ft.}; \quad r_2 = 20 - .8 = 19.2 \text{ ft.}$$

$$\text{Take } \beta, \text{ the bucket-angle,} = \frac{5}{4} 4^\circ = 5^\circ.$$

$$\text{Then} \quad 19.2 \times \pi \frac{5}{180} + \frac{1}{12} - \frac{.8}{2} \tan \theta = 2\pi \frac{19.2}{N},$$

$$\text{and} \quad \frac{20 \times 19.6}{2} \sin 5^\circ - \frac{(19.2)^2}{2} \pi \frac{5}{180} = \frac{(.8)^2}{2} \tan \theta + \frac{24\frac{1}{2}}{144}.$$

$$\text{Hence} \quad N = 164.5,$$

$$\text{and} \quad \tan \theta = 2.56, \quad \text{or} \quad \theta = 68^\circ 40'.$$

The empirical formula gives

$$N = \frac{2\pi r_1}{d} = \frac{44}{7} \frac{20}{.8} = 157.1.$$

$$\begin{aligned} \text{Again,} \quad \tan \gamma_1 &= \frac{20}{19.6} \operatorname{cosec} 5^\circ - \cot 5^\circ = .27782, \\ \text{and } \gamma_1 &= 15^\circ 32'. \end{aligned}$$

EX. 4. *One fourth* of the theoretic capacity of a bucket is filled with water. The angle between the bucket lip and the wheel's periphery is 20° , the radius to the outer periphery is 18 ft. and the depth of the crown is 12 ins. If the velocity of the water at entrance is *twice* that of the wheel's periphery, find the greatest number of buckets theoretically possible.

$$\begin{aligned} \text{The number} &< \frac{2\pi r_1 \sin \gamma_1}{md} \left(\frac{v_1}{u_1} - 1 \right) \\ &< \frac{2\frac{2}{7} 18 \sin 20^\circ}{\frac{1}{4} \cdot 1} (2 - 1) \\ &< 103.2. \end{aligned}$$

The actual number may be about two thirds of this, or 69.

16. Useful Effect.—(a) *Effect of Weight.*—The wheel should hang freely, or just clear the tail-water surface, and the total fall is measured from the surface of the water in the tail-race to the water-surface just in front of the sluices through which the water is brought on to the wheel.

Let h_1 , Fig. 272, be the vertical distance between the

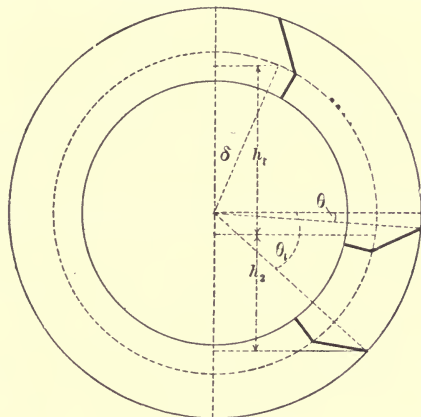


FIG. 272.

centres of gravity of the water-areas of the first and last buckets before spilling commences. Then

$$h_1 = R \cos \delta + r_1 \sin \theta, \text{ very nearly.}$$

Let h_2 be the vertical distance between the centres of gravity of the water-area of the bucket which first begins to spill and the point at which the spilling is completed. Then

$$h_2 = r_1(\sin \theta_1 - \sin \theta), \text{ very nearly.}$$

The useful work per sec. = $wQ(h_1 + kh_2)$, k being a fraction < 1 and approximately = .5.

Let A_0 be the water-area in the bucket which first begins to spill.

Between this bucket and the one which is first emptied, i.e., in the vertical distance h_2 , insert s buckets, at equal distances apart, and let their water-areas $A_1, A_2, A_3, \dots A_s$ be carefully calculated.

Let Q_m be the mean amount of water per bucket in the discharging arc.

Let A_m be the mean water-area per bucket in the discharging arc.

Then

$$A_m = \frac{A_0 + A_1 + A_2 + \dots + A_{s-1} + A_s}{s + 2}.$$

The value of k can now be easily found, since

$$k = \frac{Q_m}{Q} = \frac{A_m}{A_0}.$$

Let q be the varying amount of water in a bucket from which spilling is taking place, and at any moment let y be the vertical distance between the outer edge of the bucket and the surface of the water in the tail-race.

q is a function of y and depends upon the contour of the water in the bucket.

Let Y be the *mean* value of y between the points where spilling begins and ends, i.e., for values y_1 and y_2 of y . Then

$$Y\left(\frac{2\pi Q}{Nw}\right) = Y \int_0^q dq = \int_0^q y dq = y_1 \frac{2\pi Q}{Nw} - \int_{y_1}^{y_2} q \cdot dy,$$

since

$$\int y \cdot dq = yq - \int q \cdot dy.$$

Again, the elementary quantity of water, dq , having an initial velocity equal to that of the wheel, viz., u , falls a distance y and acquires a velocity $= \sqrt{u^2 + 2gy}$.

Thus it flows away in the tail-race, causing a loss of energy $= \frac{w \cdot dq}{g} (u^2 + 2gy) = w \cdot dq \left(\frac{u^2}{2g} + y \right)$.

Hence the *total* loss of energy between the points where spilling begins and ends

$$= \int w \cdot dq \left(\frac{u^2}{2g} + y \right) = \frac{wu^2}{2g} \int dq + w \int y \cdot dq = \left(\frac{wu^2}{2g} + Y \right) kQ.$$

Overshot and pitch-back wheels do not work well in back-water, as they lift a greater or less weight of water in rising above the surface.

If the water-level in the race is liable to variation it is better to diminish the diameter of the wheel and design it so that it may never be immersed to a greater depth than 12 ins.

(b) *Effect of Impact*.—The head h' required to produce the velocity v_1 with which the water reaches the wheel is theoretically $\frac{v_1^2}{2g}$; but as there is a loss of at least 5 per cent in the most perfect delivery, it is usual to take $h' = \frac{v_1^2}{2g}$, an average value of ν being 1.1.

Let the water enter the bucket in the direction ac , Fig. 273. Take $ac = v_1$. The water now moves round with a velocity u (assumed the same as that of the division circle), and leaves the wheel with the same velocity. Take ab in the direction of the tangent to the division circle at the point of entrance $= u$. The component bc represents the relative velocity V of the water with respect to the bucket, and this velocity is wholly destroyed. ab must necessarily be parallel to the outer arm of the bucket, so that there may be no loss of shock at entrance. Then the impulsive effect

$$= \frac{wQ}{g} \left(\frac{v_1^2}{2} - \frac{V^2}{2} - \frac{u^2}{2} \right).$$

But

$$V^2 = v_1^2 + u^2 - 2v_1u \cos \gamma,$$

γ being the angle through which the water is deviated from its original direction at the point of entrance.

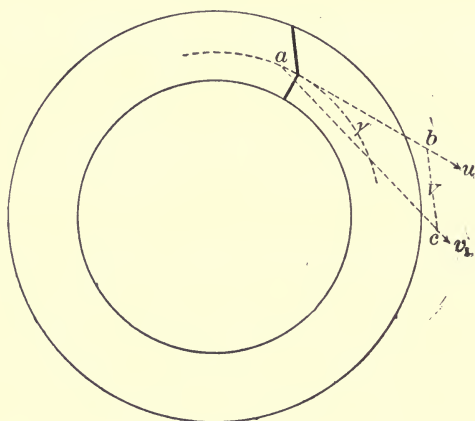


FIG. 273.

Hence the impulsive effect

$$= \frac{wQ}{g} u (v_1 \cos \gamma - u),$$

and the TOTAL USEFUL EFFECT

$$= wQ(h_1 + kh_2) + \frac{wQ}{g} u (v_1 \cos \gamma - u) - \text{loss due to journal friction.}$$

Designating the first two terms of this expression by P , the loss due to journal friction

$$= \mu \left\{ \frac{P}{u} + W \right\} \frac{\rho}{r_1} u,$$

ρ being the radius of the axle, and W the weight of the wheel.

Ex. An overshot wheel weighing 20,000 lbs., with a 12-in. crown and of 40 ft. diameter, receives 400 cu. ft. of water per minute and revolves in 6-in. bearings. The water enters the buckets at 12° from the wheel's summit, with a velocity of 16 ft. per second and at an angle of 10° with the wheel's periphery, which moves with a linear velocity of 9 ft. per second. Spilling commences and is completed at points which are respectively 140° and 160° from the wheel's summit. Determine the power of the wheel and its efficiency, taking $k = .5$ and $\mu = .04$.

Take R = radius of division circle = $19\frac{1}{2}$ ft. Then

$$h_1 = 19\frac{1}{2} \cos 12^\circ + 20 \cos 40^\circ = 34.3947662 \text{ ft.,}$$

and
$$h_2 = 20 \cos 20^\circ - 20 \cos 40^\circ = 3.472964 \text{ ft.}$$

Therefore the H.P. *due to weight*

$$\begin{aligned} &= \frac{62\frac{1}{2} \cdot 400}{33000} \left(34.3947662 + \frac{1}{2} \times 3.472964 \right) \\ &= 27.37215, \end{aligned}$$

and the H.P. *due to impact*

$$\begin{aligned} &= \frac{62\frac{1}{2}}{32} \cdot \frac{400}{33000} 9(16 \cos 10^\circ - 9) \\ &= 1.43968. \end{aligned}$$

Again, the weight of the water on the wheel

$$\begin{aligned} &= \frac{400 \cdot 62\frac{1}{2}}{60 \cdot 9} \left(20\pi \frac{128}{180} + 20\pi \cdot \frac{1}{2} \cdot \frac{20}{180} \right) \\ &= 2231.04 \text{ lbs., approx.,} \end{aligned}$$

and the total weight on the axle = 2231.04 lbs.

Thus the energy absorbed by frictional resistance in H.P.

$$= \frac{2231.04}{550} \times .04 \times 9\frac{1}{40} = .18189,$$

and hence

$$\begin{aligned} \text{the net useful work in H.P.} &= 27.37215 + 1.43968 - .18189 \\ &= 28.62994. \end{aligned}$$

The total available H.P.

$$= \frac{62\frac{1}{2} \cdot 400}{33000} \left(\frac{16^2}{64} + 20 \cos 12^\circ + 20 \right) = 33.0022,$$

and therefore the efficiency = $\frac{28.62994}{33.00223} = .8675.$

17. A **pitch-back** or **high breast** wheel is to be preferred to an overshot wheel when the surface-levels of the head- and tail-water are liable to very considerable variation.

In the pitch-back wheel the water is admitted by an adjustable sluice into the buckets on the same side as the supply-channel, Figs. 274 and 275. Thus the wheel revolves

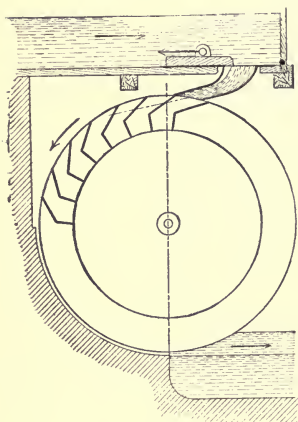


FIG. 274.

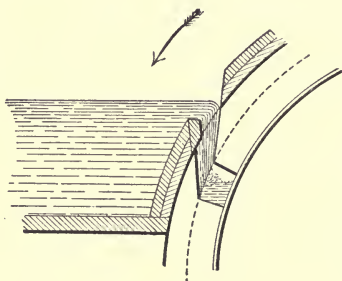


FIG. 275.

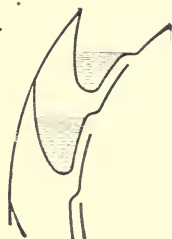


FIG. 276.

in the direction in which the water leaves, and the drowning of the wheel is prevented. Further, the buckets may be now ventilated, Fig. 277, and may therefore be placed closer together than in the unventilated overshot wheel.

The efficiency of the pitch-back is at least equal to that of the overshot.

EXAMPLES.

1. An undershot wheel works in a rectangular channel 4 ft. wide, in which the water on the up-stream side is 2 ft. deep and flows with a velocity of 12 ft. per second; the water on the down-stream side is 3 ft. deep. Find the useful work done and the efficiency.

Ans. 1000 ft.-lbs.; $\frac{3}{8}\%$.

2. Determine the maximum mechanical effect of an undershot wheel of 12 ft. diameter making 10 revolutions per minute, the fall being 3 ft. and the quantity of water passed per second 15 cu. ft.

Ans. 1423 ft.-lbs.

3. Ascertain the general proportions of a Poncelet wheel, being given: height of fall = $4\frac{1}{2}$ ft.; delivery of water = 40 cu. ft. per second; radius of exterior circumference = 9 ft.; $\gamma = 20^\circ$.

Ans. $\alpha = 143^\circ 57'$; $\psi = 128^\circ.1$; $d = 2$ ft.; $r' = 2.47$ ft.; $\lambda = 15^\circ.2$; $t = 5$ ins.; $N = 57$; $\eta = .69$.

4. Design a Poncelet wheel for a fall of 4.5 ft. and 24 cu. ft. of water per second, using the formulæ on pages 428-432, taking $\gamma = 20^\circ$, and also $\lambda = 20^\circ$ as a first approximation.

Ans. $\alpha = 143^\circ 57'$; depth of crown = 1.8 ft.; depth of stream = .372 ft; $b = 4.14$ ft.; radius of bucket = 2.26 ft.; $\psi = 128^\circ 6'$; $\lambda = 17^\circ 1'$; number of buckets = 48; mechanical effect = 8.5 H.P.; efficiency = .69.

5. An undershot water-wheel with straight floats weighing 15,000 lbs. works in a straight rectangular channel of the same width as the wheel, viz., 4 ft.; the stream delivers 28 cu. ft. of water per second, and the efficiency is $\frac{1}{4}$. Find the relation between the up-stream and down-stream velocities. If the velocity of the inflowing water is 20 ft. per second, find the velocity on the down-stream side and determine the mechanical effect of the wheel, its diameter being 20 ft., the diameter of the gudgeons being 4 ins., and the coefficient of friction .008.

Ans. 3634.06 ft.-lbs.

6. Determine the effect of a low *breast* or undershot wheel 15 ft. in diameter and making 8 revolutions per minute, the fall is 4 ft. and the delivery 20 cu. ft. per second; the velocity of the stream before coming on the wheel is double that of the wheel.

Ans. 3148 ft.-lbs.

7. 20 cu. ft. of water per second enter an undershot wheel of 30 ft. diameter, making 8 revolutions per minute, through an underflow sluice. The velocity of the entering water is twice that of the wheel's periphery. Find (a) the head of water behind the sluice; (b) the fall; (c) the theoretical mechanical effect; (d) the actual mechanical effect, disregarding axle-friction.

Ans. (a) 2.716 ft.; (b) 1.283 ft.; (c) 5.72 H.P.; (d) 2.69 H.P.

8. 20 cu. ft. of water per second enter an undershot wheel of 20 ft. diameter in a straight race, the fall being 3 ft. The depth of the entering stream is $\frac{1}{3}$ ft. The width of the wheel is $4\frac{1}{2}$ ft., and the clearance is $\frac{3}{4}$ inch. The number of the floats, of which four are immersed, is 48, and each is 1 ft. long. The weight of the wheel is 7200 lbs., the radius of the axle is $1\frac{3}{4}$ ins., and the coefficient of friction is .1. Find (a) the best speed for the wheel; (b) the corresponding mechanical effect; (c) the efficiency.

Ans. (a) 6 ft. per second; (b) 2.32 H.P., assuming the speed of wheel reduced to 5.74 ft. per second by axle-friction; (c) .34.

9. 72 cu. ft. of water are delivered to an undershot wheel with straight floats, through a channel of rectangular section and 5 ft. wide. The velocity (v_1) of the inflowing water is 24 feet per second. If the efficiency of the wheel is .25, show that the peripheral speed (u) of the wheel must be 6 ft. per second. Also determine the mechanical effect of the wheel.

Ans. 10.125 ft.-lbs. per second.

10. The water in a rectangular channel, of constant width, is $1\frac{1}{2}$ ft. deep, and impinges upon the flat buckets of an undershot wheel with a velocity of 12 ft. per sec. Show that the efficiency is greatest and equal to .181 for a peripheral speed of 8.842 ft. per sec.

11. Water enters the buckets of a low breast-wheel with a velocity of 10 ft. per sec. and in a direction making an angle of $27^\circ 44'$ with the tangent at the point of entrance, which is 4 ft. measured horizontally and 2 ft. measured vertically from the sluice where the stream-lines are horizontal. Each cubic foot of water does $563\frac{1}{2}$ ft.-lbs. of useful work per sec. when the wheel makes $4\frac{3}{8}$ revols. per min. Find the fall on the wheel, the total available fall, and the diam. of the wheel.

Ans. 8.437 ft.; 10 ft.; 24 ft.

12. A race is straight and close-fitting so that the loss of effect due to escape of water may be disregarded. A single undershot wheel with plane floats is replaced by four similar tandem wheels. If the delivery of each of the four wheels is the same, and if it is assumed that the water reaches each wheel with the same velocity with which it leaves the preceding wheel, find the total maximum delivery due to impact.

Ans. $1\frac{1}{2}$ times the delivery of the single wheel.

13. Discuss the preceding example, assuming that the delivery of each wheel is not the same, but that the total delivery is a maximum.

Ans. 1.6 times the delivery of the single wheel.

14. If n wheels of the same type are substituted for the single wheel in example 12, and if the assumptions are the same as those in example 13, show that the total delivery of the n wheels is to the delivery of the single wheel in the ratio of $2n$ to $2n + 1$, and that, theoretically, if the number is made very large, they will approximately give the entire work of the fall.

15. In a low breast-wheel of 20 ft. diameter, the water enters the bucket with a velocity of 16 ft. per second in a direction making angles of 45° with the horizontal and 15° with the wheel's periphery. The wheel makes 7 revolutions per minute and receives 5 cu. ft. per second of water. Find the mechanical effect of the wheel and the position of the sluice, which is placed where the stream-lines are horizontal.

Ans. 2075 ft.-lbs.; $AD = 2$ ft., $BD = .25$ ft.

16. The water in a head-race stands 4.66 ft. above the sole and leaves the race under a gate which is raised 6 ins. above the sole, the coefficient of velocity (v_1) being .95. The water enters a breast-wheel in a direction making an angle of 30° with the tangent to the wheel's periphery at the point of entrance. The speed (u) of the periphery is 10 ft. per second, the breadth of the wheel is 5 ft., the depth of the water in the flume is 8 ins., and the length of the flume is 8.2 ft. Find the loss of head (a) due to the destruction of the relative velocity (V) at entrance; (b) due to the velocity of flow in the tail-race; (c) in the circular flume. ($f = .018$.)

Ans. (a) 1.11 ft.; (b) 1.57 ft.; (c) .44 ft.

17. In the preceding example, find how the losses of head would be modified if the flume were lowered 1.03 ft., and if the point of entrance were raised so as to make $u = v_1 \cos 30^\circ$.

Ans. (a) .939 ft.; (b) 2.816 ft.; (c) 146 ft.

18. 20 cu. ft. of water per second enter a breast-wheel of 32 ft. diameter and having a peripheral velocity of 8 ft. per second, at an angle of $25\frac{1}{2}^\circ$ with the circumference. The depth of the crown is $1\frac{1}{4}$ ft.; the buckets are half-filled, and the fall is 9 ft. The velocity of the entering water is 12 ft. per second. The centre of the sluice-opening is .54 ft. above the point of entrance, and the width of the sluice is $3\frac{3}{4}$ ft. The wheel has 48 buckets. The distance between the wheel and breast is $\frac{1}{2}$ inch. The bucket passes through .9 ft. while receiving water, and the depth of the water-surface in the bucket below the point of entrance is 1.25 ft. Find (a) the angular distance of the point of entrance from the horizontal; (b) the fall in the breast; (c) the head of water over the sluice; (d) the velocity of the water in the bucket the moment entrance ceases; (e) the total mechanical effect, disregarding axle-friction.

Ans. (a) $53^\circ 53'$; (b) 6.525 ft.; (c) 1.935 ft.; (d) 14.9 ft.; (e) 15.59 H.P.

19. In the preceding question, if the energy absorbed by axle-friction, etc., is 743 ft.-lbs., find the efficiency of the wheel.

Ans. $\frac{3}{8}$.

20. 15 cu. ft. of water per second with a fall of $8\frac{1}{2}$ ft. are brought on a breast-wheel revolving with a linear velocity of 5 ft.; depth of shrouding = 12 in.; the buckets are half-filled, and $v_1 = 2u$; also $r_1 = 12$ ft. Find the theoretical mechanical effect, γ being 30° . *Ans.* 7040 ft.-lbs.

21. A wheel is to be constructed for a 30-ft. fall having an 8-ft. velocity at circumference and taking on the water at 12° from the summit with a velocity of 16 ft. Determine the radius of the wheel and the number of revolutions, v_1 being $2u$.

Ans. 12.9 ft.; 5.9.

22. If for the wheel in example 21 the number of revolutions is 5, and $v_1 = 2u$, the water being again taken on at 12° , find the radius and u .

Ans. 13.56 ft.; 7.1 ft. per second.

23. A breast-wheel passes 12 cu. ft. of water per second, and for the speed $u = \frac{4}{3}v_1 = 4$ ft. per second, the loss of mechanical effect, due to the relative velocity V being destroyed, is a minimum. Find this effect, γ being 30° .

Ans. 73.2 ft.-lbs.

24. In a breast-wheel $Q = 10$ cu. ft. per second; $H = 10$ ft.; $v_1 = \frac{3}{2}u$; $u = 4\frac{1}{2}$ ft. per second; $\gamma = 30^\circ$; diameter of gudgeon = 6 ins.; diameter of wheel = 30 ft.; $\mu = .08$; weight of wheel and water = 20,000 lbs. Find the mechanical effect of the wheel. (Neglect loss of effect due to escape of water from buckets and to frictional resistance along the curb.)

Ans. 5776 ft.-lbs.

25. The quantity of water laid on a breast-wheel by an overfall sluice = 6 cu. ft. per second, the total fall being 8 ft., and the velocity of the periphery 5 ft. per second; also $5v_1 = 8u$, and if d be the depth of the shrouding $2bdu = 5Q$ (in the present case $d = 12$ ins.). Find the effective fall, the height of the lip of the guide, the angle of inclination at the end of the guide-curve, the breadth of the lip of the guide-curve, and the radius of the wheel that the water may enter tangentially. If the radius is limited to 12 ft. 6 ins., find the deviation of the direction of motion of the water from that of the wheel at the point of entrance, c being $.6$.

Ans. 6.9 ft.; .325 ft.; $34^\circ 46'$; $2\frac{3}{4}$ ft.; 38.6 ft.; $28^\circ 36'$.

26. 10 cu. ft. of water per second are delivered to a breast-wheel. The total fall is 10 ft. The peripheral velocity of the wheel is 6 ft. per second. If $v_1 = 2u$ and $\gamma = 30^\circ$, find the theoretical useful effect and the theoretical efficiency.

Ans. 5358.4375 ft.-lbs.; .85735.

27. 24 cu. ft. of water enter the buckets of a 36-ft. breast-wheel, the total fall being $11\frac{1}{4}$ ft. At the point of entrance the direction of the water makes an angle of 30° with the periphery and also $2v_1 = 5u$. Find the mechanical effect of the wheel and the position of the lip of the sluice through which the water passes to the wheel.

Also, if the depth of the shrouding is 1 ft. and the buckets are only half-filled, find the width of the wheel.

The axle-bearings are 6 ins. in diameter. Taking the coefficient of friction to be .008, how much power is absorbed by frictional resistance, assuming the weight of the wheel and contents to be 30,000 lbs.?

Ans. 26.833 H.P.; $x = .1624$ ft.; $y = .5625$ ft.; 10 ft.; 4.19 H.P.

28. In an overshot wheel $r_1 = 15$ ft., $d = 10$ in., $\beta = \frac{5}{4}\psi$. If the division circle is at one half of the depth of the crown, find the angle (γ_1) between the bucket-lip and the wheel's periphery. (Take $N = 5r_1$.)

Ans. $\gamma_1 = 18^\circ 2'$.

29. An overshot wheel, in which $r_1 = 18$ ft., makes 4 revolutions per minute, and the velocity of the water on entering the buckets is twice

that of the wheel's periphery. If $\gamma_1 = 20^\circ$, find γ , and also find the relative velocity (V) of the entering water.

Ans. $\gamma = 10^\circ 9'$; $V = 7.78$ ft. per second.

30. If one fourth of the theoretic capacity of a bucket is filled by the water, find the greatest number of buckets theoretically possible, the depth of the crown being 1 ft., the radius (r_1) to the outer periphery 12 ft., the angle γ_1 20° , and the velocity of the entering water twice that of the wheel's periphery.

Ans. 103.1. Making allowance for exit of air, the number of buckets might be about two thirds of this amount, or, say, 69.

31. A wheel of 30 ft. diameter with 72 buckets makes 7 revolutions per minute, Q being 5 cu. ft. per second. The division circle is half way between the outer and inner peripheries. If $d = 1$ ft. and $v_1 = 2u$, find the effect due to impact.

Ans. 514 ft.-lbs.

32. A 30-ft. wheel weighs 24,000 lbs. and makes 6 revolutions per minute; its gudgeons are 6 ins. in diameter and the coefficient of friction is .08. The water enters the wheel with a velocity of 15 ft. per second, and in a direction making an angle of 10° with the direction of motion of the wheel at the point of entrance. The deviation from the summit of the point of entrance is 12° , of the point where spilling begins is 150° , of the point where all is spilt is 160° , and 5 cu. ft. of water enter the wheel per second, of which the partially filled buckets contain one half. Determine the total mechanical effect.

Ans. 9114 ft.-lbs.

33. The velocity of the outer periphery is $9\frac{3}{8}$ ft.; the angle between the directions of motion of stream and wheel is 15° . Find the impulsive effect of the water, v_1 being 15 ft. per second.

Ans. 91 ft.-lbs. per cu. ft. of water.

34. An overshot wheel 40 ft. in diameter makes 4 revolutions per minute and passes 300 cu. ft. of water per minute. If the gudgeons are 6 ins. in diameter and the wheel weighs 30,000 lbs., by how much will the mechanical effect be diminished? ($f = .008$.)

Ans. 25 ft.-lbs. per second.

35. The diameter of an overshot wheel = 30 ft.; $v_1 = 15$ ft.; $u = 9\frac{3}{8}$ ft.; deviation of impinging water from direction of motion of wheel (γ) = $81\frac{1}{2}^\circ$; deviation of point of entrance from summit = 12° ; deviation of point where spilling begins from the centre = $58\frac{1}{2}^\circ$; deviation of point where spilling ends = $70\frac{1}{2}^\circ$; $Q = 5$ cu. ft. Find total effect of impact and weight.

Ans. 16.9 H.P.

36. An overshot wheel with a radius of 15 ft. and a 12-in. crown takes 10 cu. ft. of water per second and makes 5 revolutions per minute. If $m = \frac{1}{3}$, find the width of the wheel and the number of the buckets.

Ans. $5\frac{1}{11}$ ft.; 75 or 90.

37. An overshot wheel of 32 ft. diameter makes 5 revolutions per minute. Find the angle between the water-surface in a bucket and the horizontal when the lip is 140° from the summit.

Ans. $4^\circ 33'$.

38. An overshot wheel of 10 ft. diameter makes 20 revolutions per minute. Find the angle between the water-surface and the horizontal when the lip is (1) 90° from the summit, (2) $45^\circ 26'$ from the summit.

Ans. (1) $34^\circ 27'$; (2) $43^\circ 18'$.

39. The water enters an overshot wheel at 12° from the summit with a velocity of 16 ft. per second and the linear velocity of the wheel's periphery is 8 ft. per second. The fall is 30 ft. Find the diameter of the wheel and the number of revolutions per minute. *Ans.* 25.4 ft.; 5.95.

40. In a 32-ft. wheel with a 12-in. crown and a peripheral velocity of 8 ft. per second, the point where spilling commences is defined by $\theta = \phi$. Find the arc over which spilling takes place, the angle between the bucket-arm and the circumference being 30° . Also find the bucket-angle. If 11 cu. ft. of water enter the wheel at 15° from the summit with a velocity of 18 ft. per second, find the mechanical effect due to impulse and to weight, k being $\frac{1}{2}$.

Ans. arc = 15.2 ft.; $\beta = 56^\circ 35'$; 2.87 H.P., 28 H.P.

41. An overshot wheel of 32 ft. diameter revolves with an angular velocity ω ; show that the angle between the horizontal and the water-surface in a bucket at 90° from the summit is $\tan^{-1} \frac{\omega^2}{2}$.

42. A water-wheel has an internal diameter of 4 ft. and an external diameter of 8 ft.; the direction of the entering water makes an angle of 15° with the tangent to the circumference. Find the angle subtended at the centre of the wheel by the bucket, which is in the form of a circular arc, and also find the radius of the bucket. *Ans.* $28^\circ 54'$; 1.2274 ft.

43. An overshot wheel 5 ft. wide, 30 ft. in diameter, having a 12-in. crown and 72 buckets, receives 10 cu. ft. of water per second and makes 5 revolutions per minute. Determine the deviation from the horizontal at which the water begins to spill, and also the corresponding depression of the water-surface.

Ans. $31^\circ 41'$; $5^\circ 51'$.

44. An overshot wheel makes $\frac{15}{2\pi}$ revolutions per minute; its mean diameter is 32 ft.; the water enters the buckets with a velocity of 8 ft. per second at a point $12^\circ 30'$ from the summit of the wheel. At the point of entrance the path of the inflowing water makes an angle of 30° with the horizontal. Show that the path is horizontal vertically above the centre. The sluice-board is placed at a point whose horizontal distance from the centre is one half that of the point of entrance. Find its position relatively to the centre and its inclination to the horizon. Also find V .

Ans. $16^\circ 6'$; 6.24 ft. per second.

45. The water enters the buckets of the wheel in the preceding example without shock. Find the elbow-angle. Also, if the buckets begin to spill at 150° from the summit, find where the bucket is empty and the number of buckets. (Depth of crown = 12. ins.; thickness of bucket = $1\frac{1}{2}$ ins.)

Ans. $125^\circ 30'$; $156^\circ 10'$; 80.

46. Given $v_1 = 15$ ft. per second, and $\delta = 20\frac{1}{4}^\circ$. Find the position of the centre of the sluice, which is 4 ins. above the point of entrance.

Ans. .0877 ft. vertically below and 1.0143 ft. horizontally from the summit. The axis of the sluice is inclined at $9^\circ 33'$ to the horizontal. (Assume $\gamma = 0^\circ$.)

47. In an overshot water-wheel $v_1 = 15$ ft.; $u = 10$ ft.; elbow-angle $= 70\frac{1}{2}^\circ$; division-angle $= 4\frac{1}{2}^\circ$; deviation from summit of point of entrance $= 12^\circ$. Find the deviation of the layer from that of the arm, so that the water might enter unimpeded; also find the inclination of the layer to the horizon, and the value of V . If the centre of the sluice-aperture is to be 4 ins. above point of entrance, find its vertical and horizontal distance from the vertex of the stream's parabolic path which is vertically above the centre of the wheel, and also find inclination of sluice-board to horizon.

Ans. $5\frac{3}{4}^\circ$; $20\frac{1}{4}^\circ$; 5.3 ft. per second; .0878 ft.; 1.04 ft.; $9^\circ 34'$.

48. A wheel makes 20 revolutions per minute; radius $= 5$ ft., angle of discharge $= 0^\circ$. Find deviation of water-surface from horizon. Also find deviation at $44^\circ 35'$ above centre.

Ans. $4^\circ 27'$; $43^\circ 16'$.

49. In an overshot wheel $Q = 18$ cu. ft.; $r_1 = 6$ ft.; $d = 1$ ft.; $b = 4$ ft.; $N = 24$; $n = 17$. At the moment spilling commences the area $cbfd = 1.025$ sq. ft.; between this point and the point where the spilling is completed three buckets are interposed, the sectional areas of the water being .501, .409, and .195 sq. ft., respectively. Find (a) the sectional area of bucket; (b) the point where the spilling commences; (c) the point where the spilling is completed; (d) the height of the arc of discharge; (e) the mechanical effect due to the fall of the water through the arc of discharge, γ being $10^\circ 46'$.

Ans. (a) .662 sq. ft.; (b) $\theta = 7^\circ 13'$, $\phi = 28^\circ 46'$; (c) $\theta = 73^\circ 23'$,

$\phi = 5^\circ 51'$; (d) 4.49 ft.; (e) 4.93 H.P.

50. In the preceding example, if the water enters with a velocity of 20 ft. per second at 20° below the summit, and if the direction of the inflowing stream makes an angle of 25° with the wheel's periphery at the point of entrance, find the mechanical effect (a) due to impulse; (b) due to the fall to the point where spilling commences.

Ans. (a) 5.08 H.P.; (b) 12.114 H.P.

51. 300 cu. ft. of water per minute enter the buckets of a 40-ft. overshot wheel with a 12-in. crown and making four revolutions per minute. The wheel has 136 buckets. At the moment when spilling commences the area $bcd f = 126.5$ sq. in. The spilling is completed when the angle between the horizontal and the radius to the lip of the bucket $= 62^\circ 30'$. Between these two positions three buckets are interposed, the sectional areas of the water in the buckets being 24.5, 14.48, and 6.6 sq. ins., respectively. The vertical distance between the water-surface in the first bucket and the centre is 18 ft. Find (a) the width of the wheel; (b) the cross-section of a bucket; (c) the angle between the horizontal and the

radius to the lip of the bucket when spilling commences; (*d*) the height of the discharging arc; (*e*) the mechanical effect due to weight.

Ans. (*a*) 2.4 ft.; (*b*) 33.28 sq. ft.; (*c*) $\theta = 52^\circ 19'$; (*d*) 1.9 ft.; (*e*) 19.73 H.P.

52. As the bucket-arm *cd* moves downward from the horizontal position, show that while the wheel moves through an angle θ the last particle of water at *c* will move through a distance approximately equal to $\frac{r(\theta r + u^2)}{u^2} (\theta - \sin \theta)$, *r* being the distance (assumed constant) of the particle of water from the axis, and *u* being the linear velocity of the wheel at the radius.

53. If the last particle of water leaves the buckets just as the lip *d* reaches the lowest point of the wheel, and if the arm is 1 ft. in length, find the angle between the lip and the wheel's periphery (1) for a wheel of 20 ft. diameter, the peripheral velocity being 5 ft. per second; (2) for a wheel of 40 ft. diameter, the peripheral velocity being 10 ft. per second; (3) for a wheel of 10 ft. diameter, the peripheral velocity being 8 ft. per second.

Ans. (1) 20° ; (2) 19.5° ; (3) 40° .

54. In an overshot wheel of 30 ft. diameter, 5 cu. ft. of water per second enter the buckets with a velocity of 16 ft. per second and the wheel's velocity at the division circle is 7 ft. per second. The point of entrance is 18° from the summit, and the angle between the directions of the inflowing water and the wheel's periphery at the point of entrance is 12° . The water begins to spill at $148\frac{1}{3}^\circ$ from the summit and the spilling is complete at $160\frac{1}{3}^\circ$ from the summit. Find the total mechanical effect due to impulse and weight. What is the tangential force at the outer periphery?

Ans. 16.28 H.P.; 1194 lbs.

55. In a 32-ft. wheel, with a 1-ft. crown and a peripheral velocity of 8 ft. per second, the point where spilling commences is defined by the relation $\theta = \phi$. Find the arc over which spilling takes place, the angle between the arm and circumference being 30° . Also find the "bucket" angle. If 11 cu. ft. of water enter the wheel at 15 ins. from the summit, and with a velocity of 18 ft. per second, show how to find the mechanical effect due to impulse and that due to weight.

Ans. $53^\circ 23'$; 4° , $1612\frac{5}{8}$ ft.-lbs.; 14,406.56 ft.-lbs per second.

56. An overshot wheel of 32 ft. diameter makes $\frac{1}{2}\frac{5}{8}$ revolutions per minute. Find the inclination to the horizontal of the water-surface in a bucket at 90° from the summit. If the wheel has 90 buckets and the arms make an angle of $22\frac{1}{2}^\circ$ with the periphery, find the depth of the crown.

Ans. $7^\circ 8'$; 11 ins.

57. An overshot wheel of 32 ft. diameter makes $\frac{1}{2}\frac{5}{8}$ revolutions per minute. Find the inclination to the horizontal of the water-surface in a bucket at 90° from the summit. If the wheel has 90 buckets and the arms make an angle of $22\frac{1}{2}^\circ$ with the periphery, find the depth of the crown.

Ans. 25.68 ft.; 5.94.

58. An overshot wheel of 36 ft. diameter and with 96 buckets has a peripheral velocity of $7\frac{1}{2}$ ft. per second. The water enters with a velocity of 15 ft. per second and acquires in the wheel a velocity of 16.49 ft. per second. Find the distance through which the float moves during impact.

Ans. 2.15 ft.

59. The sluice for a 10-ft. overshot wheel is vertically above the centre and inclined at 45° to the vertical. The water enters the buckets at a point 2 ft. vertically below the sluice and 10° from the summit of the wheel. Find the angle between the directions of motion of the entering water and of the wheel's circumference. Also find the velocity of the water as it enters the wheel.

Ans. $5^\circ 30'$; 9.68 ft. per second.

60. In an overshot wheel $v_1 = 17$ ft.; $u = 11$ ft. per second; elbow-angle = 70° ; division-angle = 5° ; water enters the first bucket at 12° from summit of wheel. Find (a) the relative velocity V so that water may enter unimpeded; (b) the direction of the entering water; (c) the diameter of the wheel, which makes 5 revolutions per minute; (d) the position and direction of the sluice, which is 2 ft. measured horizontally from the point of entrance.

Ans. 6.24 ft. per second; $\gamma = 7^\circ 13'$; 42 ft.; 45 ft.; $5^\circ 43'$.

61. In an overshot wheel the deviation of the impinging water from the direction of motion of the wheel is 10° ; the velocity (v_1) of the impinging stream = 15 ft. per second; of the circumference of the wheel (u) = $15 \cos 10^\circ$. What amount of the head is sacrificed?

Ans. 1.06 ft.

62. A 30-ft. water-wheel with 72 buckets and a 12-in. shrouding makes 5 revolutions and receives 240 cu. ft. of water per minute. Find the width and sectional area of a bucket. The fall is 30 ft.; at what point does the water enter the wheel, the inflowing velocity being $1\frac{1}{2}$ times that of the wheel's periphery? Also find the deviation of the water-surface from the horizontal at the point at which discharging commences, i.e., 140° from the summit.

Ans. 2.03'; .327 sq. ft.; $32^\circ 47'$; $4^\circ 18'$.

63. What number of buckets should be given to an overshot wheel of 40 ft. diameter and 12 ins. width in wheel, pitch-angle = 4° , thickness of bucket-lip = 1 in., water area = $24\frac{1}{2}$ sq. ins.?

Ans. 167, depth of crown being 9 ins.

CHAPTER VII.

TURBINES.

1. Reaction and Impulse Turbines.—All turbines belong to one of two classes, viz., *Reaction Turbines* and *Impulse Turbines*, and are designed to utilize more or less of the available energy of a moving mass of water.

In a *reaction* turbine a portion of the available energy is converted into kinetic energy at the inlet surface of the wheel. The water enters the wheel passages formed by suitably curved vanes, and acts upon these vanes by pressure, causing the wheel to rotate. The proportions of the turbine are such that there is a particular pressure (hence the term pressure-turbine) at the inlet surface corresponding to the best normal condition of working. Any variation from this pressure, caused, e.g., by the partial closure of the passages through which the water passes to the wheel, changes the working conditions and diminishes the efficiency. In order to avoid such a variation of pressure, it is essential that there should be a continuity of flow in every part of the turbine; the wheel passages should be kept completely filled with water, and therefore must receive the water simultaneously. Such turbines are said to have complete admission. The admission is partial when the water is received over a portion of the inlet surface only.

In an *impulse* (Girard) turbine, Figs. 277 and 278, the energy of the water is wholly converted into kinetic energy at the inlet surface. Thus the water enters the wheel with a velocity due to the total available head and therefore without

pressure, is received upon the curved vanes, and imparts to the wheel the whole of its energy by means of the impulse due to the gradual change of momentum. Care must be taken to insure that the water may be freely deviated on the curved vanes, and hence such turbines are sometimes called turbines with free deviation. For this reason the water-passages should

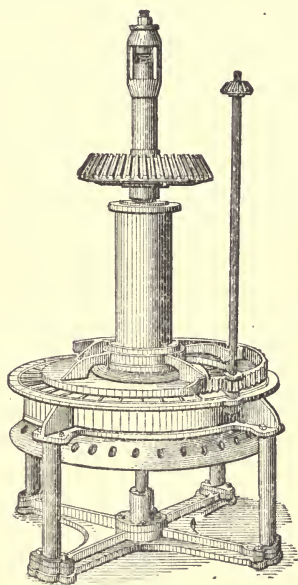


FIG. 277.

Girard Turbine for Low Falls.

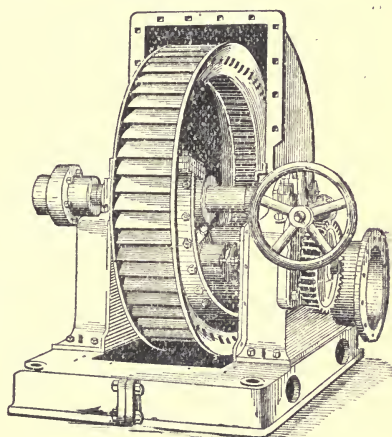


FIG. 278.

Girard Turbine for High Falls.

never be completely filled, and the water should flow through under a pressure which remains constant. In order to insure an unbroken flow through the wheel-passages and that no eddies are formed at the backs of the vanes, ventilating holes are arranged in the wheel sides, Fig. 280. Figs. 279 and 280 also show the relative path AB and the absolute path CD traversed by the water in an inward-flow and a downward-flow turbine.

If there is a sufficient head, the wheel may be placed clear

above the tail-water, when the stream will be at all times under atmospheric pressure. With low falls the wheel may be placed in a casing supplied with air from an air-pump by which the surface of the water may be kept at an invariable level below the outlet orifices, which is essential for perfectly free deviation. While the wheel-passages of a reaction turbine

should be kept completely filled with water, no such restriction is necessary with an impulse turbine. The supply may be partially checked and the water may be received by one or

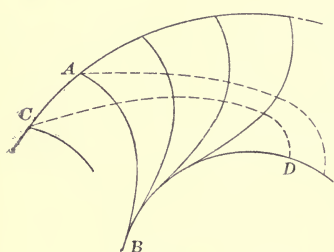


FIG. 279.

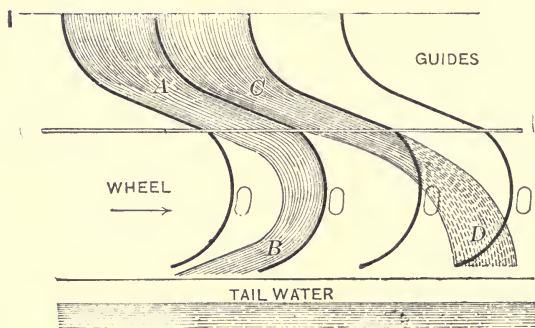


FIG. 280.

more vanes without affecting the efficiency. Thus the dimensions of an impulse turbine may vary between very wide limits, so that for high falls with a small supply a comparatively large wheel with low speed may be employed. The speed of a reaction turbine under similar conditions would be disadvantageously great, and any considerable increase of the diameter would largely increase the fluid friction and would also render the proper proportioning of the vane-angles almost impracticable. Impulse turbines may have complete or partial admission, while in reaction turbines the admission should be always

complete, as in Fig. 281, which shows the relative path AB and absolute path CD traversed by the water. When there is an ample supply of water the reaction turbine is usually to be preferred, but on very high falls its speed becomes incon-

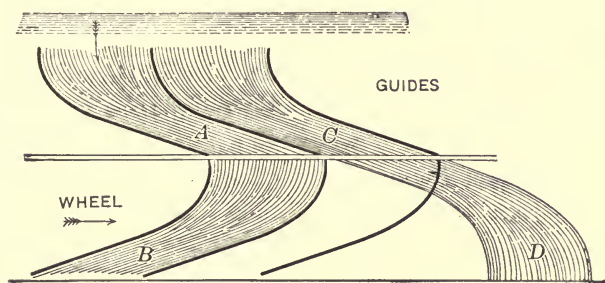


FIG. 281.

veniently great and it is then better to adopt a turbine of the impulse type. The diameter of the wheel can then be increased and the speed proportionately diminished.

The *Hurdy-gurdy* is the name popularly given to an impulse wheel which was introduced into the mining districts of California about the year 1865. Around the periphery of the wheel is arranged a series of flat iron buckets, about 4 to 6 ins. in width, which are struck normally by a jet of water often not more than three eighths of an inch in diameter. Theoretically the efficiency of such an arrangement cannot exceed 50 per cent, while in practice it rarely reaches 40 per cent. The best speed of the wheel, in accordance with both theory and practice, is about one half of that of the jet. Although the efficiency is so low, the wheel found great favor for many reasons. Any required speed could be obtained by a suitable choice of diameter; the plane of the wheel could be placed in any convenient position; the wheel could be cheaply constructed and was largely free from liability to accident. Hence it was of the utmost importance to increase, if possible, the efficiency of a wheel possessing such advantages. Obviously

a first step was to substitute cups for the flat buckets, the immediate result necessarily being a very large increase in the efficiency. This was increased still further by the adoption of double buckets, Fig. 282, that is, curved buckets divided in the middle so that the water is equally deflected on both sides.

Thus developed, the wheel is widely and most favorably known as the Pelton wheel, Fig. 282. Its efficiency is at least 80 per cent, and it is claimed that it often rises above 90 per cent. The power of the wheel does not depend upon its diameter, but upon the available quantity and head of water. The water passes to the wheel through one or more nozzles,

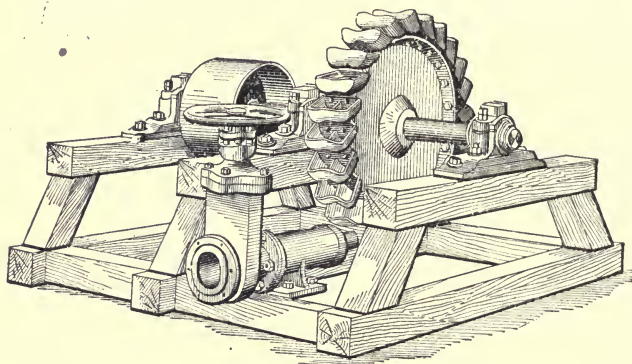


FIG. 282.

having tips bored to suit any required delivery. These tips are screwed into the nozzles and can be easily and rapidly replaced by others of larger or smaller size, so that the Pelton is especially well adapted for a varying supply of water. It is claimed that in this manner the power may be varied from a maximum down to 25 per cent of the same without appreciable loss of efficiency.

2. Actual Path of a Fluid Particle in Passing through a Turbine.—Under the combined effect of the inlet velocity v_1 and the rotation of the wheel, a fluid particle, entering at a , will traverse an actual path af cutting the outlet surface at an

angle equal to δ . This path may be approximately plotted in the following manner:

Let $af, a'f'$ be two consecutive blades.

Let q be the discharge per second through the passage between these blades.

Let xx' be a surface concentric with aa' and of radius r .

Let t be the time in seconds in which a fluid particle flows from aa' to xx' .

Let A be the area $axx'a'$.

Let d be the mean depth of this area between the crowns.

Let ω be the angular velocity of the wheel.

Then $Qt = \text{volume of water between } aa' \text{ and } xx' = Ad$.

But in the same time t the point x will have moved to z , where

$$xz = r\omega t = r\omega d \frac{A}{q}.$$

In this equation the values of ω and q are known, so that by describing any required number of cylindrical surfaces and introducing into the equation the corresponding values of r, d , and A , a series of values will be obtained for xz defining the points $z_1, z_2, z_3 \dots$ on the actual path al of the fluid particles.

Zeuner gives a somewhat more general method as follows:

Consider a fluid particle moving along the axis RM of the passage between two consecutive vanes af and $a'f'$.

If the wheel were at rest, the particle in t seconds would reach a certain point M , but the rotation of the wheel carries it to M' , where $MM' = r\omega t$, r being the radius $OM (= OM')$

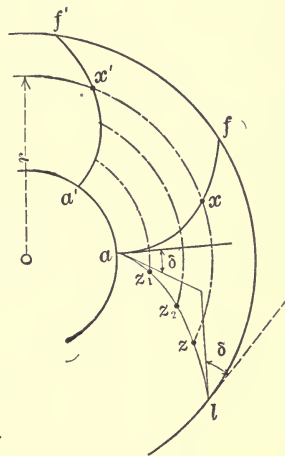


FIG. 283.

and ω the constant angular velocity about the axis at O . Thus, after t seconds, M' is the true locus of the fluid particle.

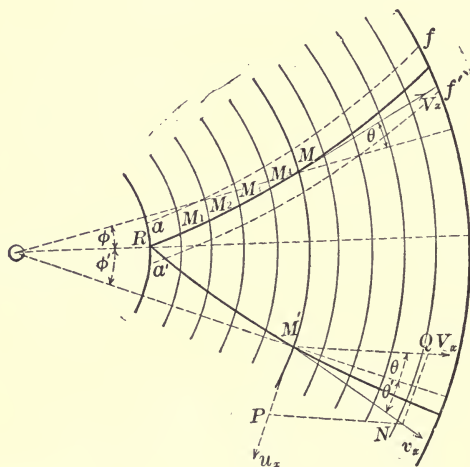


FIG. 284.

Let ϕ and ϕ' be the angular deviations of OM and OM' from OR .

Let $u_x (= M'P)$ be the linear velocity of M' .

Let $V_x (= M'Q)$ be the relative velocity of the fluid particle at M' .

Let $v_x (= M'N)$ be the absolute velocity of the fluid particle at M' .

Let θ be the angle between V_x and the radius OM or OM' .

Let θ' be the angle between v_x and OM' .

Then

$$v_x \sin \theta' = u_x - V_x \sin \theta$$

and

$$v_x \cos \theta' = V_x \cos \theta.$$

Hence

$$\frac{u_x}{V_x \cos \theta} = \tan \theta + \tan \theta'.$$

But $u_x = r\omega$; $\tan \theta = r \frac{d\phi}{dr}$; $\tan \theta' = r \frac{d\phi'}{dr}$, since $M'N$ is

necessarily tangential to the actual path RM' at M' ; and $A_x V_x \cos \theta = Q$, the volume of flow per second, A_x being the sectional area of the passage at right angles to OM . Substituting these values in the last equation,

$$\frac{\omega}{Q} A_x = \frac{d\phi}{dr} + \frac{d\phi'}{dr},$$

and therefore

$$\frac{\omega}{Q} \int_{r_1}^r A_x dr = \phi + \phi',$$

r_1 being the internal radius of the wheel.

But the expression $\int_{r_1}^r A dr$ is the volume of the passage between aa' and M and may be determined by actual measurement.

Designating this volume by U_x , then,

$$\frac{\omega}{Q} U_x = \phi + \phi',$$

an equation giving ϕ when ϕ' is known, or ϕ' when ϕ is known. Thus the actual position of the particle can be determined if its relative position is known, or its relative position can be found when its actual position is given.

Take a number of equidistant points $M_1, M_2, M_3 \dots$ along the axis of the passage, and let $\phi_1, \phi_2, \phi_3 \dots$ be the angular deviations of $OM_1, OM_2, OM_3 \dots$ from OR .

Also, let $U_1, U_2, U_3 \dots$ be the volumes of the passage between aa' and M_1, aa' and M_2, aa' and $M_3 \dots$. Then the angular deviations $\phi_1', \phi_2', \phi_3' \dots$ of the radii to the corresponding points $M_1', M_2', M_3' \dots$ on the actual path, are given by the equations

$$\frac{\omega}{Q} U_1 = \phi_1 + \phi_1',$$

$$\frac{\omega}{Q} U_2 = \phi_2 + \phi_2',$$

$$\frac{\omega}{Q} U_3 = \phi_3 + \phi_3',$$

$$\begin{array}{ccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

and the actual path can be at once plotted.

The value of $\int_{r_1}^r A_x dr$ can easily be found graphically. Thus, plot the radii $OR, OM_1, OM_2 \dots OM$ as abscissæ, and the corresponding sectional areas of the passage at $R, M_1, M_2 \dots M$ as ordinates. Joining the upper ends of these ordinates by a suitable curve, the area between this curve, the extreme ordinates and the line of abscissæ is evidently the volume required. This area may be determined with a planimeter.

3. Classification of Turbines.—The character of the construction of turbines has led to their being classified as (1) Radial-flow turbines; (2) Axial-flow turbines; (3) Mixed-flow turbines.

The water may act wholly by pressure or wholly by impulse, or partly by pressure and partly by impulse, or by reaction. In pressure wheels the water-passages are not com-

pletely filled as in reaction wheels. In impulse wheels the water spreads out in all directions, while in pressure and reaction wheels the water flows off on one side only.

In *Radial-flow* turbines the water flows through the wheel in a direction at right angles to the axis of rotation and approximately radial. The two special types of this class are the *Outward-flow* turbine, invented by Fourneyron, and the *Inward-flow* or *Vortex* turbine, invented by James Thomson. In the outward-flow turbine, Figs. 285 and 286, the water enters a cylindrical chamber and is led by means of fixed guide-blades outwards from the axis. It is distributed over the inlet-surface, passes through the curved passages of an annular wheel closely surrounding the chamber, and is finally discharged at the outer surface. The wheel works best when it is placed clear above the tail-water. A serious practical defect is the difficulty of constructing a suitable sluice for regulating the supply over the inlet-surface. When the water is insufficient to work the turbine at its full power, the exit openings may be closed to any required extent by lowering a cylindrical sluice.

A well-designed turbine of this type gives an efficiency of 70 per cent, and the maximum efficiency is about 80 per cent, but the efficiency is considerably diminished by closing the sluice. Fourneyron was led to the design of this turbine by observing the excessive loss of energy in the ordinary Scotch turbine, or reaction wheel, and introduced guide-blades in order to give the water an initial forward velocity and thus cause a diminution of the velocity of the water leaving the outlet-surface.

Boyden's turbine is a modification of the Fourneyron. The water is conducted to the guide-blades, which are inclined so as to receive the water tangentially, through a truncated cone; and the water thus acquires a gradually increasing velocity together with a spiral motion. The wheel, again, is surrounded by a *diffusor* which expands outwardly and which

should be completely submerged. The water then flows through the wheel with an increased velocity and passes away

FIG. 285.

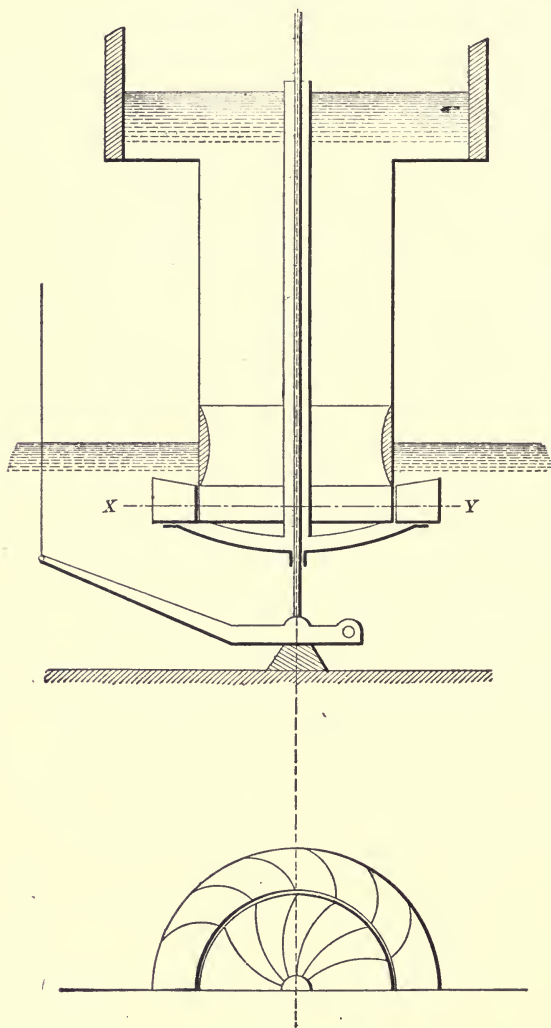
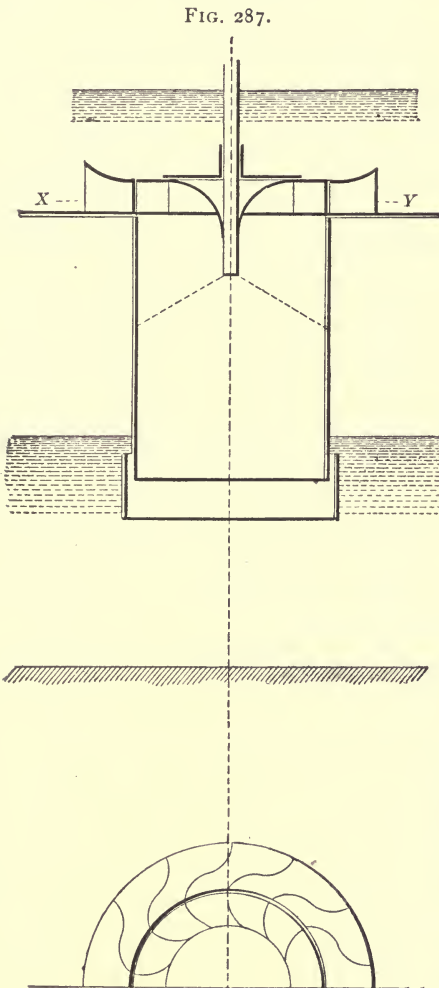


FIG. 286.

through the diffuser with a velocity which gradually diminishes. There is said to be a gain of 3 per cent effected by this arrange

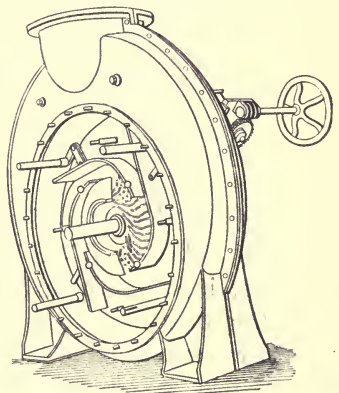
ment, while Boyden claimed for his 75-H.P. turbine an efficiency of 88 per cent.

In the *Inward-flow* or *Vortex* turbine, Figs. 287, 288, and



289, the wheel is enclosed in an annular space, into which the water flows through one or more pipes, and is usually distributed over the inlet-surface of the wheel by means of four guide-blades. The water enters the wheel, flows towards the space around the axis, and is there discharged. This turbine possesses the great advantage that there is ample space outside the

Thomson's Vortex Turbine.



wheel for a perfect system of regulating-sluiques. This turbine has attained an efficiency of $77\frac{1}{2}$ per cent.

Axial-flow turbines, Fig. 290, are also known as *Parallel-* and *Downward-flow* turbines and are sometimes called by the names of the inventors, Jonval and Fontaine. In these the water passes downward through an annular casing in a direction parallel to the axis of rotation, and is distributed by means of guide-blades over the inlet-surface of an adjacent wheel. It enters the wheel-passages and is finally discharged vertically, or nearly so, at the outlet-surface. The sluice-regulations are worse even than in the case of an outward-flow turbine, but there is this advantage, that the turbine may be placed either below the tail-water, or, if supplied with a suction-pipe, at any point not exceeding 30 ft. above the tail-water.

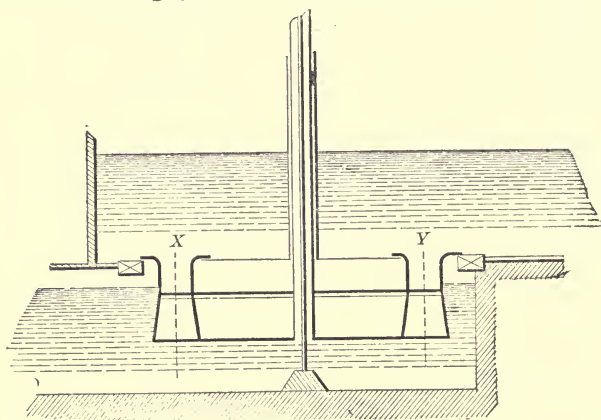


FIG. 290.

If a turbine is designed so that the pressure at the clearance between the casing and the wheel is nil, and with curved passages in the form of a freely deviated stream, it becomes what is called a *Limit* turbine. In its normal condition of working it is an *Impulse* turbine, but when drowned it is a *Reaction* turbine, with a small pressure at the clearance. For moderate falls with a varying supply its average efficiency is higher than that of a pressure turbine.

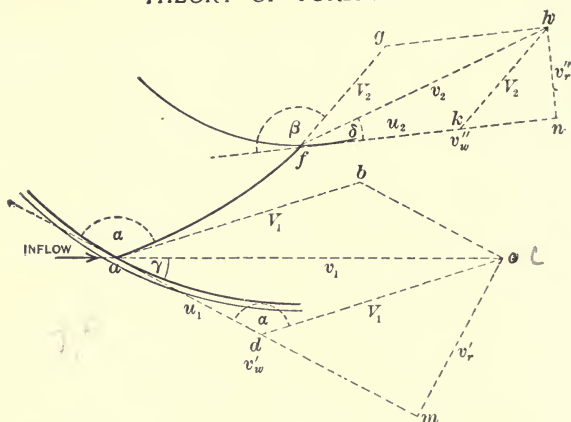
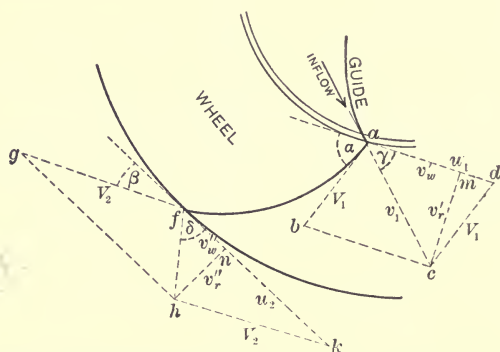
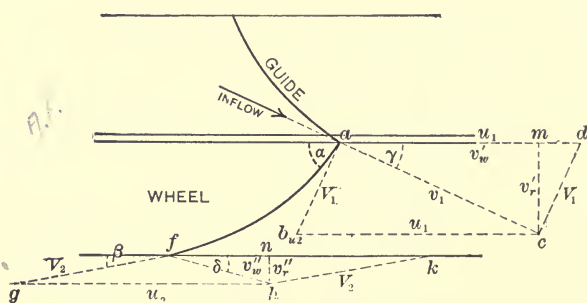
The *Mixed-* or *Combined-flow* (Schiele) turbine is a combination of the radial and axial types. The water enters in a

nearly radial direction and leaves in a direction approximately parallel to the axis of rotation. This type of turbine admits of a good mode of regulation and is cheap to construct.

The Swain turbine is a combination of the inward- and axial-flow types. The vane inlet-lips are vertical opposite the guide-blades, and at the outlet the vanes are bent into a quadrant of a circle. An efficiency of 88 per cent has been claimed for this turbine under a full load.

Comparison of Outward-flow Turbines.—Fourneyron deals with a varying supply of water by means of a circular sluice, which can be made to close off any required portion of the wheel. A similar arrangement may be added to the Cadiat turbine, which is of the outward-flow type and is fed from above through a cylindrical reservoir, the upper and lower edges of the reservoir being rounded to diminish the loss due to contraction. The objection to sluices of this kind is that the passages no longer run full when the inlet orifices are partially closed and there is therefore a considerable diminution of efficiency. In the Whitelaw turbine, p. 375, this difficulty can be obviated by changing the outlet instead of the inlet area.

The absence of guides in the Cadiat and Whitelaw turbines make their construction somewhat simpler, but their efficiency is comparatively small, that of the Cadiat being about 65 per cent, while the efficiency of the Whitelaw turbine varies from 50 to 60 per cent. On the other hand, the Fourneyron turbine has an efficiency of more than 70 per cent and is mechanically a much more perfect machine. The guides in the turbine render it possible to utilize almost the whole of the energy of the water either by equalizing the peripheral and relative speeds at the outlet, or by making the absolute velocity at the outlet radial. The Fourneyron and Cadiat turbines are specially adapted for a large supply of water and a moderate fall, say not exceeding about 30 ft., while the Whitelaw turbines are found more useful for a small supply of water and a high fall.

FIG. 291.—Enlarged Portion of Section through XY , Fig. 287.FIG. 292.—Enlarged Portion of Section through XY , Fig. 285.FIG. 293.—Enlarged Portion of a Cylindrical Section XY , Fig. 290, Developed in Plane of Paper.

4. Theory of Turbines (Figs. 291, 292, and 293).—Denote inward-flow, outward-flow, and axial-flow turbines by I. F., O. F., and A. F., respectively.

Let r_1, r_2 be the radii of the wheel inlet- and outlet-surfaces of an I. F. or O. F.

Let r_1, r_2 be the outer and inner radii of the wheel inlet-surface of an A. F.

Let R be the mean radius $\left(= \frac{r_1 + r_2}{2} \right)$ of an A. F., assumed constant throughout.

Let A_1, A_2 be the areas of the wheel inlet- and outlet-orifices.

Let d_1, d_2 be the depths of the same in an I. F. or O. F.

Let d_1, d_2 be the widths of the same in an A. F.

Let h be the depth of the wheel in an A. F.

Let H_1 be the effective head over the inlet-surface of the wheel. This is the total head over the inlet-surface diminished by the head consumed in frictional resistance in the supply-channel, and by the head lost in bends, sudden changes of section, etc.

Then $H_1 + h$ is the total head over the outlet of an A. F. available for work.

Let H_2 be the fall from the outlet-surface to the surface of the water in the tail-race. If the turbine is submerged, then H_2 is negative.

Let v_1, v_2 be the absolute velocities of the water at the inlet- and outlet-surfaces.

Let u_1, u_2 be the absolute velocities of the inlet- and outlet-surfaces. In an A. F. turbine $u_1 = u_2$.

Let V_1, V_2 be the velocities of the water relatively to the wheel at the inlet- and outlet-surfaces.

Let the angular velocity of the wheel $= \omega = \frac{u_1}{r_1} = \frac{u_2}{r_2}$.

Let η designate the hydraulic efficiency of the turbine.

Let the water enter the wheel in the direction ac , making an angle γ with the tangent ad . Take ac to represent v_1 , and ad to represent u_1 . Complete the parallelogram bd . The side ab represents V_1 , and in order that there may be *no shock at entrance*, ab must be tangential to the vane at a . Again, at f draw fg , a tangent to the vane, and fk , a tangent to the wheel's periphery.

Take fg and fk to represent V_2 and u_2 respectively. Complete the parallelogram gk . The diagonal fh must represent in direction and magnitude the absolute velocity v_2 with which the water leaves the wheel. Let the angle $hfk = \delta$.

Draw cm perpendicular to ad , and hn perpendicular to gk .

The *tangential* component, viz., am or fn , of the velocity of the water as it enters or leaves the wheel is termed *velocity of whirl* (v_w).

The *radial* component, viz., cm or hn , of the velocity of the water as it enters or leaves the wheel is termed *velocity of flow* (v_r).

$$\begin{aligned}\text{Take} \quad v_w' &= am, & v_w'' &= fn, \\ v_r' &= cm, & v_r'' &= hn.\end{aligned}$$

Let the angle $bad = 180^\circ - \alpha$.

Let the angle $gfk = 180^\circ - \beta$.

Thus α and β are the angles which the vane (or blade) tips (or lips) make with the wheel's peripheries.

Then, at the inlet-surface,

$$v_w' = v_1 \cos \gamma = ac \cos \gamma = am = ad \pm dm = u_1 - V_1 \cos \alpha, \quad (1)$$

$$v_r' = v_1 \sin \gamma = cm = V_1 \sin \alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and at the outlet-surface,

$$v_w'' = v_2 \cos \delta = fn = fk \pm kn = u_2 - V_2 \cos \beta, \quad . \quad (3)$$

$$v_r'' = v_2 \sin \delta = hn = V_2 \sin \beta. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Let Q be the quantity of water which passes per second through the turbine. Then, disregarding the thickness of the vanes, in an *I. F. or O. F. turbine*

$$v_r' A_1 = v_r' \cdot 2\pi r_1 d_1 = Q = v_r'' \cdot 2\pi r_2 d_2 = v_r'' A_2, \quad (5)$$

and therefore

$$v_r' r_1 d_1 = \frac{Q}{2\pi} = v_r'' r_2 d_2.$$

Also, if $d_1 = d_2 = d$,

$$v_r' r_1 = \frac{Q}{2\pi d} = v_r'' r_2.$$

In an *A. F. turbine*

$$v_r' A_1 = v_r' \cdot 2\pi R \cdot d_1 = Q = v_r'' \cdot 2\pi R \cdot d_2 = v_r'' A_2, \quad (6)$$

and therefore

$$v_r' d_1 = \frac{Q}{2\pi R} = v_r'' d_2.$$

If $d_1 = d_2 = d$,

$$v_r' = \frac{Q}{2\pi R d} = v_r''.$$

Allowance may be made for vane thickness as follows:

Let θ be the angle between the vane of thickness BC and the wheel's periphery AB . Then the space occupied by the vane along the wheel's periphery is $AB = BC \operatorname{cosec} \theta$.

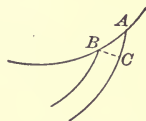


FIG. 294.

Let n be the number of the guide-vanes, and t their thickness.

Let n_1 be the number of the wheel-vanes, and t_1, t_2 their thickness at the inlet- and outlet-surfaces respectively.

Then, in a radial-flow turbine,

$$A_1 = \frac{Q}{v_r'} \{ 2\pi r_1 - nt \operatorname{cosec} \gamma - n_1 t_1 \operatorname{cosec} \alpha \} \quad (7)$$

and

$$A_2 = \frac{9}{10} d_2 \{ 2\pi r_2 - n_1 t_2 \operatorname{cosec} \beta \}, \quad . \quad . \quad . \quad . \quad (8)$$

$\frac{9}{10}$ being a fraction depending on practical considerations.

In an axial-flow turbine R is to be substituted for r_1 and r_2 in the values of A_1 and A_2 .

n_1 may be made equal to $n + 1$ or $n + 2$.

Work and Efficiency.—As the water flows through the wheel, let v be the velocity of flow at any point N distant r

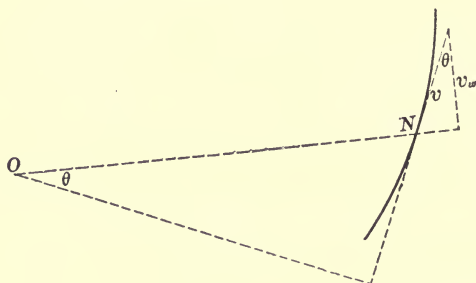


FIG. 295.

(= ON) from the axis O , and let p be the length of the perpendicular from O upon the direction of v . Then

$$\begin{aligned} \frac{wQ}{g}v &= \text{momentum of moving mass of water} \\ &= \text{impulse on wheel} \\ &= F, \text{ suppose.} \end{aligned}$$

Therefore, also,

$$\frac{wQ}{g}vp = Fp = \text{moment of couple producing rotation,}$$

and the useful work of the couple per second

$$= Fp\omega = \frac{wQ}{g}vp\omega.$$

But if v_w is the component of v at N perpendicular to the radial line ON ,

$$v_w = v \cos \theta = \frac{vp}{r},$$

and therefore the useful work of the couple per second

$$= \frac{wQ}{g} v_w r \omega.$$

Thus in an *I. F.* or *O. F.* turbine

$$\text{the useful effect at inlet} = \frac{wQ}{g} v_w' r_1 \omega, = \frac{wQ}{g} v_w' u_1,$$

$$\text{the useful effect at outlet} = \frac{wQ}{g} v_w'' r_2 \omega, = \frac{wQ}{g} v_w'' u_2,$$

and the **USEFUL WORK** per second done by the water on the wheel between inlet and outlet

$$= \frac{wQ}{g} (v_w' r_1 - v_w'' r_2) \omega, \quad . \quad . \quad . \quad . \quad (9)$$

$$= \frac{wQ}{g} (v_w' u_1 - v_w'' u_2). \quad . \quad . \quad . \quad . \quad (10)$$

The **EFFICIENCY** is given by the relation

$$\eta \times wQH_1 = \text{the useful work per sec.}$$

$$= \frac{wQ}{g} (v_w' u_1 - v_w'' u_2),$$

or

$$\eta g H_1 = v_w' u_1 - v_w'' u_2, \quad . \quad . \quad . \quad . \quad (11)$$

which is the fundamental equation governing the design of *I. F.* or *O. F.* turbines.

In an A. F. turbine

$$\text{the useful effect at inlet} = \frac{wQ}{g} v_w' R \omega = \frac{wQ}{g} v_w' u,$$

$$\text{the useful effect at outlet} = \frac{wQ}{g} v_w'' R \omega = \frac{wQ}{g} v_w'' u,$$

and the USEFUL WORK per second done by the water on the wheel between inlet and outlet

$$= \frac{wQ}{g}(\mathbf{v}_w' - \mathbf{v}_w'')R\omega, \quad . \quad . \quad . \quad . \quad (12)$$

$$= \frac{wQ}{g}(\mathbf{v}_w' - \mathbf{v}_w'')u_1. \quad . \quad . \quad . \quad . \quad (13)$$

The efficiency is given by the relation

$$\begin{aligned} \eta \times wQ(H_1 + h) &= \text{the useful work per sec.} \\ &= \frac{wQ}{g}(v_w' - v_w'')u_1, \end{aligned}$$

or

$$\eta g(\mathbf{H}_1 + \mathbf{h}) = (\mathbf{v}_w' - \mathbf{v}_w'')u_1, \quad . \quad . \quad . \quad . \quad (14)$$

which is the fundamental equation governing the design of A. F. turbines.

Again, *disregarding hydraulic resistances*, each pound of water on leaving the turbine carries away $\frac{v_2^2}{2g}$ ft.-lbs. of energy.

Hence

the USEFUL WORK in an I. F. or O. F. turbine

$$= wQ\left(\mathbf{H}_1 - \frac{\mathbf{v}_2^2}{2g}\right), \quad . \quad . \quad . \quad . \quad (15)$$

the corresponding EFFICIENCY being $1 - \frac{\mathbf{v}_2^2}{2g\mathbf{H}_1}$, $. \quad . \quad (16)$

and the USEFUL WORK in an A. F. turbine

$$= wQ\left(\mathbf{H}_1 + \mathbf{h} - \frac{\mathbf{v}_2^2}{2g}\right), \quad . \quad . \quad . \quad . \quad (17)$$

the corresponding EFFICIENCY being $1 - \frac{\mathbf{v}_2^2}{2g(\mathbf{H}_1 + \mathbf{h})}$. (18)

Assuming that the velocity of whirl at outlet, viz., v_w'' , is nil and that H is the portion of H_1 , or of $H_1 + h$, which is transformed into useful work, then

$$gH = u_1 v_w' = u_1(u_1 - v_r' \cot \alpha),$$

which may be written in the form

$$1 = \frac{u_1}{\sqrt{gH}} \left(\frac{u_1}{\sqrt{gH}} - \frac{v_r'}{\sqrt{gH}} \cot \alpha \right),$$

a quadratic giving

$$\frac{u_1}{\sqrt{gH}} = \frac{v_r'}{\sqrt{gH}} \frac{\cot \alpha}{2} + \sqrt{\left(\frac{v_r'}{\sqrt{gH}} \right)^2 \frac{\cot^2 \alpha}{4} + 1}.$$

This result has been employed in preparing the following Table of values of $\frac{u_1}{\sqrt{gH}}$ corresponding to different values of $\frac{v_r'}{\sqrt{gH}}$ and of α :

$\frac{v_r'}{\sqrt{gH}}$	$\alpha =$										
	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
1.0	3.983	2.189	1.618	1.329	1.142	I	.874	.752	.618	.457	.251
0.9	3.629	2.047	1.547	1.188	1.128	I	.887	.773	.647	.488	.271
0.8	3.289	1.909	1.477	1.090	1.114	I	.900	.795	.677	.524	.304
0.7	2.952	1.776	1.409	1.225	1.095	I	.908	.819	.709	.563	.339
0.6	2.621	1.647	1.344	1.188	1.083	I	.923	.842	.744	.607	.382
0.5	2.301	1.523	1.281	1.155	1.069	I	.935	.886	.781	.657	.435
0.45	2.145	1.463	1.250	1.139	1.062	I	.942	.879	.800	.683	.465
0.4	1.991	1.405	1.220	1.122	1.055	I	.949	.890	.820	.712	.499
0.35	1.847	1.346	1.190	1.107	1.048	I	.954	.902	.840	.744	.541
0.3	1.705	1.293	1.161	1.090	1.041	I	.961	.917	.860	.773	.586
0.25	1.569	1.240	1.132	1.074	1.034	I	.967	.930	.882	.806	.636
0.2	1.440	1.188	1.105	1.059	1.027	I	.973	.943	.905	.842	.694
0.15	1.318	1.138	1.078	1.044	1.020	I	.980	.966	.927	.878	.758
0.10	1.204	1.090	1.051	1.029	1.013	I	.987	.972	.951	.917	.830

Allowance may be made for the principal hydraulic resistances by taking

$f_2 \frac{v_1^2}{2g}$ as the loss of head before entering the wheel and

$f_4 \frac{V_2^2}{2g}$ as the loss of head before entering in the wheel-passages.

Then the total loss of head

$$= f_2 \frac{v_1^2}{2g} + f_4 \frac{V_2^2}{2g} + \frac{v_2^2}{2g} \quad \dots \quad (19)$$

The values of the empirical coefficients f_2 and f_4 may vary, the former from .025 to .20, and the latter from .10 to .20.

Ex. 1. Water enters an O. F. turbine of $3\frac{1}{2}$ ft. exterior and $1\frac{1}{4}$ ft. interior diameter with a whirling velocity of 20 ft. per second, and leaves in the reverse direction with a whirling velocity of 10 ft. per second. The wheel makes 240 revolutions per minute. Find the useful head.

$$u_1 = \frac{\pi \cdot 1\frac{1}{4} \cdot 240}{60} = 22 \text{ ft. per sec.,}$$

$$u_2 = 2u_1 = 44 \text{ ft. per sec.}$$

Then, if H is the useful head,

wQH = work done in driving the wheel

$$= \frac{wQ}{g} (u_1 v_w' - u_2 v_w'')$$

$$= \frac{wQ}{g} (22 \times 20 - 44 \times (-10)) = wQ \cdot \frac{880}{32},$$

and $H = 27\frac{1}{2}$ ft.

Ex. 2. A turbine with a radial inlet-lip receives 10 cu. ft. of water per second at a radius of 2 ft., and makes 105 revolutions per minute. The water enters at 60° with the wheel's periphery, and leaves without velocity of whirl. If the efficiency is .88, find the effective head and the H.P. of the turbine.

Since $\alpha = 90^\circ$,

$$v_w' = u_1 = \frac{\pi \cdot 4 \cdot 105}{60} = 22 \text{ ft. per sec.,}$$

and

$$.88 = \text{the efficiency} = \frac{u_1 v_w'}{g H_1} = \frac{u_1^2}{32 H_1} = \frac{484}{32 H_1},$$

or

$$H_1 = 17.1875 \text{ ft.}$$

$$\text{The H.P.} = \frac{62\frac{1}{2} \cdot 10}{550} \times .88 H_1 = 17.1875.$$

Ex. 3. The wheel of an A. F. turbine of 3 ft. interior diameter has a 6-in. width of orifice opening and is 1 ft. deep. It passes 33 cu. ft. of water per second under the head of 24 ft. over the inlet, and the water leaves the wheel in a direction given by $\text{cosec } \delta = 1.015$. Determine the efficiency.

By the condition of continuity,

$$\frac{\pi}{4}(4^2 - 3^2)v_r'' = 33 = 5\frac{1}{2}v_r'',$$

or

$$v_r'' = 6 \text{ ft. per sec.}$$

Therefore

$$v_2 = v_r'' \text{ cosec } \delta = 6 \times 1.015 = 6.09 \text{ ft. per sec.,}$$

and

$$\text{the efficiency} = 1 - \frac{v_2^2}{2g(24 + 1)} = 1 - \frac{(6.09)^2}{1600} = .9768.$$

$$\text{The H.P.} = \frac{62\frac{1}{2}}{32} \frac{33}{550} \times 25 \times .9768 = 2.929.$$

Ex. 4. Find the outlet lip-angle (β) from the following data: radius to inlet = *twice* that to outlet surface; linear speed of inlet surface = *one-half* that equivalent to the effective head; inlet *velocity of flow* = *one-eighth* of that equivalent to the effective head; sectional area of waterway is constant from inlet to outlet; the water leaves without velocity of whirl.

$$\text{Then } u_1 = \frac{1}{2} \sqrt{2gH_1} = 4 \sqrt{H_1} = 2u_2.$$

By condition of continuity,

$$A_1 v_r' = A_2 v_r'' = A_1 v_r,$$

and

$$v_2 = v_r = \frac{1}{8} \sqrt{2gH_1} = \sqrt{H_1}.$$

Hence

$$\cot \beta = \frac{u_2}{v_2} = \frac{2\sqrt{H_1}}{\sqrt{H_1}} = 2.$$

Ex. 5. The wheel of a turbine, passing 10 cu. ft. of water per second under a head of 32 ft., is 6 ins. deep and its inlet-surface has a diameter of 2 feet. The inlet-lip is radial and the efficiency may be assumed to be unity. Find the guide-vane lip-angle and the power of the turbine.

$$1 = \text{the efficiency} = \frac{u_1 v_w'}{32 \times 32} = \frac{u_1^2}{1024}.$$

Therefore $u_1 = 32 \text{ ft. per sec.} = v_w'.$

By condition of continuity,

$$\pi \cdot 2 \cdot \frac{1}{2} \cdot v_r' = 10,$$

or $v_r' = \frac{35}{11} = 3\frac{2}{11} \text{ ft. per sec.}$

Hence $\tan \gamma = \frac{v_r'}{u_1} = \frac{3\frac{2}{11}}{32} = .09943,$

and $\gamma = 5^\circ 41'.$

The H. P. $= 62\frac{1}{2} \cdot 10 \cdot \frac{32}{550} = 36\frac{4}{11}.$

Ex. 6. In an impulse radial-flow turbine the inlet- and outlet-orifice areas are equal and the water leaves without velocity of whirl. Disregarding hydraulic resistances, show that the velocity of whirl is $\cos^2 \gamma$.

By condition of continuity,

$$A_2 v_2 = A_3 v_r'' = Q = A_1 v_r' = A_1 v_1 \sin \gamma.$$

Therefore $v_2 = v_1 \sin \gamma,$

and

$$\text{the efficiency} = 1 - \frac{v_2^2}{v_1^2} = 1 - \sin^2 \gamma = \cos^2 \gamma.$$

Ex. 7. In a radial-flow impulse turbine the peripheral and relative speeds at outlet are equal. Show that the direction of the water at inlet bisects the angle between the rim and the inlet-lip. Also show that $r_1^2 d_1 \sin 2\gamma = r_2^2 d_2 \sin \beta$.

$$V_2^2 - V_1^2 = u_2^2 - u_1^2.$$

But $V_2 = u_2,$ and therefore $V_1 = u_1,$

so that $2\gamma = 180^\circ - \alpha.$

By condition of continuity,

$$2\pi v_1 d_1 v_r' = Q = 2\pi r_2 d_2 \cdot v_r'',$$

or $r_1 d_1 \cdot V_1 \sin \alpha = r_2 d_2 \cdot V_2 \sin \beta,$

or $r_1 d_1 u_1 \sin 2\gamma = r_2 d_2 \cdot u_2 \sin \beta = r_2 d_2 \cdot \frac{r_2}{r_1} u_1 \sin \beta.$

or $r_1^2 d_1 \sin 2\gamma = r_2^2 d_2 \sin \beta.$

Application of Torricelli's Principle.—If $\frac{p_1}{w}$, $\frac{p_2}{w}$ are the pressure-heads at the inlet- and outlet-surfaces of a turbine wheel, the effective head over the inlet-orifices is $H_1 - \frac{p_1 - p_2}{w}$.

Hence, *disregarding hydraulic resistances*,

$$\text{IN A REACTION TURBINE } \frac{v_1^2}{2g} = H_1 - \frac{p_1 - p_2}{w}. \quad (20)$$

In turbines of the *impulse* type $p_1 = p_2$, and the water is usually under atmospheric pressure only both at inlet and outlet. Thus

$$\text{IN AN IMPULSE TURBINE } \frac{v_1^2}{2g} = H_1. \quad (21)$$

Allowance may be made for the loss of head at entrance into the wheel by substituting $\frac{1}{c_v^2} \frac{v_1^2}{2g}$ for $\frac{v_1^2}{2g}$ in these two equations, the average value of the empirical coefficient c_v being about .949, or $c_v^2 = .9$.

Application of Bernoulli's Principle.

IN A REACTION I. F. OR O. F. TURBINE

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} - \frac{u_2^2 - u_1^2}{2g},$$

the last term being the work per pound of water due to centrifugal force. Therefore

$$\begin{aligned} \frac{V_2^2 - V_1^2}{2g} - \frac{u_2^2 - u_1^2}{2g} &= \frac{p_1 - p_2}{w} \quad (22) \\ &= H_1 - \frac{v_1^2}{2g}, \end{aligned}$$

or

$$\frac{v_1^2}{2g} + \frac{V_2^2 - V_1^2}{2g} - \frac{u_2^2 - u_1^2}{2g} = H_1, \quad (23)$$

which may also be written in the form

$$\frac{u_1 v_1 \cos \gamma}{g} + \frac{V_2^2 - u_2^2}{2g} = H_1, \quad . \quad . \quad . \quad (24)$$

since, from the triangle acd ,

$$2u_1 v_1 \cos \gamma = u_1^2 + v_1^2 - V_1^2. \quad . \quad . \quad . \quad (25)$$

IN AN IMPULSE I. F. OR O. F. TURBINE

$$p_1 = p_2, \quad \text{and} \quad v_1^2 = 2gH_1.$$

Therefore eq. (23) becomes

$$\frac{V_2^2 - V_1^2}{2g} = \frac{u_2^2 - u_1^2}{2g}. \quad . \quad . \quad . \quad (26)$$

IN AN I. F. TURBINE $u_1 > u_2$ and the term $\frac{u_2^2 - u_1^2}{2g}$ is

negative. Hence eq. (23) shows that as the inlet velocity v_1 increases or diminishes the speed of the turbine diminishes or increases, and that therefore the centrifugal force tends to maintain a steady motion. A diminution in v_1 also necessarily leads to a corresponding diminution in the loss of head due to hydraulic resistances. For these reasons the centrifugal head should be made as large as is practicable, and the ratio $\frac{u_1}{u_2} = \frac{r_1}{r_2}$ is usually made equal to 2.

IN AN O. F. TURBINE $u_1 < u_2$ and the term $\frac{u_2^2 - u_1^2}{2g}$ is *positive*. Hence the speed of the turbine increases and diminishes with v_1 , and the centrifugal force is adverse to steady motion, tending both to augment a variation from the normal speed and to increase frictional losses of head. The centrifugal head should therefore be made as small as is practicable, and a common value of the ratio $\frac{u_1}{u_2} = \frac{r_1}{r_2}$ is $\frac{4}{5}$.

IN A REACTION A. F. TURBINE each fluid particle in passing from inlet to outlet remains at the same distance from

the axis, and therefore no work is done by centrifugal force, but an additional head, h , equal to the depth of the wheel, is gained. Then

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} - h,$$

or

$$\frac{V_2^2 - V_1^2}{2g} = \frac{p_1 - p_2}{w} + h = H_1 + h - \frac{v_1^2}{2g}. \quad (27)$$

Therefore

$$\frac{v_1^2}{2g} + \frac{V_2^2 - V_1^2}{2g} = H_1 + h, \quad (28)$$

which may also be written in the form

$$\frac{u_1 v_1 \cos \gamma}{g} + \frac{V_2^2 - u_2^2}{2g} = H_1 + h, \quad (29)$$

since $u_1 = u_2$.

IN AN IMPULSE A. F. TURBINE

$$p_1 = p_2, \quad \text{and} \quad v_1^2 = 2gH_1.$$

Therefore eq. (28) becomes

$$\frac{V_2^2 - V_1^2}{2g} = h. \quad (30)$$

In order to secure the advantages of centrifugal force, Belanger proposed that the wheel-passages should be so formed that the path of a fluid particle would gradually approach the axis of rotation.

Lip (or Tip) Angles.—The angles α and β which the wheel-blade tips at inlet and outlet make with the wheel's peripheries are generally obtained as follows:

From the triangle acd ,

$$\frac{\sin(\alpha + \gamma)}{\sin \alpha} = \frac{u_1}{v_1} = \cos \gamma + \cot \alpha \sin \gamma,$$

and therefore

$$\cot(180^\circ - \alpha) = -\cot \alpha = \cot \gamma - \frac{u_1}{v_1} \operatorname{cosec} \gamma. \quad (31)$$

From the triangle fkh ,

$$\frac{\sin (\beta + \delta)}{\sin \beta} = \frac{u_2}{v_2} = \cos \delta + \cot \beta \sin \delta,$$

and therefore

$$\cot (180^\circ - \beta) = -\cot \beta = \cot \delta - \frac{u_2}{v_2} \operatorname{cosec} \delta. \quad (32)$$

Conditions Governing the Efficiency of Turbines. — The whole of the water's energy should, if possible, be employed in doing useful work on the wheel, and the water should therefore leave the wheel without velocity, or v_2 should be nil. This condition cannot of course be realized in practice, as no water would then pass through the wheel and consequently no work could be done. For purposes of efficiency it is usual to make v_2 small by adopting one of the following hypotheses:

EITHER *that the velocity of whirl at outlet is nil,*

OR *that at the outlet the relative velocity of the water and the peripheral linear velocity of the wheel are equal.*

FIRST consider the hypothesis "that the velocity of whirl at outlet is nil." Then

$$v_{w2}'' = 0. \quad \dots \dots \dots (33)$$

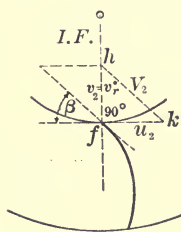


FIG. 296.

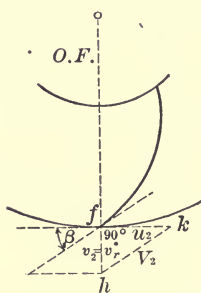


FIG. 297.

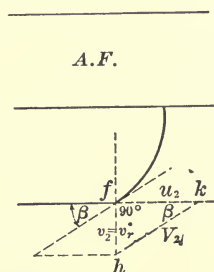


FIG. 298.

Thus the direction of v_2 is radial in an I. F. or O. F. turbine, Figs. 296 and 297, and vertical in an A. F. turbine,

Fig. 298, and therefore the angle hfk ($= \delta$) in the outlet triangle of velocities must be a right angle. Hence

$$v_2 = v_r'' = u_2 \tan \beta = V_2 \sin \beta, \quad . \quad . \quad . \quad (34)$$

and

$$V_2^2 - u_2^2 = v_2^2 = u_2^2 \tan^2 \beta. \quad . \quad . \quad . \quad (35)$$

Also, eq. (5) gives

$$v_1 \sin \gamma A_1 = v_2 A_2 = u_2 \tan \beta A_2. \quad . \quad . \quad . \quad (36)$$

General Deductions.

IN AN I. F. OR O. F. TURBINE

$$\frac{u_1}{r_1} = \frac{u_2}{r_2} = \omega.$$

Also, disregarding blade thickness,

$$A_1 = 2\pi r_1 d_1 \text{ and } A_2 = 2\pi r_2 d_2.$$

Relation between the lip-angles.

By eq. (36) and the triangle acd ,

$$\frac{r_1^2 d_1 \sin \gamma}{r_2^2 d_2 \tan \beta} = \frac{u_1}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha}, \quad (37)$$

or

$$\frac{r_1^2 d_1}{r_2^2 d_2} \cot \beta = \cot \gamma + \cot \alpha. \quad . \quad (38)$$

IN AN A. F. TURBINE

$$u_1 = u_2 = R'\omega.$$

Also, disregarding blade thickness,

$$A_1 = 2\pi R d_1 \text{ and } A_2 = 2\pi R d_2.$$

Relation between the lip-angles.

By eq. (36) and the triangle acd ,

$$\frac{d_1 \sin \gamma}{d_2 \tan \beta} = \frac{u_1}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha}, \quad (37)$$

or

$$\frac{d_1}{d_2} \cot \beta = \cot \gamma + \cot \alpha. \quad (38)$$

REACTION TURBINES.

IN AN I. F. OR O. F. TURBINE.

Speed of turbine.

By eqs. (24), (35), (37),

$$u_2^2 = \frac{2gH_1 \cot \beta}{\tan \beta + 2\frac{d_2}{d_1} \cot \gamma}. \quad (39)$$

Velocity of efflux.

$$\begin{aligned} v_2^2 &= u_2^2 \tan^2 \beta \\ &= \frac{2gH_1 \tan \beta}{\tan \beta + 2\frac{d_2}{d_1} \cot \gamma}. \quad (40) \end{aligned}$$

IN AN A. F. TURBINE.

Speed of turbine.

By eqs. (28), (35), (37),

$$u_2^2 = \frac{2g(H_1 + h) \cot \beta}{\tan \beta + 2\frac{d_2}{d_1} \cot \gamma}. \quad (39)$$

Velocity of efflux.

$$\begin{aligned} v_2^2 &= u_2^2 \tan^2 \beta \\ &= \frac{2g(H_1 + h) \tan \beta}{\tan \beta + 2\frac{d_2}{d_1} \cot \gamma}. \quad (40) \end{aligned}$$

Amount Q of water passing through the turbine per second, blade thickness being disregarded.

$$Q = 2\pi r_2 d_2 v_r'' = 2\pi r_2 d_2 v_2$$

$$= 2\pi r_2 d_2 \sqrt{\frac{2g H_1 \tan \beta}{\tan \beta + 2 \frac{d_2}{d_1} \cot \gamma}}. \quad (41)$$

The useful work (disregarding hydraulic resistances)

$$= \omega Q \left(H_1 - \frac{v_2^2}{2g} \right)$$

$$= \frac{\omega Q H_1}{1 + \frac{1}{2} \frac{d_1}{d_2} \tan \beta \tan \gamma}. \quad (42)$$

The corresponding efficiency

$$= \frac{1}{1 + \frac{1}{2} \frac{d_1}{d_2} \tan \beta \tan \gamma}. \quad (43)$$

It is sometimes assumed, but, generally speaking, as a guide only, that the inlet-lip is radial, Figs. 299, 300, so that

$$\alpha = 90^\circ \text{ and } u_1 = v_w'.$$

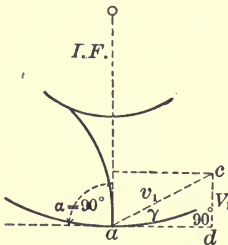


FIG. 299.

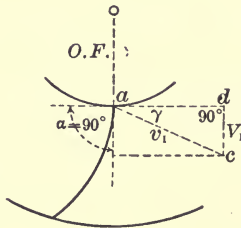


FIG. 300.

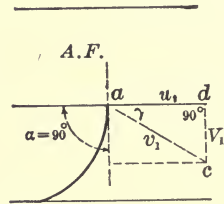


FIG. 301.

Then

$$\text{the efficiency} = \frac{u_1 v_w'}{g H_1} = \frac{u_1^2}{g H_1}. \quad (44)$$

Amount Q of water passing through the turbine per second, blade thickness being disregarded.

$$Q = 2\pi R d_2 v_r'' = 2\pi R d_2 v_2$$

$$= 2\pi R d_2 \sqrt{\frac{2g (H_1 + h) \tan \beta}{\tan \beta + 2 \frac{d_2}{d_1} \cot \gamma}}. \quad (41')$$

The useful work (disregarding hydraulic resistances)

$$= \omega Q \left(H_1 + h - \frac{v_2^2}{2g} \right)$$

$$= \frac{\omega Q (H_1 + h)}{1 + \frac{1}{2} \frac{d_1}{d_2} \tan \beta \tan \gamma}. \quad (42)$$

The corresponding efficiency

$$= \frac{1}{1 + \frac{1}{2} \frac{d_1}{d_2} \tan \beta \tan \gamma}. \quad (43)$$

It is sometimes assumed, but, generally speaking, as a guide only, that the inlet-lip is vertical, Fig. 301, so that

$$\alpha = 90^\circ \text{ and } u_1 = v_w'.$$

Then

$$\text{the efficiency} = \frac{u_1 v_w'}{g (H_1 + h)}$$

$$= \frac{u_1^2}{g (H_1 + h)}. \quad (44')$$

An approximate estimate of the speed of the turbine may now be obtained by making the efficiency perfect, when

$$u_1^2 = gH_1. \quad (45)$$

By eqs. (20), (37), (39), the difference between the inlet and outlet pressure-heads

$$\begin{aligned} &= \frac{p_1 - p_2}{w} = H_1 - \frac{v_1^2}{2g} \\ &= H_1 \left\{ 1 - \frac{\left(\frac{r_2 d_2}{r_1 d_1} \right)}{\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right)} \right\} \quad (46) \end{aligned}$$

If the turbine is above the surface of the tail-water, there will be no inflow of air

if $p_1 > p_2$, i.e., if

$$\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right) > \left(\frac{r_2 d_2}{r_1 d_1} \right)^2.$$

If the turbine is drowned with a head h' of water over the outlet, there will be no back-flow of water

if $p_1 > p_2 + \omega h'$, i.e., if

$$\frac{H_1 - h'}{H_1} > \frac{\left(\frac{r_2 d_2}{r_1 d_1} \right)^2}{\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right)}.$$

An approximate estimate of the speed of the turbine may now be obtained by making the efficiency perfect, when

$$u_1^2 = g(H_1 + h). \quad (45)$$

By eqs. (20), (37), (39), the difference between the inlet and outlet pressure-heads

$$\begin{aligned} &= \frac{p_1 - p_2}{w} = H_1 - \frac{v_1^2}{2g} \\ &= H_1 - \frac{\left(\frac{d_2}{d_1} \right)^2 (H_1 + h)}{\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right)} \quad (46) \end{aligned}$$

If the turbine is above the surface of the tail-water, there will be no inflow of air

if $p_1 > p_2$, i.e., if

$$\frac{H_1}{H_1 + h} > \frac{\left(\frac{d_2}{d_1} \right)^2}{\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right)}.$$

If the turbine is drowned with a head h' of water over the outlet, there will be no back-flow of water

if $p_1 > p_2 + \omega h'$, i.e., if

$$\frac{H_1 - h'}{H_1 + h} > \frac{\left(\frac{d_2}{d_1} \right)^2}{\sin^2 \gamma \left(1 + 2 \frac{d_2}{d_1} \cot \gamma \cot \beta \right)}.$$

IMPULSE TURBINES.

Speed of turbine.

By eqs. (21), (37),

$$u_2 = \frac{r_2}{r_1} u_1 = \frac{r_1 d_1 \sin \gamma}{r_2 d_2 \tan \beta} \sqrt{2gH_1}. \quad (47)$$

Velocity of efflux.

$$v_2 = u_2 \tan \beta = \frac{r_1 d_1}{r_2 d_2} \sin \gamma \sqrt{2gH_1}. \quad (48)$$

Speed of turbine.

By eqs. (21), (37),

$$u_2 = u_1 = \frac{d_1 \sin \gamma}{d_2 \tan \beta} \sqrt{2gH_1}. \quad (47)$$

Velocity of efflux.

$$v_2 = u_2 \tan \beta = \frac{d_1}{d_2} \sin \gamma \sqrt{2gH_1}. \quad (48)$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$Q = 2\pi r_1 d_1 v_r' = 2\pi r_1 d_1 v_1 \sin \gamma$$

$$= 2\pi r_1 d_1 \sin \gamma \sqrt{2gH_1}. \quad (49)$$

The useful work (disregarding hydraulic resistances)

$$= \omega Q \left(H_1 - \frac{v_2^2}{2g} \right)$$

$$= \omega Q H_1 \left(1 - \frac{r_1^2 d_1^2}{r_2^2 d_2^2} \sin^2 \gamma \right). \quad (50)$$

The corresponding efficiency

$$= 1 - \frac{r_1^2 d_1^2}{r_2^2 d_2^2} \sin^2 \gamma. \quad (51)$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$Q = 2\pi R d_1 v_r' = 2\pi R d_1 v_1 \sin \gamma$$

$$= 2\pi R d_1 \sin \gamma \sqrt{2gH_1}. \quad (49)$$

The useful work (disregarding hydraulic resistances)

$$= \omega Q \left(H_1 + h - \frac{v_2^2}{2g} \right)$$

$$= \omega Q (H_1 + h) \left(1 - \frac{H_1}{H_1 + h} \frac{d_1^2}{d_2^2} \sin^2 \gamma \right) \quad (50)$$

The corresponding efficiency.

$$= 1 - \frac{H_1}{H_1 + h} \frac{d_1^2}{d_2^2} \sin^2 \gamma. \quad (15)$$

An expression can also be easily obtained giving the efficiency (eq. 51) of the A. F. turbine independent of the head, H_1 . Thus, by eqs. (28), (35), and (47),

$$\frac{d_1}{d_2} \frac{\sin 2\gamma}{\tan \beta} + \frac{d_1^2 \sin^2 \gamma}{d_2^2 \tan^2 \beta} = \frac{H_1 + h}{H_1}.$$

It may be assumed, as a first approximation, that in impulse turbines the whole of the water's energy at inlet is transformed into useful work. Then

$$\frac{v_1^2}{2g} = \frac{u_1 v_1'}{g} = \frac{u_1 v_1 \cos \gamma}{g}.$$

Therefore

$$v_1 = 2u_1 \cos \gamma = 2V_1 \cos \gamma.$$

SECOND. Consider the hypothesis "that at the outlet the relative velocity of the water and the peripheral linear velocity of the wheel are equal." Then

$$u_2 = V_2. \quad (52)$$

The triangle of velocities, fkh , at outlet is now therefore an isosceles triangle, in which $fk = kh$, and the angle $hfk = \delta = 90^\circ - \frac{\beta}{2}$. Therefore

$$v_2 = 2u_2 \sin \frac{\beta}{2} = 2V_2 \sin \frac{\beta}{2}. \quad (53)$$

Eq. (5), again, gives

$$A_1 v_1 \sin \gamma = A_2 V_2 \sin \beta = A_2 u_2 \sin \beta. \quad (54)$$

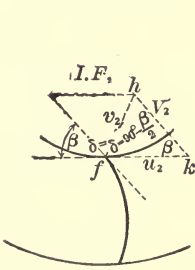


FIG. 302.

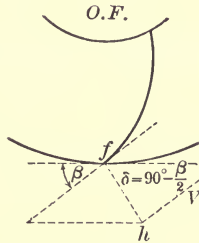


FIG. 303.

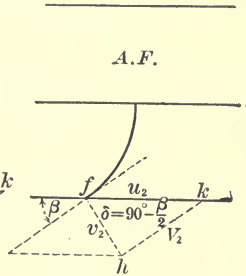


FIG. 304.

General Deductions.

IN AN I. F. OR O. F. TURBINE

$$\frac{u_1}{r_1} = \frac{u_2}{r_2} = \omega.$$

Also, disregarding blade thickness,

$$A_1 = 2\pi r_1 d_1; \quad A_2 = 2\pi r_2 d_2.$$

Relation between the lip angles.

By eq. (54) and the triangle acd , Figs. 302, 303,

$$\frac{r_1^2 d_1 \sin \gamma}{r_2^2 d_2 \sin \beta} = \frac{u_1}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha}, \quad (55)$$

or

$$\frac{r_1^2 d_1}{r_2^2 d_2} \operatorname{cosec} \beta = \cot \gamma + \cot \alpha. \quad (56)$$

IN AN A. F. TURBINE

$$u_1 = u_2 = R\omega.$$

Also, disregarding blade thickness,

$$A_1 = 2\pi R d_1; \quad A_2 = 2\pi R d_2.$$

Relation between the lip angles.

By eq. (54) and the triangle acd , Fig. 304,

$$\frac{d_1 \sin \gamma}{d_2 \sin \beta} = \frac{u_1}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha}, \quad (55)$$

or

$$\frac{d_1}{d_2} \operatorname{cosec} \beta = \cot \gamma + \cot \alpha. \quad (56)$$

REACTION TURBINES.

IN AN I. F. OR O. F. TURBINE.

Speed of turbine.

By eqs. (24), (52),

$$u_1 v_1 \cos \gamma = g H_1, \quad (57)$$

and hence, by eq. (55),

$$u_2^2 = \frac{r_2^2}{r_1^2} u_1^2 = g H_1 \frac{d_1}{d_2} \frac{\tan \gamma}{\sin \beta}. \quad (58)$$

Velocity of efflux.

$$\begin{aligned} v_2^2 &= 4u_2^2 \sin^2 \frac{\beta}{2} \\ &= 2g H_1 \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma. \quad (59) \end{aligned}$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$\begin{aligned} Q &= 2\pi r_2 d_2 v_r'' = 2\pi r_2 d_2 v_2 \cos \frac{\beta}{2} \\ &= 2\pi r_2 \sqrt{g H_1 d_1 d_2 \sin \beta \tan \gamma}. \quad (60) \end{aligned}$$

The useful work (disregarding hydraulic resistances)

$$\begin{aligned} &= w Q H_1 \left(1 - \frac{v_1^2}{2g} \right) \\ &= w Q H_1 \left(1 - \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma \right). \quad (61) \end{aligned}$$

The corresponding efficiency

$$= 1 - \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma. \quad (62)$$

By eqs. (20), (57), (58), the difference between the inlet and outlet pressure-heads

$$\begin{aligned} \frac{p_1 - p_2}{w} &= H_1 - \frac{v_1^2}{2g} \\ &= H_1 \left(1 - \frac{r_2^2 d_2 \sin \beta}{r_1^2 d_1 \sin 2\gamma} \right). \quad (63) \end{aligned}$$

IN AN A. F. TURBINE.

Speed of turbine.

By eqs. (28), (52),

$$u_1 v_1 \cos \gamma = g(H_1 + h), \quad (57)$$

and hence, by eq. (55),

$$u_2^2 = u_1^2 = g(H_1 + h) \frac{d_1}{d_2} \frac{\tan \gamma}{\sin \beta}. \quad (58)$$

Velocity of efflux.

$$\begin{aligned} v_2^2 &= 4u_2^2 \sin^2 \frac{\beta}{2} \\ &= 2g(H_1 + h) \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma. \quad (59) \end{aligned}$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$\begin{aligned} Q &= 2\pi R d_2 v_r'' = 2\pi R d_2 v_2 \cos \frac{\beta}{2} \\ &= 2\pi R \sqrt{g(H_1 + h) d_1 d_2 \sin \beta \tan \gamma}. \quad (60) \end{aligned}$$

The useful work (disregarding hydraulic resistances)

$$\begin{aligned} &= w Q \left(H_1 + h - \frac{v_1^2}{2g} \right) \\ &= w Q (H_1 + h) \left(1 - \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma \right). \quad (61) \end{aligned}$$

The corresponding efficiency

$$= 1 - \frac{d_1}{d_2} \tan \frac{\beta}{2} \tan \gamma. \quad (62)$$

By eqs. (20), (57), (58), the difference between the inlet and outlet pressure-heads

$$\begin{aligned} \frac{p_1 - p_2}{w} &= H_1 - \frac{v_1^2}{2g} \\ &= H_1 - (H_1 + h) \frac{d_2 \sin \beta}{d_1 \sin 2\gamma}. \quad (63) \end{aligned}$$

If the turbine is above the surface of the tail-water, there will be no in-flow of air

if $p_1 > p_2$, i.e., if

$$\frac{\sin 2\gamma}{\sin \beta} > \frac{r_2^2 d_2}{r_1^2 d_1}.$$

If the turbine is drowned with a head h' of water over the outlet, there will be no back-flow of water

if $p_1 > p_2 + \omega h'$, i.e., if

$$\frac{H_1 - h'}{H} > \frac{r_2^2 d_2}{r_1^2 d_1} \frac{\sin \beta}{\sin 2\gamma}.$$

If the turbine is above the surface of the tail-water, there will be no in-flow of air

if $p_1 > p_2$, i.e., if

$$\frac{\sin 2\gamma}{\sin \beta} > \frac{H_1 + h}{H_1} \frac{d_2}{d_1}.$$

If the turbine is drowned with a head h' of water over the outlet, there will be no back-flow of water

if $p_1 > p_2 + \omega h'$, i.e., if

$$\frac{\sin 2\gamma}{\sin \beta} > \frac{H_1 + h}{H_1 - h'} \frac{d_2}{d_1}.$$

IMPULSE TURBINES.

IN AN I. F. OR O. F. TURBINE.

By eqs. (29), (52),

$$u_1 = V_1. \quad (64)$$

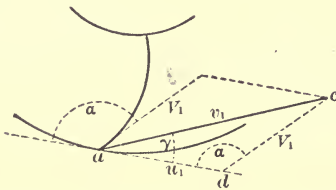


FIG. 305.

IN AN A. F. TURBINE.

By eqs. (30), (52),

$$2gh = u_2^2 - V_1^2 = u_1^2 - V_1^2. \quad (64)$$

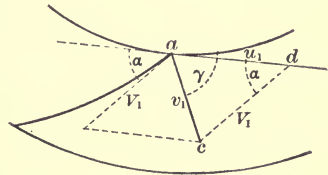


FIG. 306.

Then the inlet triangle of velocities acd , as well as the outlet triangle fkh , is also an isosceles triangle, Figs. 305, 306, and

$$u_1 = da = dc = V_1.$$

Therefore

$$\alpha = 180^\circ - 2\gamma. \quad (65)$$

Therefore

$$u_1^2 - 2gh = V_1^2$$

$$= u_1^2 + v_1^2 - 2u_1v_1 \cos \gamma,$$

and

$$u_1v_1 \cos \gamma = \frac{v_1^2}{2} + gh = g(H_1 + h). \quad (65)$$

Relation between the lip angles.

By eq. (54) and the isosceles triangle acd ,

$$\frac{r_1^2 d_1 \sin \gamma}{r_2^2 d_2 \sin \beta} = \frac{u_1}{v_1} = \frac{1}{2} \sec \gamma,$$

or

$$\frac{r_1^2 d_1}{r_2^2 d_2} = \frac{\sin \beta}{\sin 2\gamma} \quad . \quad . \quad (66)$$

Speed of turbine.

$$\begin{aligned} u_2 &= \frac{r_2}{r_1} u_1 = \frac{r_2}{r_1} \cdot \frac{v_1}{2} \sec \gamma \\ &= \frac{r_2}{r_1} \frac{\sec \gamma}{2} \sqrt{2gH_1}. \quad (67) \end{aligned}$$

Velocity of efflux.

$$v_2 = 2u_2 \sin \frac{\beta}{2} = \frac{r_2}{r_1} \frac{\sin \frac{\beta}{2}}{\cot \gamma} \sqrt{2gH_1}. \quad (68)$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$\begin{aligned} Q &= 2\pi r_1 d_1 v_r' = 2\pi r_1 d_1 v_1 \sin \gamma \\ &= 2\pi r_1 d_1 \sin \gamma \sqrt{2gH_1}. \quad (69) \end{aligned}$$

The useful work (disregarding hydraulic resistances)

$$\begin{aligned} &= wQ \left(H_1 - \frac{v_2^2}{2g} \right) \\ &= wQH_1 \left(1 - \frac{r_2^2}{r_1^2} \frac{\sin^2 \frac{\beta}{2}}{\cos^2 \gamma} \right) \quad . \quad . \quad (70) \end{aligned}$$

$$= wQH_1 \left(1 - \frac{d_1}{d_2} \tan \gamma \tan \frac{\beta}{2} \right). \quad (71)$$

The corresponding efficiency

$$= 1 - \frac{r_2^2}{r_1^2} \frac{\sin^2 \frac{\beta}{2}}{\cos^2 \gamma} \quad . \quad . \quad (72)$$

$$= 1 - \frac{d_1}{d_2} \tan \gamma \tan \frac{\beta}{2} \quad . \quad (73)$$

Relation between the lip angles.

By eqs. (54), (65),

$$\frac{d_1 \sin \gamma}{d_2 \sin \beta} = \frac{u_1}{v_1} = \frac{H_1 + h \sec \gamma}{H_1} \frac{1}{2},$$

or

$$\frac{d_1}{d_2} \frac{H_1}{H_1 + h} = \frac{\sin \beta}{\sin 2\gamma} \quad . \quad (66)$$

Speed of turbine.

$$\begin{aligned} u_2 &= u_1 = \frac{H_1 + h \sec \gamma}{H_1} \frac{v_1}{2} \\ &= \frac{H_1 + h \sec \gamma}{H_1} \frac{1}{2} \sqrt{2gH_1}. \quad (67) \end{aligned}$$

Velocity of efflux.

$$v_2 = \frac{H_1 + h}{H_1} \sin \frac{\beta}{2} \sec \gamma \sqrt{2gH_1}. \quad (68)$$

Quantity Q of water passing through the turbine per second, blade thickness being disregarded.

$$\begin{aligned} Q &= 2\pi R d_1 v_r' = 2\pi R d_1 v_1 \sin \gamma \\ &= 2\pi R d_1 \sin \gamma \sqrt{2gH_1}. \quad (69) \end{aligned}$$

The useful work (disregarding hydraulic resistances)

$$\begin{aligned} &= wQ \left(H_1 + h - \frac{v_2^2}{2g} \right) \\ &= wQ(H_1 + h) \left(1 - \frac{H_1 + h}{H} \sin^2 \frac{\beta}{2} \sec^2 \gamma \right) \quad (70) \end{aligned}$$

$$= wQ(H_1 + h) \left(1 - \frac{d_1}{d_2} \tan \gamma \tan \frac{\beta}{2} \right). \quad (71)$$

The corresponding efficiency

$$= 1 - \frac{H_1 + h}{H} \sin^2 \frac{\beta}{2} \sec^2 \gamma \quad (72)$$

$$= 1 - \frac{d_1}{d_2} \tan \gamma \tan \frac{\beta}{2} \quad . \quad (73)$$

5. Remarks on the Efficiency.—The expressions giving the efficiency in the preceding deductions are all independent of the head, and it follows that turbines work equally well above and below water.

The efficiency, again, increases as the ratio $\frac{d_1}{d_2}$ diminishes, but it should be remembered that the value of d_1 must not be too small, as this might cause a contraction at entrance and a corresponding loss of energy. The wheel-passages should always run full bore, and therefore d_2 must not be too large.

Finally, the efficiency increases as the angles β and γ diminish.

6. Practical Values.—The following are the values which experience indicates as giving good results in practice, but they should be only regarded as guides:

Let v be the theoretical velocity due to the head H_1 , so that $v^2 = 2gH_1$. Then

In an I. F. reaction turbine

$$v_r' = v_r'' = \frac{v}{8},$$

$$u_1 = \frac{r_1}{r_2} u_2 = .56v,$$

$$r_2 = d_2 = \frac{r_1}{2}.$$

γ usually varies from 10° to 30° , an average value being 20° .

If $u_2 = V_2$, β usually varies from 135° to 150° , an average value being 145° .

If $v_w'' = 0$, β usually varies from 30° to 45° , an average value being 35° .

In an O. F. reaction turbine

$$v_r' = \frac{v}{4},$$

$$v_r'' = .21v \text{ to } .17v,$$

$$u_1 = \frac{r_1}{r_2} u_2 = .56v.$$

Let n be the number of the guide-blades.

Let n_1 be the number of the wheel-blades.

Then

$$r_2 - r_1 = \frac{8r_2}{n_1}$$

= 4 \times shortest distance between wheel-blades,

$$\frac{2r_1}{n} = \text{shortest distance between guide-blades,}$$

$$n = \frac{1}{2}n_1 \text{ to } \frac{3}{4}n_1.$$

$$\text{The H. P.} = .17r_2^2 H_1^{\frac{3}{2}};$$

γ usually varies from 20° to 50° , an average value being 25° .

β usually varies from 20° to 30° , an average value being 25° .

In an A. F. reaction turbine

$$v_r' = v_r'' = .15v \text{ to } .2v,$$

$$u_1 = u_2 = .56v.$$

γ usually varies from 15° to 50° , an average value being 25° .

β usually varies from 12° to 30° , an average value being 25° .

For a delivery of 30 to 60 cu. ft. and a fall of 25 to 40 ft.

$$\gamma = 15^\circ \text{ to } 18^\circ \quad \text{and} \quad \beta = 13^\circ \text{ to } 16^\circ.$$

For a delivery of 40 to 200 cu. ft. and a fall of 5 to 30 ft.

$$\gamma = 18^\circ \text{ to } 24^\circ \quad \text{and} \quad \beta = 16^\circ \text{ to } 24^\circ.$$

For a delivery of more than 200 cu. ft. and for falls of less than about 5 or 6 ft.

$$\gamma = 24^\circ \text{ to } 30^\circ \quad \text{and} \quad \beta = 24^\circ \text{ to } 28^\circ.$$

Denoting $\sqrt{A_1} \sin \gamma$ by A' ,

R may vary from $\frac{3}{2}A'$ to $2A'$ if $A' < 2$ sq. ft.

“ “ “ “ $\frac{5}{4}A'$ to $\frac{3}{2}A'$ if $A' > 2$ sq. ft. and < 16 sq. ft.

“ “ “ “ A' to $\frac{5}{4}A'$ if $A' > 200$ sq. ft.

In A. F. *impulse* turbines R is often made to vary from $\frac{5}{4}A'$ to $2A'$.

In reaction and impulse turbines the blade thickness varies from $\frac{1}{8}$ to $\frac{3}{8}$ in. if the blades are of wrought iron, and from $\frac{1}{2}$ to $\frac{5}{8}$ in. if they are of cast iron. The tips of cast-iron blades are usually tapered.

EX. 1. An axial-flow impulse turbine passes 170 cu. ft. of water per second under the head of 8.6 ft. over the inlet, and it may be assumed that the whole of this head is transformed into useful work. The depth of the wheel is .9 ft., its mean diameter is 8.4 ft., and the outlet-lip makes an angle of 72° with the vertical. The turbine has 62 guide- and 60 wheel-vanes, all the vanes being $\frac{1}{8}$ in. thick. The outlet velocity of whirl is nil. Find the direction of motion of the water at inlet, the slope of the wheel vane at inlet, the H.P., the speed, and the inlet and outlet orifice areas and widths.

First. Disregard hydraulic resistances.

$$\text{Then} \quad \frac{v_1^2}{2g} = 8.6 = \frac{u_1 v_{w'}}{g} = \frac{u_1 v_1 \cos \gamma}{g},$$

$$\text{and} \quad v_1 = 2u_1 \cos \gamma = 8 \sqrt{8.6} = 23.4606 \text{ ft. per sec.}$$

$$\text{Also,} \quad V_1^2 = v_1^2 + u_1^2 - 2v_1 u_1 \cos \gamma = u_1^2 = u_2^2.$$

Therefore $V_1 = u_1 = u_2$, and the triangle acd is isosceles, so that

$$\alpha = 180^\circ - 2\gamma.$$

$$\text{Again,} \quad \frac{v_1^2}{2g \times 9.5} = \text{the efficiency} = 1 - \frac{v_2^2}{2g} = \frac{8.6}{9.5} = .905,$$

$$\text{and} \quad v_2 = 8 \sqrt{.9} = 7.58946 \text{ ft. per sec.}$$

Therefore $u_1 = u_2 = v_1 \cot 18^\circ = 23.358$ ft. per sec.,

and $\sec \gamma = \frac{2u_1}{v_1} = 1.99125,$

so that $\gamma = 59^\circ 52',$

and $\alpha = 180^\circ - 2\gamma = 60^\circ 16'.$

The H.P. = $\frac{62\frac{1}{2} \times 170 \times 9.5}{550} \times .905 = 166.136.$

The speed in revolutions per min. = $\frac{60 \times 23.358}{\pi \times 8.4} = 53.08.$

The inlet area = $\frac{170}{v_{r'}} = \frac{170}{v_1 \sin \gamma} = 8.38$ sq. ft.

The outlet area = $\frac{170}{v_2} = \frac{170}{7.589} = 22.4$ sq. ft.

$8.38 = d_1 \left\{ \pi \times 8.4 - \frac{62}{24} \operatorname{cosec} 59^\circ 52' - \frac{60}{24} \operatorname{cosec} 60^\circ 16' \right\} = d_1 \times 20.53396,$

and $d_1 = .408$ ft.

$22.4 = d_2 \left\{ \pi \times 8.4 - \frac{60}{24} \operatorname{cosec} 18^\circ \right\} = d_2 \times 18.30983,$

and $d_2 = 1.223$ ft.

Second. Take the hydraulic resistances into consideration.

$$\frac{v_1^2}{2g} = \frac{8}{9}(8.6) = \frac{u_1 v_w}{g} = \frac{u_1 v_1 \cos \gamma}{g}.$$

Therefore $v = 2u_1 \cos \gamma = 22.1189$ ft. per sec.

The triangle acd is therefore isosceles, and

$$V_1 = u_1 = u_2,$$

so that $\alpha = 180^\circ - 2\gamma.$

Also,

$$\frac{11}{10} V_1^2 = \frac{11}{10} u_1^2 \sec^2 \gamma = V_1^2 + 2gh = u_1^2 + 57.6 = \frac{11}{10} u_1^2 \sec^2 \gamma.$$

Therefore $u^2(1.1 \sec^2 \gamma - 1) = 57.6,$

and $u = 16.325$ ft. per sec. = $u_2.$

Then $\cos \gamma = \frac{v}{2u} = \frac{22.1189}{32.65} = .677453,$

and $\gamma = 47^\circ 21'.$

Hence, too, $\alpha = 180^\circ - 2\gamma = 85^\circ 18'.$

The speed in revolutions per min. = $\frac{60 \times 16.325}{\pi \times 8.4} = 37.1.$

The efficiency = $\frac{\frac{8}{3}(8.6)}{9.5} = .8046.$

$$\text{The H.P.} = \frac{62\frac{1}{2} \times 170 \times 9.5}{550} \times .8046 = 147.676.$$

$$\text{The inlet area} = \frac{170}{v_1 \sin \gamma} = \frac{170 \operatorname{cosec} 47^\circ 21'}{22.1189} = 10.449 \text{ sq. ft.}$$

$$\text{The outlet area} = \frac{170}{v_2} = \frac{170}{u_2 \tan 18^\circ} = 32.05 \text{ sq. ft.}$$

$$10.449 = \frac{9}{10} d_1 \left\{ \pi \times 8.4 - \frac{62}{24} \operatorname{cosec} 47^\circ 21' - \frac{60}{24} \operatorname{cosec} 85^\circ 18' \right\}$$

$$= 18.3402 \times d_1,$$

and $d_1 = .57 \text{ ft.}$

$$32.05 = \frac{9}{10} d_2 \left\{ \pi \times 8.4 - \frac{60}{24} \operatorname{cosec} 18^\circ \right\}$$

$$= d_2 \times 14.85885,$$

and $d_2 = 2.157 \text{ ft.}$

Ex. 2. An A. F. reaction turbine of 7 ft. mean diameter passes 198 cu. ft. of water per second under a total head of 13.5 ft., the depth of the wheel being 1 ft. At inlet the lip angle (α) is 90° , and at outlet the peripheral and relative velocities are equal ($V_2 = u_2 = v_1$). The width of the wheel is 1 ft. at inlet and 1.25 ft. at outlet. Determine the direction and magnitude of the velocity of the water at entrance, the lip angle at outlet, the speed in revolutions per minute, the efficiency and the H P. Disregard hydraulic resistances.

By the condition of continuity,

$$\pi \cdot 7 \cdot 1 \cdot v_r' = 198 = \pi \cdot 7 \cdot 1\frac{1}{4} v_r'',$$

and therefore $v_r' = 9 \text{ ft. per sec.}, v_r'' = 7\frac{1}{2} \text{ ft. per sec.}$

Again,

$$64 \times 13.5 - v_1^2 = 864 - v_r'^2 - u_1^2 = V_2^2 - V_1^2 = u_1^2 - v_r'^2,$$

or $2u_1^2 = 864$, or $u_1 = 12\sqrt{3} \text{ ft. per sec.} = u_2$,

$$\cot \gamma = \frac{u_1}{v_r'} = \frac{12\sqrt{3}}{9} = 2.309, \text{ and } \gamma = 23^\circ 25',$$

$$\sin \beta = \frac{v_r''}{V_2} = \frac{7.2}{u_1} = \frac{7.2}{12\sqrt{3}} = \frac{1}{5}\sqrt{3} = .3464, \text{ and } \beta = 20^\circ 5'.$$

Therefore $\delta = \frac{1}{2}(180^\circ - 20^\circ 5') = 79^\circ 77\frac{1}{2}'$,

$$v_2 = 2u_2 \sin \frac{\beta}{2} = 24\sqrt{3} \times .1744 = 7.25 \text{ ft per sec.}$$

$$\text{The efficiency} = 1 - \frac{v_2^2}{64 \times 13.5} = 1 - .0608 = .9391.$$

$$\text{The H.P.} = \frac{62\frac{1}{2} \times 13.5 \times 198}{550} \times .9391 = 285.25.$$

$$\text{Revolutions per min.} = \frac{60 \times 12 \sqrt{3}}{\pi \times 7} = 56.68.$$

Ex. 3. To construct an O. F. turbine from the following data: the fall (H_1) = 5 ft.; the interior diameter ($2r_1$) = 1.8 ft.; the exterior diameter ($2r_2$) = 2.45 ft.; Q = 30 cu. ft. per second; $\gamma = 30^\circ$; the efficiency (η) = .9. Also, disregard hydraulic resistances.

First. Take $v_w'' = 0$. Then

$$.9 = 1 - \frac{v_2^2}{64 \times 5} = \frac{u_1 v_1 \cos 30^\circ}{32 \times 5}.$$

Therefore $v_2 = 4 \sqrt{2}$ ft. per sec.,

and $u_1 v_1 = 96 \sqrt{3}$.

Again, by the condition of continuity (eq. 5),

$$\pi \times 1.80 \times d_1 v_1 \sin 30^\circ = 30 = \pi \times 2.45 \times d_2 v_2.$$

Taking $d_1 = d_2$,

$$.9 v_1 = 2.45 v_2 = 9.8 \sqrt{2},$$

and $v_1 = \frac{98}{9} \sqrt{2}$ ft. per sec.

Therefore $u_1 = \frac{96 \sqrt{3}}{\frac{98}{9} \sqrt{2}} = \frac{216}{49} \sqrt{6}$ ft. per sec.,

and $u_2 = \frac{2.45}{1.8} u_1 = 6 \sqrt{6}$ ft. per sec.

Hence $\frac{\sin(\alpha + 30^\circ)}{\sin \alpha} = \frac{u_1}{v_1} = \frac{972 \sqrt{3}}{2401} = \frac{\sqrt{3}}{2} + \frac{1}{2} \cot \alpha$,

or $\cot(180^\circ - \alpha) = \sqrt{3} \cdot \frac{457}{2401} = .32966$,

and $\alpha^\circ = 108^\circ 15' = \text{inlet-tip angle}.$

Also, $\tan \beta = \frac{v_2}{u_2} = \frac{4 \sqrt{2}}{6 \sqrt{6}} = .3849$,

and $\beta = 21^\circ 3' = \text{outlet-tip angle}.$

Disregarding the thickness of the vanes,

the inlet area $= A_1 = \frac{30}{v_1'} = \frac{30}{v_1 \sin 30^\circ} = \frac{135}{49} \sqrt{2} = 3.8963$ sq. ft.,

and $d_1 = \frac{3.8963}{\pi \times 1.8} = .6886$ ft. $= d_2$;

the outlet area $= A_2 = \frac{30}{v_2} = \frac{30}{4 \sqrt{2}} = 5.303$ sq. ft.

The number of revolutions per min. = $\frac{60 \times 6 \sqrt{6}}{\pi \times 2.45} = 114.52$.

Second. Take $V_2 = u_2 = \frac{2.45}{1.8} u_1$. Then the triangle fkh is isosceles.

Therefore $v_2 = 2u_2 \sin \frac{\beta}{2} = v_w'' \operatorname{cosec} \frac{\beta}{2}$,

and $v_2^2 = 2u_2 v_w''$.

Again, $1 - \frac{v_2^2}{64 \times 5} = .9 = \frac{u_1 v_w' - u_2 v_w''}{32 \times 5}$

$$= \frac{u_1 v_1 \cos 30^\circ - \frac{v_2^2}{2}}{160}.$$

Therefore $v_2 = 4 \sqrt{2}$ ft. per sec.,

and $u_1 v_1 = \frac{320}{3} \sqrt{3} = v_1 \frac{2.45}{1.8} u_2$

$$= v_1 \frac{2.45}{1.8} \frac{v_2}{2} \operatorname{cosec} \frac{\beta}{2} = v_1 \frac{49}{18} \sqrt{2} \operatorname{cosec} \frac{\beta}{2}.$$

By the condition of continuity,

$$\begin{aligned} \frac{22}{7} d_1 v_1 \sin 30^\circ \times 1.8 &= 3\pi = \frac{22}{7} d_2 v_2 \times 2.45 = \frac{22}{7} d_2 v_2 \cos \frac{\beta}{2} \times 2.45 \\ &= \frac{22}{7} d_2 9.8 \sqrt{2} \cos \frac{\beta}{2}. \end{aligned}$$

Taking $d_1 = d_2$,

$$.9v_1 = 9.8 \sqrt{2} \cos \frac{\beta}{2}.$$

Hence $\frac{320}{3} \sqrt{3} = \frac{9.8 \sqrt{2}}{.9} \cos \frac{\beta}{2} \cdot \frac{49}{18} \sqrt{2} \operatorname{cosec} \frac{\beta}{2}$,

or, $\cot \frac{\beta}{2} = \frac{4320 \sqrt{3}}{2401} = 3.1167$,

and $\beta = 35^\circ 34' = \text{outlet-tip angle}.$

Hence, also

$v_1 = 14.6634$ ft. per sec., $u_2 = 9.261$ ft. per sec., and $u_1 = 6.7963$ ft. per sec.

Again, $\frac{\sin(\alpha + 30^\circ)}{\sin \alpha} = \frac{u_1}{v_1} = \frac{1.8}{2.45} \frac{u_2}{v_1} = \frac{1.8}{2.45} \frac{v_2}{2 \sin \frac{\beta}{2}} \cdot \frac{.9}{9.8 \sqrt{2} \cos \frac{\beta}{2}}$,

or $\cos 30^\circ + \cot \alpha \sin 30^\circ = \frac{32.4}{2.45 \times 49} \operatorname{cosec} \beta = .46399$,

or $\cot(180^\circ - \alpha) = .804$,

and $\alpha = 128^\circ 48' = \text{inlet-tip angle}.$

Disregarding the thickness of the vanes,

$$\text{the inlet area} = A_1 = \frac{30}{v_r'} = \frac{30}{v_1 \sin 30^\circ} = \frac{54}{9.8 \sqrt{2}} \sec \frac{\beta}{2} = 4.092 \text{ sq. ft.},$$

$$\text{and} \quad d_1 = \frac{4.092}{\pi \times 1.8} = .7233 \text{ ft.} = d_2;$$

$$\text{the outlet area} = \frac{30}{v_2 \cos \frac{\beta}{2}} = \frac{15}{2 \sqrt{2}} \times 1.05018 = 5.5694 \text{ sq. ft.}$$

$$\text{The number of revolutions per min.} = \frac{60 \times 6.796}{\pi \times 1.8} = 72.$$

Ex. 4. An I. F. reaction turbine of 24 ins. exterior and 12 ins. interior diameter passes 400 gallons of water per second. The inlet and outlet orifice areas are equal and the depth of the latter is 1.25 ft. The guide-vane lip has a slope of 1 in 5 and the inlet-lip is radial. Disregarding vane thickness and hydraulic resistances, find the total head over the inlet and also the efficiency, the outlet velocity of whirl being nil.

By the condition of continuity,

$$A_1 v_r' = A_2 v_r' = 400 \div 6\frac{1}{2} = 64 = A_2 v_r'' = A_2 v_2.$$

Therefore

$$v_r' = v_r'' = v_2 = 64 \div \pi \cdot 1 \cdot 1\frac{1}{2} = 8\frac{8}{5} \text{ ft. per sec.},$$

$$\text{and the head equivalent to } v_2 = \frac{v_2^2}{2g} = \left(\frac{56}{55}\right)^2 = 1.036694 \text{ ft.}$$

Again,

$$u_1 = v_r' \cot \gamma = 5v_r' = 40\frac{8}{11} \text{ ft. per sec.} = 2u_2,$$

$$\text{and the useful head} = \frac{u_1 v_{w'}}{g} = \frac{u_1^2}{g} = (40\frac{8}{11})^2 \div 32 = 51\frac{191}{22} \text{ ft.}$$

Hence

$$\text{the total head} = 1.036694 + 51.834710 = 52.871404 \text{ ft.},$$

$$\text{and the efficiency} = \frac{51.834710}{52.871404} = .98.$$

$$\text{Also, the speed in revolutions per min.} = \frac{60 \times 40\frac{8}{11}}{\pi \times 2} = 388.76.$$

$$\text{The H.P.} = \frac{62\frac{1}{2} \times 64 \times 52.871404}{550} \times .98 = 376.98.$$

$$\tan \beta = \frac{v_2}{u_2} = 8\frac{8}{5} \div 20\frac{4}{11} = .4, \quad \text{and} \quad \beta 21^\circ 48'.$$

Ex. 5. In the preceding example show how the results will be modified if, instead of the outlet velocity of whirl being nil, the relative and peripheral velocities at outlet are equal.

As before,

$$v_r' = v_r'' = 8\frac{8}{5} \text{ ft. per sec.,}$$

$$u_1 = 40\frac{8}{11} \text{ ft. per sec.} = 2u_2 = 2\sqrt{2}.$$

$$\text{The speed in revolutions per min.} = \frac{60 \times 40\frac{8}{11}}{\pi \times 2} = 388.76.$$

$$\text{Again, } V_2^2 - V_1^2 = u_2^2 - u_1^2 + 2gH_1 - v_1^2,$$

$$\text{or } u_2^2 - v_r'^2 = u_2^2 - u_1^2 + 2gH_1 - u_1^2 - v_r'^2,$$

$$\text{and } H_1, \text{ the total head,} = \frac{u_1^2}{g} = 51\frac{1\frac{01}{11}}{11} \text{ ft.}$$

Also,

$$\sin \beta = \frac{v_r''}{V_2} = \frac{v_r''}{u_2} = 8\frac{8}{5} \div 20\frac{4}{11} = .4, \text{ and } \beta = 23^\circ 35'.$$

$$\text{The efficiency} = 1 - \frac{v_2^2}{2gH_1} = 1 - \frac{4u_2^2 \sin^2 \frac{\beta}{2}}{2gH_1} = 1 - \frac{u_2^2(1 - \cos \beta)}{32H_1}$$

$$= 1 - \frac{(20\frac{4}{11})^2 \times .0835209}{32 \times 51.834710} = .979.$$

$$\text{The H.P.} = \frac{62\frac{1}{2} \times 64 \times 51.83471}{550} \times .979 = 369.109.$$

Ex. 6. A vortex impulse turbine, without guide-vanes but with 32 wheel-vanes of $\frac{3}{4}$ -in. thickness, has an exterior diameter of 2.625 ft., an interior diameter of .21 ft., and passes 30 cu. ft. of water per second under a head of 560 ft. The water enters at an angle of 30° with the wheel's periphery, and the relative and peripheral velocities at outlet are equal. The wheel depth at outlet is 3 times the depth at inlet. Allowance is made for hydraulic resistances by taking .94 as a coefficient of velocity at inlet, and by adding 10 per cent to the head equivalent to the relative velocity at outlet.

$$v_1 = .94 \sqrt{64 \cdot 560} = 177.955 \text{ ft. per sec.}$$

$$\text{Also, } u_2^2 - u_1^2 = \frac{11}{10} V_2^2 - V_1^2 = \frac{11}{10} u_2^2 - V_1^2,$$

$$\text{or } V_1^2 = u_1^2 + \frac{1}{10} u_2^2 = u_1^2 + \frac{1}{10} \left(\frac{2.1}{2.625} \right)^2 u_1^2 = \frac{266}{250} u_1^2.$$

$$\text{Therefore } \sqrt{\frac{250}{266}} = \frac{u_1}{V_1} = \frac{\sin(\alpha + 30^\circ)}{\sin 30^\circ},$$

or $\sin(\alpha + 30) = \frac{1}{2} \sqrt{\frac{250}{266}} = .48472,$

or $\alpha + 30^\circ = 151^\circ, \text{ and } \alpha = 121^\circ.$

Again, $u_1 = v_1 \frac{\sin(\alpha + 30^\circ)}{\sin \alpha} = 177.955 \frac{\sin 29^\circ}{\sin 59^\circ}$
 $= 100.634 \text{ ft. per sec.,}$

and $u_2 = 80.5072 \text{ ft. per sec.}$

The speed in revolutions per min. $= \frac{60 \times 100.634}{\pi \times 2.625} = 731.88.$

By the condition of continuity,

$$A_1 v_1' = A_1 v_1 \sin \gamma = 30 = A_2 v_2'' = A_2 V_2 \sin \beta = A_2 u_2 \sin \beta.$$

Therefore

$$\frac{9}{10} d_1 \left\{ \pi \times 2.625 - 32 \times \frac{3}{48} \right\} = A_1 = \frac{30}{v_1} \operatorname{cosec} 30^\circ = \frac{60}{177.955}$$

$$= .337941 \text{ sq. ft.,}$$

and $d_1 = .06 \text{ ft.} = \frac{d_2}{3}.$

Also,

$$\frac{9}{10} \times .18 \left\{ \pi \times 2.1 - 32 \times \frac{3}{48} \operatorname{cosec} \beta \right\} = A_2 = \frac{30}{u_2} \operatorname{cosec} \beta$$

$$= \frac{30}{80.5072} \operatorname{cosec} \beta,$$

or $1.0692 = \operatorname{cosec} \beta (.324 + .372637) = \operatorname{cosec} \beta \times .696637,$

and $\operatorname{cosec} \beta = 1.534, \text{ or } \beta = 40^\circ 41'.$

Therefore, also, $\delta = 69^\circ 39\frac{1}{2}'.$

Again, $v_w' = 177.955 \cos 30^\circ = 154.113 \text{ ft. per sec.,}$

and

$$v_w'' = u_2(1 - \cos \beta) = 80.5072 \times .241676 = 19.4723 \text{ ft. per sec.}$$

Hence

$$\text{the efficiency} = \frac{100.634 \times 154.113 - 80.5072 \times 19.4723}{32 \times 560}$$

$$= \frac{13941.33}{17920} = .78.$$

The H.P. $= \frac{62\frac{1}{2} \times 30 \times 560}{550} \times .78 = 1485.2.$

7. Theory of the Suction (or Draft) Tube.—Vortex and axial-flow turbines sometimes have their outlet-orifices opening into a suction (or draft) tube which extends downwards and discharges *below* the surface of the tail-water. By such an arrangement the turbine can be placed at any convenient height above the tail-water and thus becomes easily accessible, while at the same time a shorter length of shafting will suffice. The suction tube is usually cylindrical and of constant diameter, so that there is an abrupt change of section at the outlet-surface of the turbine, producing a corresponding loss of energy by eddies, etc. This loss may be prevented by so forming the tube at the upper end that there is no abrupt change of section, and by gradually increasing the diameter downwards. The cost of construction is greater, but the action of the tube is much improved.

Let h' be the head above the inlet-orifices of the wheel.

Let h'' be the head between the inlet-orifices and the surface of the tail-water.

Let L_1 be the loss of head up to the inlet-surface.

Let L_2 be the loss of head between the wheel and the tube-outlet.

Let v_4 be the velocity of discharge from the outlet at bottom of tube.

Let P be the atmospheric pressure.

Then, assuming that there is no sudden change of section at the outlet-surface,

$$h' + \frac{P}{w} - \frac{p_1}{w} = \frac{v_1^2}{2g} + L_1,$$

$$h'' + \frac{p_2}{w} + \frac{v_2^2}{2g} = \frac{v_4^2}{2g} + L_2 + \frac{P}{w},$$

and therefore

$$\frac{p_1 - p_2}{w} = h' + h'' - \frac{v_1^2 - v_2^2 + v_4^2}{2g} - L_1 - L_2$$

$$= H - \frac{v_1^2}{2g}(1 - \mu_2 + \mu_4 + \mu_5 + \mu_6),$$

where $H = h' + h'' =$ total head above tail-water surface, and v_2^2, v_4^2, L_1, L_2 are expressed in the forms

$$\mu_2 v_1^2, \mu_4 v_1^2, \mu_5 \frac{v_1^2}{2g}, \mu_6 \frac{v_1^2}{2g},$$

$\mu_2, \mu_4, \mu_5, \mu_6$ being empirical coefficients.

Again, the effective head

$$H_1 = \frac{v_1^2}{2g} + \frac{p_1 - p_2}{2} = H + \frac{v_1^2}{2g}(\mu_2 - \mu_4 - \mu_5 - \mu_6),$$

and is entirely independent of the position of the turbine in the tube.

Also, if A_4 is the area of the outlet from the suction-tube,

$$A_4 v_4 = Q = A_1 v_1 \sin \gamma,$$

so that v_1 can be expressed in terms of v_4 , and hence $\frac{p_1 - p_2}{w}$

is also independent of the position of the turbine in the tube.

Suppose the velocity of flow to be so small that v_4, v_2, L_2 may be each taken equal to *nil*. Then

$$h'' + \frac{p_2}{w} = \frac{P}{w};$$

and since the minimum value of p_2 is also *nil*, the maximum theoretical height of the wheel above the tail-water surface is equal to the head due to one atmosphere. Again,

$$\begin{aligned} g(h' + h'') &= gH = v_w' u_1 - v_w'' u_2 + \frac{v_4^2}{2} \\ &= v_1 \cos \gamma u_1 - u_2(u_2 - V_2 \cos \beta) + \frac{v_4^2}{2}. \end{aligned}$$

But

$$A_1 v_1 \sin \gamma = Q = A_2 v_2 \sin \delta = A_2 V_2 \sin \beta = A_4 v_4;$$

and hence, taking

$$\begin{aligned} v_2 &= \sqrt{\mu_2} \cdot v_1, \quad V_2 = \sqrt{\mu_3} \cdot v_1, \quad v_4 = \sqrt{\mu_4} \cdot v_1, \\ gH &= v_1(u_1 \cos \gamma + \sqrt{\mu_3} \cdot u_2 \cos \beta) - u_2^2 + \frac{\mu_4 v_1^2}{2}, \end{aligned}$$

and therefore

$$\begin{aligned}\frac{2}{\mu_4}(gH + u_2^2) &= v_1^2 + 2v_1 \frac{u_1 \cos \gamma + \sqrt{\mu_3} \cdot u_2 \cos \beta}{\mu_4} \\ &= v_1^2 + 2v_1 u_2 \cdot \frac{\frac{r_1}{r_2} \cos \gamma + \sqrt{\mu_3} \cdot \cos \beta}{\mu_4} \\ &= v_1^2 + 2v_1 u_2 B,\end{aligned}$$

where $B = \frac{1}{\mu_4} \left(\frac{r_1}{r_2} \cos \gamma + \sqrt{\mu_3} \cos \beta \right)$.

Hence it follows that v_1 increases with u_2 , i.e., with the speed of the turbine, if

$$\frac{u_2^2}{gH} > \frac{B^2 \mu_4}{2 + B^2 \mu_4}.$$

A suction-tube is not used with an outward-flow turbine, but a similar result is obtained by adding a surrounding stationary casing with bell-mouth outlet. A similar diffuser might be added with effect to a Jonval turbine working without a suction-tube below the tail-water. The theory of the diffuser is similar to that of the suction-tube.

8. Losses and Mechanical Effect.—The losses may be enumerated as follows:

I. The loss (L_1) of head in the channel by which the water is taken to the turbine.

$$L_1 = f_1 \frac{l}{m} \frac{v_0^2}{2g},$$

f_1 being the coefficient of friction with an average value of .0067, l the length of the channel of approach, m its mean hydraulic depth, and v_0 the mean velocity in the channel.

L_1 is generally inappreciable in the case of turbines of the inward- and axial-flow types, as they are usually supplied with water from a large reservoir in which v_0 is sensibly nil.

If A_0 is the sectional area of the supply-channel, then

$$A_0 v_0 = Q = A_1 v_1 \sin \gamma,$$

and

$$L_1 = f_1 \frac{l}{m} \left(\frac{A_1 \sin \gamma}{A_0} \right)^2 \frac{v_1^2}{2g}.$$

II. The loss (L_2) of head in the guide-passages.

This loss is made up of:

(a) The loss due to resistance at the entrance into the passages;

(b) The loss due to the friction between the fluid and the fixed blades;

(c) The loss due to the curvature of the blades;

(d) The loss of head on leaving the guide-passages.

These four losses may be included in the expression

$$L_2 = f_2 \frac{v_1'^2}{2g},$$

f_2 being a coefficient which has been found to vary from .025 to .2 and upwards. An average value of f_2 is .125, but this is somewhat high for good turbines.

NOTE.—In *impulse* turbines f_2 has been found to vary from .11 to .17.

III. The loss (L_3) due to shock at entrance into the wheel.

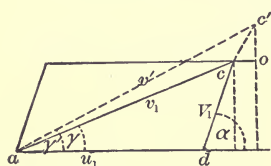


FIG. 307.

In order that there may be no shock at entrance, the relative velocity (V_1) must be tangential to the lip of the vane. For any other velocity ($v_1' = ac'$) and direction ($dac' = \gamma'$) of the water at entrance, evidently

$$\begin{aligned} L_3 = \text{the loss of head} &= \frac{(cc')^2}{2g} = \frac{(c'o)^2 + (co)^2}{2g} \\ &= \frac{(v' \sin \gamma' - v_1 \sin \gamma)^2}{2g} + \frac{(v' \cos \gamma' - v_1 \cos \gamma)^2}{2g} \\ &= \frac{(v' \sin \gamma' - V_1 \sin \alpha)^2}{2g} + \frac{(v' \cos \gamma' - \bar{v}_1 - V_1 \cos \alpha)^2}{2g}. \end{aligned}$$

Generally co is small, and L_3 is always nil when the turbine is working at full pressure and at the normal speed.

This loss of head in shock caused by abrupt changes of section, and also at an angle, may be avoided by causing the section to vary gradually, and by substituting a continuous curve for the angle.

IV. The loss (L_4) of head due to friction, etc., in passing through the wheel-passages, including the loss due to leakage in the space between the guides and the inlet-surface. This loss may be expressed in the form

$$L_4 = f_4 \frac{V_2^2}{2g} = f_4 \left(\frac{A_1 \cos \gamma}{A_2 \sin \beta} \right)^2 \frac{v_1^2}{2g},$$

where f_4 varies from .10 to .20.

NOTE.—The loss of head due to skin-friction often governs the dimensions of a turbine, and renders it advisable, in the case of high falls, to employ small high-speed turbines.

V. The loss of head (L_5) due to the abrupt change of section between the outlet-surface and the suction-tube.

As in III, $v_2 (= fh)$ is suddenly changed into $v_2' (= fh')$, and the loss of head is

$$L_5 = \frac{(hh')^2}{2g} = \frac{(hx)^2 + (h'x)^2}{2g} = \frac{(hx)^2}{2g},$$

since $h'x$ is very small and may be disregarded. Thus

$$L_5 = \frac{(V_2 \sin \beta - v_3')^2}{2g},$$

v_3' being the component of $v_2' (fh')$ in the direction of the axis of the suction-tube.

If there is no abrupt change of section between the outlet-surface and the tube, L_5 is nil.

VI. The loss of head (L_6) due to friction in the suction-tube. Assume that the velocity v_3 of flow in the tube is equal

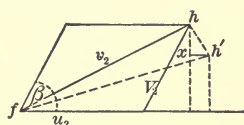


FIG. 308.

to v_2' , the velocity with which the water leaves the turbine. Also let A_3 be the sectional area of the tube. Then

$$L_6 = f_6 \frac{l'}{m'} \frac{v_3^2}{2g} = f_6 \frac{l'}{m'} \left(\frac{A_1 \sin \gamma^2 v_1^2}{A_3} \right) \frac{v_1^2}{2g},$$

$f_6 (= f_1)$ being the coefficient of friction with an average value of .0067, l' the length of the tube, and m' its mean hydraulic depth.

VII. The loss (L_7) of head due to entrance to sluice at base of tube. This loss may be expressed in the form

$$L_7 = f_7 \frac{v_4^2}{2g} = f_7 \left(\frac{A_1 \sin \gamma}{A_4} \right)^2 \frac{v_1^2}{2g},$$

the average value of f_7 being about .03.

VIII. The loss (L_8) of head due to the energy carried away by the water on leaving the suction-tube.

$$L_8 = \frac{v_4^2}{2g},$$

and v_4 usually varies from $\frac{1}{6} \sqrt{2gH}$ to $\frac{2}{7} \sqrt{2gH}$.

In good turbines the loss should not exceed 6 per cent. It might be reduced to 3 per cent, or even to 1 per cent, but this would largely increase the skin-friction.

IX. The loss of head (L_9) produced by the friction of the bearings.

$$L_9 = \mu W \frac{\rho}{r_1} u_1,$$

μ being the coefficient of journal friction, W the weight of the turbine and of the water it contains, and ρ the radius of the journal.

Hence the total loss of head

$$= L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_9 = L,$$

and the total *mechanical effect*

$$= wQ \left(\frac{v_1^2}{2g} - L \right).$$

NOTE.—If there is no suction-tube, $L_5 = 0 = L_6 = L_7 = L_8$, and the total loss becomes

$$L_1 + L_2 + L_3 + L_4 + L_9 + \frac{v_2^2}{2g} + \left\{ \begin{array}{l} \text{fall from outlet-surface to} \\ \text{tail-water surface.} \end{array} \right.$$

EXAMPLE.—A vortex turbine, with a draft-tube of the same sectional area as that of the outlet-orifice openings, passes 100 cu. ft. of water per second under the head of $9\frac{1}{2}$ ft. The exterior and interior diameters are in the ratio of 5 to 4, and the outlet- and inlet-areas are in the ratio of 9 to 10. The direction of the water at the inlet and the outlet lip angle are given by $\sin \gamma = .25 = \sin \beta$. The water leaves the tube through a sluice having a sectional area 10 per cent greater than that of the outlet-orifice area. The outlet velocity of whirl is nil, i.e., $\delta = 90^\circ$.

Disregard the losses L_1, L_3, L_5, L_6 , and L_9 .

The loss of head to inlet	$= f_2 \frac{v_1^2}{2g}$
“ “ “ “ in wheel-passages	$= f_4 \frac{V^2}{2g}$
“ “ “ “ at sluice-entrance	$= f_7 \frac{v_4^2}{2g}$
“ “ “ “ carried away by water	$= \frac{v_4^2}{2g}$

Hence the total loss of head

$$= \frac{V^2}{2g} \left\{ f_2 \left(\frac{v_1}{V} \right)^2 + f_4 + (1 + f_7) \left(\frac{v_4}{V} \right)^2 \right\}.$$

But, by the condition of continuity,

$$A_1 v_1 \sin \gamma = Q = A_2 V_2 \sin \beta = A_4 v_4.$$

Therefore

$$\frac{v_1}{V_2} = \frac{A_2}{A_1} \cdot \frac{\sin \beta}{\sin \gamma} = .9; \quad \frac{v_4}{V_2} = \frac{A_2}{A_4} \sin \beta = \frac{.25}{1.1} = \frac{1}{4.4}.$$

Hence the total loss of head

$$= \frac{V_2^2}{2g} \left\{ .1 \times .81 + .126 + \frac{1.03}{(4.4)^2} \right\} = \frac{V_2^2}{2g} \times .26,$$

taking $f_2 = .1$, $f_4 = .126$, and $f_7 = .03$.

Again, the “*useful*” head

$$\begin{aligned} &= \frac{u_1 v_w'}{32} = \frac{1}{32} \cdot \frac{5}{4} \cdot u_2 \cdot v_1 \cos \gamma \\ &= \frac{5}{128} V_2 \cos \beta \cdot \frac{A_2 V_2}{A_1} \cdot \frac{\sin \beta \cos \gamma}{\sin \gamma} = \frac{V_2^2}{2g} \cdot \frac{135}{64} = 2.109375 \text{ ft.} \end{aligned}$$

Therefore

$$9.5 = \frac{V_2^2}{2g} (.26 + 2.1094) = \frac{V_2^2}{2g} \times 2.3694,$$

or

$$\frac{V_2^2}{2g} = 4 \text{ ft., approx., and } V_2 = 16 \text{ ft. per sec.}$$

The *useful work* per lb. of water

$$= 4 \times \frac{135}{64} = 8.4375 \text{ ft.-lbs.}$$

The *work consumed* in hydraulic resistances per lb.

$$= 4 \times .26 = 1.04 \text{ ft.-lbs.}$$

The *total work* per lb. of water = 9.4775.

$$\text{The efficiency} = \frac{8.4375}{9.4775} = .89.$$

$$\text{The H.P.} = \frac{62\frac{1}{2} \cdot 100 \cdot 9\frac{1}{2}}{550} \times .89 = 96.1.$$

$$A_2 = \frac{100}{V_2} \operatorname{cosec} \beta = \frac{400}{16} = 25 \text{ sq. ft. and } A_1 = \frac{A_2}{.9} = 27.78 \text{ sq. ft.}$$

Again,

$$v_1 = .9V_2 = 14.4 \text{ ft. per sec.},$$

$$u_2 = V_2 \cos \beta = 16\sqrt{\frac{15}{16}} = 15.492 \text{ ft. per sec.},$$

and

$$u_1 = \frac{5}{4}u_2 = 19.365 \text{ ft. per sec.}$$

Also,

$$\frac{u_1}{v_1} = \frac{\sin(\alpha + \gamma)}{\sin \alpha} = \cos \gamma + \cot \alpha \sin \gamma,$$

or

$$\cot \alpha = \frac{u_1}{v_1} \operatorname{cosec} \gamma - \cot \gamma = \frac{19.365}{14.4} \times 4 - 3.873 = 1.5061.$$

and

$$\alpha = 33^\circ 35'.$$

If the diameter of the tube is equal to that of the outlet-surface, viz., 4 ft., and if its lower edge is rounded so that $f_7 = 0$, then

$$\begin{aligned} \text{energy per lb. of water carried away} &= \frac{v_3^2}{2g} \\ &= \frac{v_2^2 \left(\frac{A_2}{A_3}\right)^2}{2g} = \frac{V_2^2 \sin^2 \beta \left(\frac{25}{\pi \cdot 16}\right)^2}{2g} = \frac{V_2^2}{2g} \frac{1}{16} \left(\frac{175}{352}\right)^2 = \frac{V_2^2}{2g} \times .015448. \end{aligned}$$

The loss in shock in draft-tube

$$= \frac{(v_2 - v_3)^2}{2g} = \frac{v_2^2 \left(1 - \frac{A_2}{A_3}\right)^2}{2g} = \frac{V_2^2}{2g} \frac{1}{16} \left(1 - \frac{175}{352}\right)^2 = \frac{V_2^2}{2g} \times .04705.$$

Thus the total loss now becomes

$$\frac{V_2^2}{2g} (.081 + .126 + .05448 + .04705) = \frac{V_2^2}{2g} \times .2695.$$

$$\text{As before, the useful head} = \frac{V_2^2}{2g} \times 2.1094.$$

Therefore the total head $= \frac{V_2^2}{2g} \times 2.3789,$

and the efficiency $= \frac{2.1094}{2.3789} = .886.$

Also,

$$9.5 = \frac{V_2^2}{2g} \times 2.3789, \text{ or } \frac{V_2^2}{2g} = 3.993 \text{ ft., and } V_2 = 15.987 \text{ ft. per sec.}$$

If there is no draft-tube, $\frac{v_2^2}{2g}$ must be substituted for $(1 + f_7) \frac{v_2^2}{2g}$, and $\frac{v_2^2}{2g} = \frac{V_2^2}{2g} \frac{1}{16}.$

Thus the total loss of head is now

$$\frac{V_2^2}{2g} (.081 + .126 + .0625) = \frac{V_2^2}{2g} \times .2695,$$

which exceeds the loss of head *with* a draft-tube by $\frac{V_2^2}{2g} \times .0095 = .038$ ft., which is less than four hundredths of a foot and is practically inappreciable.

EXAMPLES.

1. A downward-flow turbine of 24 ins. internal diameter passes 10 cu. ft. of water per second under a head of 31 ft.; the depth of the wheel is 1 ft. and its width 6 ins. Find the efficiency, assuming the whirling velocity at outlet to be nil. *Ans.* .997.

2. A downward-flow turbine of 5 ft. external diameter passes 20 cu. ft. of water per second under a head of 4 ft., the depth of the wheel being 1 ft. The water enters the wheel at an angle at 60° with the vertical, the receiving-lip of the wheel-vanes is vertical, and the velocity of whirl at outlet is nil. Find the internal diameter and the speed in revolutions per minute. *Ans.* 4.6 ft.; 46.53.

3. A downward-flow turbine has an internal diameter of 24 ins.; the breadth of the wheel is 6 ins.; the turbine passes 33 cu. ft. per second under an effective head of 16 ft. Assuming the whirling velocity at outlet to be nil, find the efficiency and power of the turbine. If the vane-lip at inlet is vertical, find the direction of the vane at outlet, and the speed of the turbine in revolutions per minute.

Ans. .931; 55.865 H.P.; $\beta = \gamma = 21^\circ 2'$; 166.7.

4. Discuss the preceding example on the assumption that the peripheral speed at outlet (u_2) is equal to the speed of the water at that point relatively to the wheel (V_2).

Ans. .928; 55.715 H.P.; $\beta = 21^\circ 47'$ and $\gamma = 20^\circ 21'$.

5. An axial-flow impulse turbine of 5 ft. mean diameter passes 170 cu. ft. of water per second under an effective head of 8.6 ft.; the depth of the wheel is .9 ft. At what angle should the water enter the wheel to give an efficiency of 81 per cent, the width of the wheel being constant and disregarding hydraulic resistances? $v_w'' = 0$. *Ans.* $= 27^\circ 16'$.

Also find (a) the velocity with which the water enters the wheel; (b) the speed of the turbine in revolutions per minute; (c) the directions of the vane-edges at inlet and outlet; (d) the velocity of the water as it leaves the wheel; (e) the power of the turbine.

Ans. (a) 23.46 ft. per second; (b) 45.08; (c) $\alpha = 130^\circ 10'$; $\beta = 42^\circ 19'$; (d) 10.748 ft. per second; (e) 148.65 H.P.

If, instead of assuming that the whirling velocity at exit is nil, it is assumed that the peripheral speed (u_2) of the wheel at the mean radius is equal to the relative velocity (V_2) of the water at exit, show how the several results are affected.

Ans. $\gamma = 25^\circ 8'$; (a) 23.46 ft. per second; (b) 54.638; (c) $\alpha = 124^\circ 49'$, $\beta = 44^\circ 6'$; (d) 10.748 ft. per second; (e) 148.65 H.P.

Also show how the results are affected when it is assumed that the hydraulic resistances necessitate an increase of $12\frac{1}{2}$ per cent in the head equivalent to the velocity with which the water enters the wheel, and an increase of 10 per cent in the head equivalent to the relative velocity (V_2) at outlet.

Ans. When $v_w'' = 0$ (a) 22.12 ft. per second; (b) 44.21; (c) $\alpha = 147^\circ 50'$, $\beta = 27^\circ 44'$; (d) 10.748 ft. per second; (e) 148.65 H.P.
When $u_2 = V_2$ (a) 22.119 ft. per second; (b) 50.97; (c) $\alpha = 123^\circ 19'$, $\beta = 47^\circ 28'$; (d) 10.748 ft. per second; (e) 148.65 H.P.

If the turbine has 65 guide-blades of .2-in. thickness and 63 wheel-vanes of .4-in. thickness, find the widths of the inlet and outlet openings.

Ans. If $v_w'' = 0$, $d_1 = 4.214$ ft., $d_2 = 2.83$ ft.

If $u_2 = V_2$, $d_1 = 1.78$ ft., $d_2 = 1.48$ ft.

6. The efficiency of an axial-flow turbine of 4 ft. mean diameter is 90 per cent, and it passes 12 cu. ft. per second under an effective head of 40 ft. At the mean radius the water enters at an angle of 30° with the wheel's face, and the whirling velocity at outlet is nil. Find (a) the velocity with which the water enters and leaves the wheel; (b) the directions of the vane at inlet and outlet; (c) the sectional areas of the inlet- and outlet-orifices; (d) the speed of the wheel in revolutions per minute; (e) the power of the turbine.

Ans. (a) 32 ft. per second, 16 ft. per second; (b) $\alpha = 49^\circ 6'$, $\beta = 21^\circ 3'$; (c) .75 sq. ft.; (d) 198.39; (e) $49\frac{1}{4}$ H.P.

7. An axial-flow turbine of 5 ft. mean radius passes 212 cu. ft. of water per second under a total effective head of 12.1 ft. At the mean radius, the direction of the inflowing water makes an angle of 70° with the vertical, and the vane-lip at the outlet makes an angle of 17° with the wheel's periphery. If the whirling velocity at the outlet-surface is nil, find (a) the velocity with which the water must enter the wheel to give an efficiency of .953 per cent. Also find (b) the direction of the vane-lip at outlet; (c) the speed of the wheel in revolutions per minute; (d) the widths and areas of the inlet- and outlet-orifices; (e) the power of the turbine.

Ans. (a) 19.9 ft. per second; (b) $\alpha = 81^\circ 25'$; (c) 37.67; (d) .991 ft., 31.148 sq. ft., 1.81 ft., 35.14 sq. ft.; (e) 277.709.

If the turbine has 41 guide-blades and 40 wheel-vanes, all of .25 in. thickness, find the widths of the inlet- and outlet-openings.

Ans. 1.23 ft.; 1.37 ft.

8. Write down the equations for Jouval's modification of Euler's turbine.

9. An axial-flow impulse turbine passes 170 cu. ft. of water per second under an effective head of 9.5 ft., the depth of the wheel being .9 ft. and its mean radius 4.2 ft. The vane-lip at the outlet makes an angle of 72° with the vertical. Assuming that the whole of the effective head is transformed into useful work, and that the whirling velocity at the outlet-surface is nil, find (a) the inclination to the horizontal of the outlet-lip of

the guide-vane; (b) the direction of the inlet-lip of the wheel-vane; (c) the efficiency; *first* neglecting hydraulic resistances, and *second* taking these resistances into account.

Ans. First. (a) $59^{\circ} 52'$; (b) $60^{\circ} 16'$; (c) .905.

Second. (a) $47^{\circ} 21'$; (b) $85^{\circ} 18'$; (c) .804.

10. In the preceding example find the inlet- and outlet-orifice areas in the two cases.

Ans. First. 8.38 sq. ft.; 22.4 sq. ft.

Second. 10.45 sq. ft.; 32.08 sq. ft.

If there are 62 wheels and 66 guide-vanes, the thickness of the latter being .2 in. and of the former .4 in., find the width of the inlet-orifices.

Ans. First. .409 ft.; 1.26 ft. Second. .508 ft.; 1.81 ft.

11. An axial-flow turbine passes 200 cu. ft. of water per second under a head of 14 ft., the depth of the wheel being 1 ft. The mean radius of the wheel is 3 ft.; the areas of the inlet- and outlet-surfaces are in the ratio of 7 to 8; the water enters the wheel at an angle of 21° to the wheel face, and the outlet edge of the vane makes an angle of 16° with the face. Find the speed, efficiency, and power of the turbine, and also the direction of the inlet-lip of the vanes, $v_w'' = 0$.

Ans. 73.69 revolutions per minute; .954; 325.243 H.P.;

$\alpha = 65^{\circ} 57'$.

12. A downward-flow turbine of $3\frac{1}{11}$ ft. mean diameter and of the impulse type is supplied with $5\frac{1}{2}$ cu. ft. of water per second under a head of 400 ft. and makes 500 revolutions per minute. The water enters the wheel at an angle of $\sin^{-1}.6$ with the horizontal, and the depth of the wheel is 1 ft. The water leaves the turbine with a velocity of 60 ft. per second. Determine the whirling velocity at outlet, the direction in which the water leaves the turbine, the efficiency, and the horse-power.

Ans. 17.725 f/s; $72^{\circ} 49'$; .86; 214.8.

13. In an A. F. impulse turbine of 4 ft. diameter, 1 ft. deep, and with a 6-in. width of opening at inlet and outlet, the efficiency (η) = .8; $\beta = 30^{\circ}$; $\gamma = 30^{\circ}$; $V_1 = u_2$. Determine the inlet-lip angle (α), the effective fall, the delivery (Q) (disregarding thickness of vanes), the H.P. and the number of revolutions per minute.

Ans. $\alpha = 75^{\circ}$; 1.366 ft.; 29.39 cu. ft. per second; 6.322 H.P.;

44.63.

14. An axial-flow reaction turbine of 7 ft. mean diameter passes 198 cu. ft. of water per second under a total head of 13.5 ft., the depth of the wheel being 1 ft. At the inlet-surface the vane-lip is vertical and the water leaves the wheel vertically. If the inlet width of the wheel is 1 ft. and the outlet width $1\frac{1}{2}$ ft., find the direction in which the water enters the wheel, the direction of the lip at outlet, the inlet and outlet areas, the H.P. of the turbine, and its efficiency.

Ans. $24^{\circ} 4'$; $19^{\circ} 40'$; 22 and $27\frac{1}{2}$ sq. ft.; 285.525; .94.

15. An axial-flow turbine is to be used for raising water. Explain

how the vanes should be arranged, and show how to determine the efficiency.

16. In an A. F. impulse turbine, working under a head of 100 ft., the direction in which the water enters at the mean radius makes an angle of $23^{\circ} 16'$ with the vertical and leaves the wheel without velocity of whirl. The depth of the wheel is 1 foot, and the inlet velocity (v_1) is equal to the linear velocity (u_1) of the wheel's surface at the mean radius. The mean diameter of the wheel is $3\frac{1}{2}$ ft., and its width is 6 ins. Find the blade angles at inlet and outlet, the efficiency, the speed in revolutions per minute, the amount of water passing through the turbine per second, and the H.P.

Ans. $56^{\circ} 38'$; $64^{\circ} 52'$; .782; $436\frac{4}{11}$; $404\frac{1}{8}$ cu. ft.; $3592\frac{3}{4}$.

17. Water is delivered to an O. F. turbine at a radius of 24 in. with a whirling velocity of 20 ft. per second, and leaves in a reverse direction at a radius of 4 ft. with a whirling velocity of 10 ft. per second. If the linear velocity of the inlet-surface is 20 ft. per second, find the head equivalent to the work done in driving the wheel.

Ans. 24.8 ft.

18. An outward-flow turbine of 9.5 in. internal diameter works under an effective head of 270 ft. Find the speed in revolutions per minute, assuming that the whirling velocity at the inlet-surface relatively to the wheel is nil and that the efficiency is unity.

Ans. 2242.

19. An outward-flow turbine, whose external and internal diameters are 8 ft. and $5\frac{1}{2}$ ft. respectively, makes 26 revolutions per minute under an effective head of 4 ft. The water enters the wheel in a direction making an angle of 30° (γ) with the direction of motion at the point of entrance. Determine the angles of the *moving vane* at ingress and egress, the efficiency being .85. Also find the energy per pound of water carried away by the water as it leaves the turbine, $v_w'' = 0$.

Ans. $\alpha = 130^{\circ} 2'$; $\beta = 29^{\circ} 38'$; .6 ft.-lbs.

20. A radial outward-flow turbine of the impulse type passes $8\frac{1}{2}$ cu. ft. of water per second under an effective head of 560 ft.; the width of the wheel is $7\frac{1}{2}$ in.; the radius to the outlet-surface is 1.15 times the radius to the inlet-surface; the linear velocity of the inlet-surface is 87 ft. per second; the direction of the water at entrance makes an angle of 17° with the wheel's periphery. Find (a) the efficiency; (b) the lip angles; (c) the areas of the inlet- and outlet-orifices, neglecting *first* hydraulic resistances, and *second* taking these resistances into account ($v_w'' = 0$).

Ans. First. (a) .879; (b) $\alpha = 149^{\circ} 31'$ and $\beta = 33^{\circ} 21'$; (c) .1535 sq. ft., and .1291 sq. ft. Second. (a) .767; (b) $\alpha = 153^{\circ} 44'$ and $\beta = 28^{\circ} 55'$; (c) .176 sq. ft. and .154 sq. ft.

21. Construct a Fourneyron turbine for a fall of 5 ft. with 30 cu. ft. of water per second, $\alpha = 80^{\circ}$, $\gamma = 30^{\circ}$, $\frac{r_2}{r_1} = 1.35$. Assume $u_2 = V_2$, and neglect hydraulic resistances.

Ans. $\beta = 16^{\circ} 42'$; $A_1 = 4.29$ sq. ft.; $A_2 = 5.8189$ ft.; $\eta = .915$; if $r_1 = 1.8$ ft., then $d_1 = d_2 = .38$ ft.

22. In an impulse outward-flow turbine of 10 B.H.P., working under a head of 9 ft., $\gamma = 22\frac{1}{2}^\circ$; $180^\circ - \alpha = 37\frac{1}{2}^\circ$; $\beta = 45^\circ$; $9(r_2 - r_1) = r_1$; $d_1 = .2r_1$. There is a loss of 5 per cent due to friction in the velocity at entrance. Find the efficiency (η), the volume of water passed per second, and the diameter of the turbine.

Ans. .705; 13.869 cu. ft.; 2.249 ft.

23. A turbine delivers 1 cu. ft. of water per second. The water leaves the outlet periphery radially ($v_w'' = 0$). The vane-lip at inlet is radial ($\alpha = 90^\circ$). The direction of inflow makes an angle of 60° with the wheel's periphery. The radius of inlet-surface is 2 ft. The number of revolutions per minute is 100. If the efficiency is 90 per cent, find the head and the effective work done.

Ans. 15.243 ft.; 1.5625 H.P.

24. One cubic foot of water per second enters a radial O.F. impulse wheel of 2 ft. external and $1\frac{1}{2}$ ft. internal diameter, at an angle of 60° with the radius, and leaves without whirl. The effective head is 400 ft. The peripheral speed at the outlet-surface is $20\sqrt{3}$ ft. per second. Determine α , v_2 , the outlet and inlet areas and depths, the H.P. and efficiency.

Ans. 1.15 sq. ins., 1.8 sq. ins.; 183 ins., .39 ins.; 12.8; .28.

25. In a radial-flow reaction turbine with radial inlet-lips, if $d_2 = 2d_1$ and $\gamma = \tan^{-1} 4$, show that the reciprocal of the efficiency is $1 + \tan \beta$ if the whirling velocity at outlet is nil.

26. An O.F. impulse turbine of $3\frac{1}{2}$ ft. exterior and 3 ft. interior diameter passes 100 cu. ft. of water per second under a head of 625 ft. At entrance the direction of the water makes an angle of 30° with the periphery. If the relative and peripheral speeds at outlet are equal, determine the direction and magnitude of the velocity of the water on leaving the wheel, the efficiency, and the speed in revolutions per minute. Disregard hydraulic resistances.

Ans. If $d_1 = d_2$, $v_2 = 91.065$ ft. per sec.; $\delta = 70^\circ 14\frac{1}{2}'$; $\eta = .79$; $N = 734.8$.

If $A_1 = A_2$, $v_2 = 109.435$ " " ; $\delta = 66^\circ 02'$; $\eta = .70$; $N = 734.8$.

27. A radial impulse turbine passes $8\frac{1}{2}$ cu. ft. of water under an effective head of 560 ft. The direction of the entering water is inclined at 17° to the wheel's periphery. The linear speed of the inlet-surface is 87 ft. per second. Assuming that the velocity of whirl at the outlet is nil, and disregarding hydraulic resistances, find (a) the efficiency; (b) the velocity with which the water enters the wheel; (c) the velocity of the water as it leaves the wheel; (d) the sectional areas of the inflowing and outflowing stream; (e) the direction of the vane-lip at inlet; (f) the power of the turbine.

The radii of the inlet- and outlet-surfaces are $4\frac{1}{2}\frac{1}{4}$ ft. and $4\frac{1}{8}$ ft. respectively. Find (g) the direction of the vane edge at outlet.

Ans. (a) .879; (b) 189.31 ft. per sec.; (c) 65.86 ft. per sec.;

(d) .15356 sq. ft., .129 sq. ft.; (e) $\alpha = 149^\circ 31'$;

(f) 475.43 H.P.; (g) $\beta = 41^\circ 3'$.

28. In the preceding example show how the results are affected by

taking .94 as the coefficient of velocity in calculating the velocity with which the water enters the wheel, and assuming that $\frac{1}{10} \frac{V_2^2}{2g}$ is the frictional loss of head in the passages.

Ans. (a) .826; (b) 177.955 ft. per sec.; (c) 36.7348 ft. per sec.;
(d) .163 sq. ft.; .2314 sq. ft.; (e) $\alpha = 147^\circ 57'$;
(f) 446.9 H.P.; (g) $\beta = 25^\circ 54'$.

29. In an I.F. turbine the radius of the inlet-surface is twice that of the outlet-surface; the linear velocity of the inlet-surface is one half that due to the head; the water enters the wheel with a velocity of flow (v_r') equal to one eighth that due to the head, and the sectional area of the water-way is constant from inlet to outlet. Find the angle between the discharging-lip of the vane and the wheel's periphery, the whirling velocity at the outlet-surface being nil.

Ans. $\cot^{-1} 2$.

30. In a vortex turbine the depth of the inlet-orifices is one eighth of the diameter of the wheel $\left(= \frac{D_1}{8} \right)$ and $\frac{25}{32}$ of the depth of the outlet-orifices. The width of the wheel is one tenth of the diameter $\left(= \frac{D_1}{10} \right)$. The inlet-lip of the vanes is radial, and the water enters at an angle of 30° with the inlet periphery. Find the size, speed, and efficiency of the turbine in terms of the supply of water Q and the effective head H . Also find the direction of the outlet edge of the vanes.

Ans. I. Assume $v_w'' = 0$. Then $r_1 = .458 \frac{Q^{\frac{1}{3}}}{H^{\frac{1}{4}}}$;

No. of revolutions per minute $= 109.5 \frac{H^{\frac{1}{4}}}{Q^{\frac{1}{3}}}$;

$\eta = .863$; $\beta = 35^\circ 11'$.

II. Assume $u_2 = V_2$. Then $r_1 = .44 \frac{Q^{\frac{1}{3}}}{H^{\frac{1}{4}}}$;

No. of revolutions per minute $= 122.39 \frac{H^{\frac{1}{4}}}{Q^{\frac{1}{3}}}$;

$\eta = .8146$; $\beta = 44^\circ 48'$.

31. A vortex turbine, with a wheel of 2 ft. diameter and 6 ins. breadth, passes 10 cu. ft. of water per second under a head of 32 feet. Find the inclination of the guides and the power of the turbine. Assume $u_2 = V_2$, $\alpha = 90^\circ$, and the efficiency $= 1$. *Ans.* $5^\circ 41'$; $36 \frac{4}{11}$ H.P.

32. An inward-flow turbine has an internal radius of 12 ins. and an external radius of 24 ins.; the water enters at 15° with the tangent to the circumference, and is discharged radially; the velocity of radial flow is 5 ft. at both circumferences; the velocity of outer periphery of wheel is 16 ft. per second. Find the angles of the vanes at the inner and outer circumferences, and the useful work done per pound of fluid.

Ans. $\beta = 32^\circ$; $d = 118^\circ 1'$; 9.35 ft.-lbs.

33. For a supply of 64 cu. ft. per second, under a head of 81 ft., determine the speed, size, H.P., and efficiency of a vortex turbine in which $d_1 = r_1 = 3d_2 = 5 \times$ width of wheel, assuming that there is no velocity of whirl at outlet.

34. A radial I. F. reaction turbine, with or without draft-pipe, passes 113 cu. ft. of water under an effective head of 13 ft. The radius of the inlet-surface is 1.169 times the radius of the outlet-surface, and the ratio of the outlet to the inlet area is .92. The vane-lip at outlet makes an angle of 15° with the wheel's periphery, and the water enters at an angle of 12° with the wheel's periphery. The sectional area of the draft-tube (if there is one) at the point of discharge is 1.035 times the sectional area of the outlet-orifice. Show that the useful work per pound of water is 11.117 ft.-lbs., and that the work consumed in hydraulic resistance (Art. 8, page 531) is nearly 1.882 ft.-lbs.; also find A_1 , A_2 , v_2 , and the efficiency.

Ans. (a) 28.2975 sq. ft.; 26.03 sq. ft.; (b) 4.34 ft. per sec.; .855.

35. In the preceding example, if the radius of the outlet-surface is 4 ft., find (a) the speed of the wheel in revolutions per minute; also find (b) the depth of the wheel at inlet and outlet, the guide-vanes being 40 and the wheel-vanes 41 in number, and the thickness of the former being $\frac{3}{16}$ inch and of the latter $\frac{1}{4}$ inch. *Ans.* (a) 38.656; (b) 1.23 ft., 1.35 ft.

36. In example 34 find the efficiency if the diameter of the draft-tube is made the same as the diameter of the outlet-surface, the lower edge of the tube being rounded. What will be the "loss in shock" in the tube per pound of water? *Ans.* .864; .077 ft.-lbs.

37. An inward-flow turbine has an external diameter of 3 ft. and an internal diameter of 2 ft. It passes 12 cu. ft. of water per second under an effective head of 40 ft. The water enters the wheel at an angle of 30° with the wheel's periphery, and the depth of the outlet-orifices is twice the depth of the inlet-orifices. The efficiency of the turbine is .9. Disregarding friction, find (a) the vane-angles at inlet and outlet; (b) the velocity with which the water leaves the wheel; (c) the speed of the turbine in revolutions per minute; (d) the velocity with which the water enters the wheel; (e) the areas of the outlet- and inlet-orifices; (f) the power of the turbine ($v_w'' = 0$).

Ans. (a) $\alpha = 105^\circ 09'$, $\beta = 35^\circ 35'$; (b) 16 ft. per sec.; (c) 198.39; (d) $42\frac{3}{8}$ ft. per sec.; (e) .5625 sq. ft.; .75 sq. ft.; (f) $49\frac{1}{11}$ H.P.

38. In an inward-flow reaction turbine of 6.27 H.P. the radial velocity of flow is constant from inlet to outlet and is 12 ft. per second. The water, with a velocity of 60 ft. per second, enters at $11^\circ 32'$ with the wheel's periphery, which has a linear speed of 50 ft. per second. The diameters of the outlet- and inlet-surfaces are 1 and 2 ft. respectively. Find the tip angles, the head, the efficiency, and the quantity of water passing through the turbine per second.

Ans. $\alpha = 126^\circ 12'$; $\beta = 151^\circ 19'$; 91.86 ft.; 60%; 1 cu. ft.

39. An inward-flow radial impulse turbine of 4.5 ft. and 4 ft. external and internal radii passes $8\frac{1}{2}$ cu. ft. of water per second under an effective head of 560 ft. The direction of the entering water is inclined at 17° to the wheel's periphery, and the wheel has the same depth at the inlet- and outlet-surfaces. If the peripheral speed at the outlet-surface (u_2) is equal to the relative velocity of the water (V_2) with respect to the wheel, find (a) the efficiency; (b) the speed of the turbine in revolutions per minute; (c) the sectional areas of the stream at inlet and outlet; (d) the direction of the vane-outlet edge; (e) the velocity of the water as it leaves the wheel; (f) the power of the turbine.

Ans. (a) .873; (b) 209.94; (c) .15357 sq. ft., .13651 sq. ft.; (d) $\beta = 45^\circ 2'$; (e) 67.39 ft. per second; (f) 472.33 H.P.

40. In the preceding example examine how the results will be affected when hydraulic resistances are taken into account, allowing .94 as a coefficient of velocity for the water on entering the wheel, and assuming that the head equivalent to the relative velocity (V_2) on leaving the wheel is increased by 10 per cent.

Ans. (a) .865; (b) 193.185 revolutions per minute; (c) .163 sq. ft., .145 sq. ft.; (d) $\beta = 46^\circ 18'$; (e) 63.653 ft. per second; (f) 467.83 H.P.

41. An I. F. turbine of 4 ft. external diameter works under an effective head of 250 ft. Find the speed of the wheel in revolutions per minute, v_w'' being 0, the efficiency *unity*, and $\alpha = 90^\circ$. *Ans.* 427.

42. An I. F. turbine of 4 ft. external and 3 ft. internal diameter makes 360 revolutions per minute. The sectional area of flow is 3 sq. ft. and is the same in every part of the turbine. The direction of the inflowing water makes an angle of 30° with the wheel's periphery. Assuming that the whirling velocity at the outlet-surface is nil, find (a) the efficiency; (b) the H.P.; and (c) the delivery in cubic feet per minute. The total head is 200 ft. *Ans.* (a) .86; (b) 2476.8; (c) 7593.

43. An inward-flow turbine being required for an available head of 20 ft. and a discharge of 800 cu. ft. per minute, determine (a) the size and (b) the speed of the wheel; (c) the inclinations of the guide- and wheel-vanes; and (d) the efficiency of the turbine, assuming $r_2 = \frac{1}{2}r_1 =$ depth of wheel; $v_r' = \frac{1}{3}\sqrt{2gH}$; $v_w'' = 0$, $\alpha = 90^\circ$, and $d_1 = d_2$.

Ans. (a) $r_2 = .487$ ft., $r_1 = .974$ ft.; (b) 240 revolutions per minute; (c) $\gamma = 10^\circ 21'$, $\beta = 36^\circ 8'$; (d) $93\frac{3}{4}$ per cent.

44. A vortex turbine passes Q cu. ft. of water per second under an effective head of H ft. The inlet-lip of the vanes is radial, and the direction of the entering water makes an angle of $20^\circ 17'$ with the wheel's periphery. The areas of the inlet- and outlet-orifices are $\frac{\pi D_1^2}{8}$ and $\frac{\pi D_2^2}{5}$

respectively, and the width of the wheel is $\frac{D_1}{10}$, D_1 being the diameter of the inlet-surface. If the whirling velocity at the outlet-surface is nil, find (a) the efficiency; (b) the direction of the outlet edge of the vane;

(c) the velocity with which the water enters and leaves the wheel; (d) the speed of the wheel in revolutions per minute; (e) the diameters of the inlet- and outlet-surfaces.

Ans. (a) .863; (b) $\beta = 35^\circ 10'$; (c) $6.0677H^{\frac{1}{2}}$, $2.9627H^{\frac{1}{2}}$;

(d) $109.52 \frac{H^{\frac{1}{2}}}{Q^{\frac{1}{2}}}$; (e) $.915 \frac{Q^{\frac{1}{2}}}{H^{\frac{1}{2}}}$, $.733 \frac{Q^{\frac{1}{2}}}{H^{\frac{1}{2}}}$.

45. A vortex turbine passes 11 cu. ft. of water per second under a head of 35 ft.; the diameter of the outlet-surface is 2 ft. and its breadth 6 ins. Find the power of the turbine, disregarding friction and assuming that the whirling velocity at the outlet-surface is nil.

Ans. 43.5 H.P.

46. Find the H.P. developed by an I. F. turbine, of 3 ft. external and $1\frac{1}{2}$ ft. internal diameter, passing 900 tons of water per hour. The velocity of whirl at inlet ($v_{w'}$) is equal to that of the periphery and is 45 ft. per second; the outlet velocity of whirl is $16\frac{2}{3}$ ft. per second.

Ans. $46\frac{7}{8}$.

47. A turbine with radial vanes passes 3600 gallons per hour under an effective head of 36 ft. Find the peripheral speed and the inlet area so that the efficiency may be a maximum.

CHAPTER VIII.

CENTRIFUGAL PUMPS.

1. General Statement.—If an hydraulic motor is driven in the reverse direction, and supplied with water at the point from which the water originally proceeded, the motor becomes a pump. All turbines are reversible, and may therefore be converted into pumps, but no pump has yet been constructed of an inward-flow type. The ordinary centrifugal pump is an outward-flow machine.

Before the pump can be put into action it must be filled, and this can be effected through an opening (closed by a plug) in the casing when the pump is under water, or, if the pump is above water, by creating a vacuum in the pump-case by means of an air-pump or a steam-jet pump, when the water must necessarily rise in the suction-tube.

At first the water rotates as a solid mass, and delivery commences when the speed is such that the head due to centrifugal force $\left(\frac{u_2^2 - u_1^2}{2g}\right)$ exceeds the lift. This speed may be afterwards reduced, providing a portion of the energy is utilized at exit.

As soon as the pump, which is keyed on to a shaft driven by a belt or other means, commences to work, the water rises in the suction-tube and enters the eye of the pump-disc on one side, or divides and enters on both sides.

As in turbines, the wheel-blade tips are so curved as to receive, at a specified normal speed, the inflowing water without shock. The water leaves the disc with a more or less

considerable velocity, and impinges upon the fluid mass flowing around the volute, or spiral casing surrounding the disc, towards the discharge-pipe. This volute should have a section

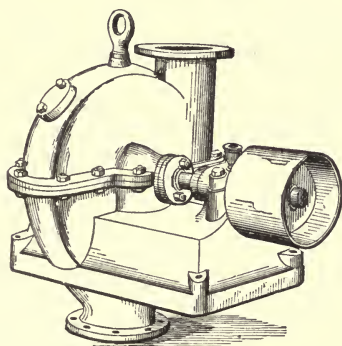


FIG. 309.

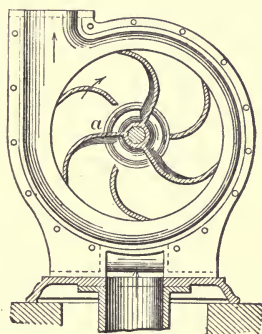


FIG. 312.

FIG. 310.

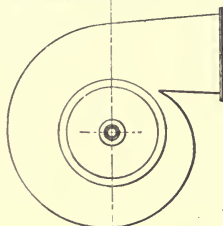
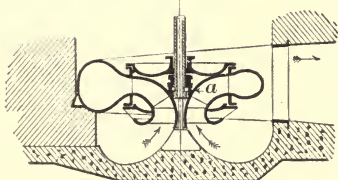


FIG. 311.

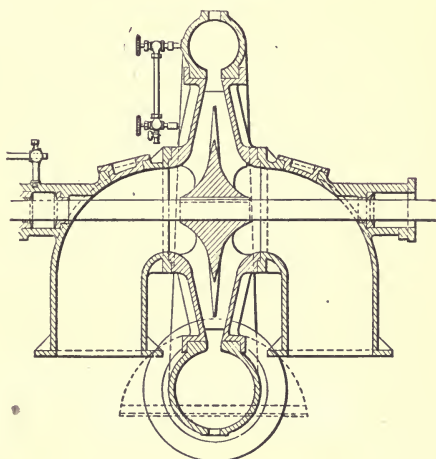


FIG. 313.

gradually increasing to the point of discharge, in order that the delivery across any transverse section of the volute may be uniform. This volute is also so designed as to compel rotation in one direction only, with a velocity corresponding to the velocity of whirl ($v_{w''}$) on leaving the fan. There are examples

of pumps in which the delivery is effected in all directions, and the water is guided to the outlet by a number of spiral blades.

A centrifugal pump is more economical and less costly for short lifts than a reciprocating pump, and has been known to give good and economic results for lifts as great as 40 ft. *to 100*

With compound centrifugal pumps very much greater lifts are economically possible.

There are three main differences between centrifugal pumps and turbines:

1st. The gross lift with a pump is greater, on account of frictional resistances, than the fall in the case of a turbine.

2d. The water enters the pump-fan chamber without any velocity of whirl ($v_w' = 0$), and leaves the fan with a velocity of whirl (v_w'') which should be reduced to a minimum in the act of lifting, but which is by no means small. In a turbine, on the other hand, the water has a considerable velocity of whirl (v_w') at entrance, while at exit the velocity of whirl (v_w'') is reduced to a minimum, and is generally *nil*.

3d. In a turbine the direction of the water as it flows into the wheel is controlled by guide-blades; whereas in the case of a pump, the direction of the water, as it flows towards the discharge-pipe, is controlled by a single guide-blade, which forms the outer surface of the volute, or chamber, into which the water flows on leaving the fan.

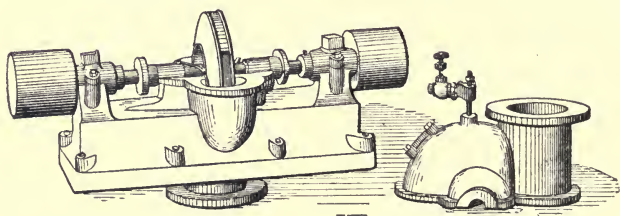


FIG. 314.—Experimental Centrifugal Pump in the Hydraulic Laboratory, McGill University.

Experiment seems to indicate that the efficiency of a centrifugal pump increases as the inlet-tip angle diminishes, and that

it is therefore advantageous to make this angle as small as is practicable, but opinions on this point differ. The real influence of the tip angles on the efficiency is yet to be determined, and it is doubtful whether the ordinary hypothesis of radial flow ($\gamma = 90^\circ$) at inlet without shock is even approximately correct.

The inlet velocity and therefore also the pump's efficiency may be increased by the use of a suction-tube with a gradually diminishing section, e.g., a tube in the form of the frustum of a cone. A still greater advantage may be obtained by giving the discharge-pipe a gradually increasing section. In this case the velocity of discharge gradually diminishes and the pressure-head is proportionately increased, so that there is a gain of head available for increasing the pumping power. The velocity in the discharge-pipe should not be too great, as it may lead to a very sensible loss of energy. Generally speaking, a velocity of 3 to 6 ft. per second has been found to give the most favorable results.

It is claimed by some authorities that an advantage may be gained by the addition of a vortex or whirlpool chamber surrounding the pump-disc. In support of this contention it is urged that the water discharged from the disc continues to rotate in this chamber, and that a portion of the kinetic energy is thus converted into pressure energy, which would otherwise be largely wasted in eddies in the volute or discharge-pipe (see Art. 21, Chap. I). The water leaves the vortex-chamber with a diminished whirling velocity which cannot be very different in direction and magnitude from the velocity of the mass of water in the volute. The vortex-chamber is sometimes provided with guide-blades following the direction of free vortex stream-lines (equiangular spirals) so as to prevent irregular motion.

Centrifugal pumps work under different conditions from turbines, and hence there are corresponding differences necessary in their design. They work best for the particular lift for which they are designed, and any variation from this lift causes a rapid reduction in the efficiency.

FIG. 315.

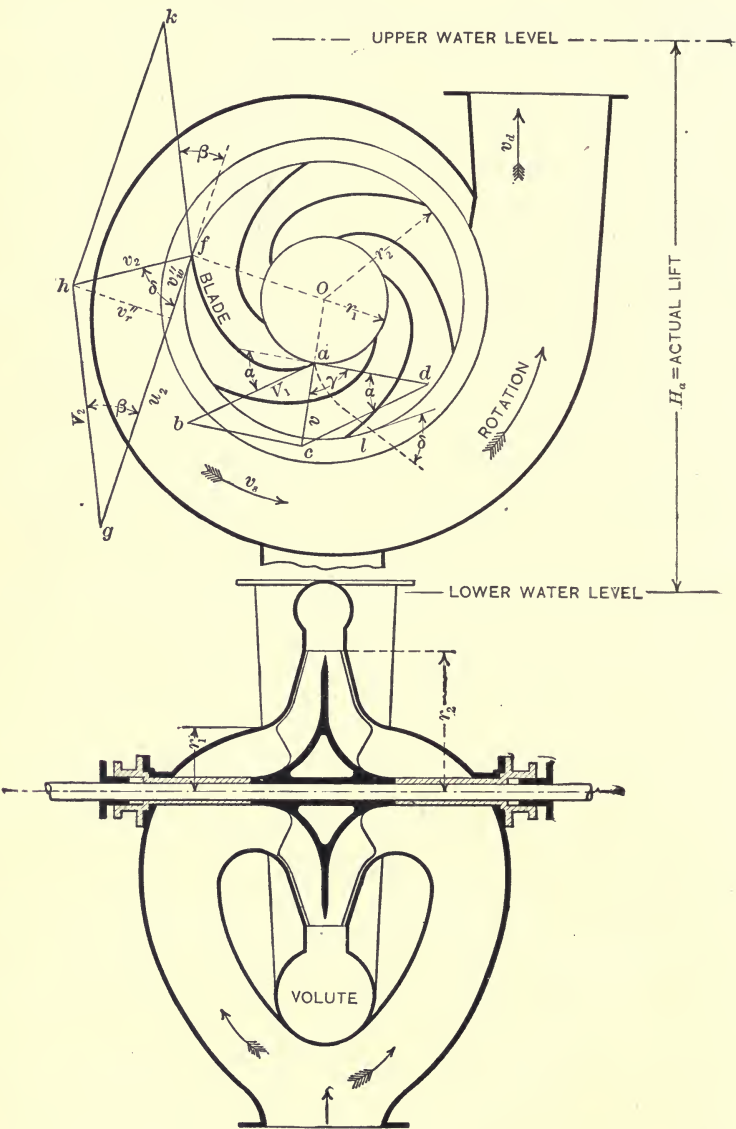


FIG. 316.

2. Analysis of the Centrifugal Pump.—Designate the velocities, angles, and pressure-heads at the inlet- and outlet-surfaces of the wheel, Figs. 315 and 316, by the same symbols as in the case of the turbine, Art. 4.

Let Q be the delivery of the pump in cu. ft. per sec.

Let H_g be the gross lift including the head equivalent to the total hydraulic resistance (h_r), the actual lift (H_a), and the head equivalent to the velocity of delivery (v_d), viz., $\frac{v_d^2}{2g}$.

Then

$$H_g = h_r + H_a + \frac{v_d^2}{2g}.$$

h_r includes the heads equivalent to the resistance in the suction-pipe (h_1), in the delivery-pipe (h_2), and in the wheel-passages (h_3), so that

$$h_r = h_1 + h_2 + h_3.$$

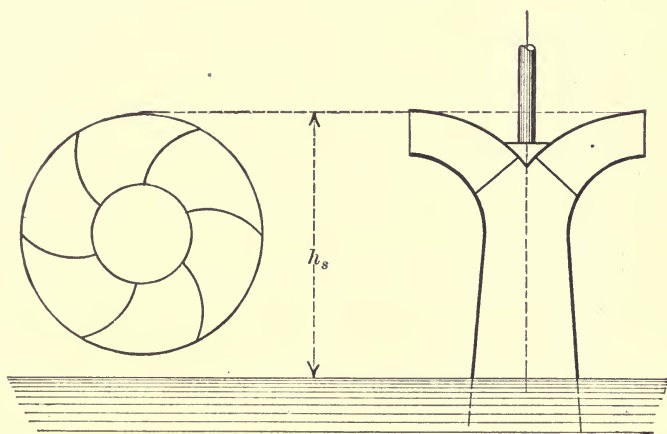


FIG. 317.

FIG. 318.

H_a includes the height of suction (h_s) and the height of delivery (h_d), so that

$$H_a = h_s + h_d.$$

The height of suction (h_s) is generally taken to be the vertical distance between the lower water-level and the axis of the pump, but this is incorrect and may lead to serious errors. The true height of suction, i.e., the height to which the water must be raised before the pump will commence to do work, should be measured from the lower water-level to the top of the impellor or to the top of the wheel according as the axis of the pump is horizontal or vertical.

The actual loss in hydraulic resistances between the suction-level and the eye of the pump may be determined by the following method suggested by Albert F. Hall. A long gauge-glass AB , with a cast-iron cap CD , is fitted into the top of the suction-pipe. The water rises in the tube to a certain level aa , and the pressure in this tube can be directly measured by means of the gauge G .

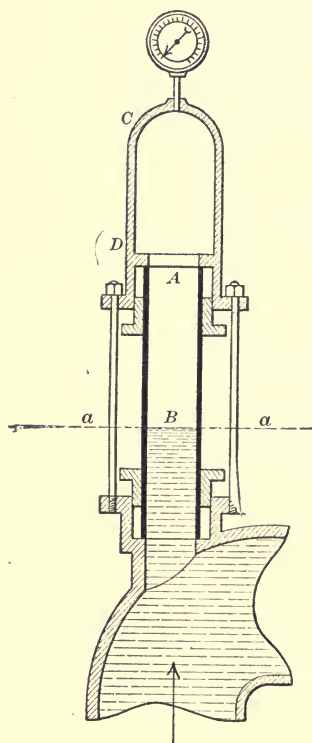


FIG. 319.

If H_B is the barometric head and H_G the gauge-reading, in feet, then $H_B - H_G$ is the actual dynamic head at aa . Hence if H' is the static head, i.e., the vertical distance between the suction-level and aa , $(H_B - H_G) - H'$ is the loss due to the several hydraulic resistances.

The air-chamber thus constructed seems to cause a steadier flow of water, and experiment shows that the variation of level at aa is small and is only about $\frac{1}{4}$ to $\frac{1}{2}$ inch. In pumps which are fed on both sides, Fig. 313, the steadiness of the level is increased by placing an air-chamber on each suction-bend, connecting the two at the upper end by a horizontal

pipe. A valve in the middle of the pipe may communicate with a vacuum pump, and each chamber may also be controlled by a separate valve.

The water apparently flows through the bend, past the orifice, as over an elastic cushion.

The *total work* done on the pump per second

$$= \frac{wQ}{g} (v_w'' u_2 - v_w' u_1) = wQH_g, \quad . \quad . \quad . \quad (1)$$

and therefore

$$v_w'' u_2 - v_w' u_1 = gH_g. \quad . \quad . \quad . \quad (2)$$

$$\text{The efficiency } \eta = \frac{H_a}{H_g} = \frac{gH_a}{v_w'' u_2 - v_w' u_1}, \quad . \quad . \quad . \quad (3)$$

and $gH_a = \eta(v_w'' u_2 - v_w' u_1)$ is the fundamental equation governing the design of a centrifugal pump.

The water spreads out more or less radially from the eye of the pump and, for simplicity of calculation, it is often assumed that $\gamma = 90^\circ$. Then

$$v_w' = 0, \quad v_1 = v_r' \quad \text{and} \\ gH_a = \eta u_2 v_w''.$$

Again, eq. (2) becomes

$$v_w'' u_2 = gH_g = (u_2 - v_r'' \cot \beta) u_2,$$

which may be written in the form

$$\frac{u_2^2}{gH_g} - \frac{u_2}{\sqrt{gH_g}} \frac{v_r''}{\sqrt{gH_g}} \cot \beta = 1,$$

a quadratic giving

$$\frac{u_2}{\sqrt{gH_g}} = \frac{v_r''}{\sqrt{gH_g}} \frac{\cot \beta}{2} \pm \sqrt{\left(\frac{v_r''}{\sqrt{gH_g}} \right)^2 \frac{\cot^2 \beta}{4} + 1}. \quad (4)$$

By means of this result the following Table has been prepared and gives the values of $\frac{u_2}{\sqrt{gH_g}}$, corresponding to different

values of $\frac{v_r''}{\sqrt{gH_g}}$ and β :

$\frac{v_r''}{\sqrt{gH_g}}$	$\beta =$										
	15°	165°	30°	150°	45°	135°	60°	120°	75°	105°	90°
1.0	3.983	.251	2.189	.457	1.618	.618	1.330	.752	1.144	.876	1
.9	3.633	.275	2.047	.489	1.547	.647	1.293	.773	1.127	.887	1
.8	3.290	.304	1.909	.524	1.477	.677	1.257	.795	1.112	.898	1
.7	2.951	.339	1.775	.563	1.409	.709	1.222	.818	1.098	.910	1
.6	2.620	.382	1.647	.607	1.344	.744	1.188	.842	1.083	.922	1
.5	2.301	.434	1.522	.656	1.281	.781	1.154	.866	1.069	.935	1
.4	1.992	.500	1.405	.712	1.220	.820	1.121	.890	1.055	.948	1
.3	1.706	.586	1.293	.773	1.161	.861	1.090	.917	1.041	.961	1
.2	1.441	.695	1.188	.842	1.105	.905	1.059	.944	1.027	.973	1
.1	1.204	.830	1.090	.917	1.051	.951	1.029	.971	1.013	.987	1

Remarks on the Angles γ and α , and on the Curve of the Blade.—The assumption of a radial flow from the eye, i.e., that $\gamma = 90^\circ$, cannot of course be true and, possibly, is not even approximately accurate, but is solely made for the purpose of securing simplicity in the calculations. In fact, the water flows towards the eye with a uniform motion parallel to the axis of rotation, while at every point between the inlet and outlet of the wheel the motion of a fluid particle is the resultant of a constant angular acceleration (Art. 21, Chap. I) and of a radial acceleration due to centrifugal force, viz., $r\omega^2$.

At the inlet the tip angle α may be varied between wide limits, but its value should be such as to make the efficiency as great as possible: But α cannot be expressed as a function of the mechanical and hydraulic resistances, and it is therefore impossible to find, analytically, the value of α which will make these resistances a minimum. The best value for α can only be determined by a very extensive series of experiments. According to the best practice, however, α usually lies between 50° and 60° .

The curve of the blade is in some cases a circular arc. In many first-class wheels it is a cycloid developed by a circle of a diameter equal to one fourth of the diameter of the wheel, the cycloidal arc, at the innermost point, being tangential to

the hub. Brix, again, has deduced the following formula, giving the angle ϕ between the blade and the direction of rotation at any point distant r from the axis:

$$\cot \phi = \frac{2\pi d\omega}{Q} \left\{ r^2 - \frac{gT}{\omega^2} - \frac{r_1 v_r'}{\omega} \right\} \left\{ 1 - \frac{nt \operatorname{cosec} \phi}{2\pi r} \right\},$$

where $2\pi r d_1 = (2\pi r - nt)d$; T = work of pump to radius r ; r_1 = radius to inner end of blade; v_r = radial velocity at inlet; n = number of blades; t = thickness of blade.

By plotting the values of ϕ corresponding to different values of r the curve of the blade may be defined.

It is essential that there should be no dissipation of energy in eddy motion at the inlet, and the direction of the relative velocity, V_1 , should therefore be tangential to the blade-tip at a , Fig. 315. Then,

$$\text{from the triangle } adc, V_1^2 = v_1^2 + u_1^2 - 2v_1u_1 \cos \gamma, \quad (5)$$

$$\text{“ “ “ } fkh, V_2^2 = v_2^2 + u_2^2 - 2v_2u_2 \cos \delta, \quad (6)$$

and therefore

$$\frac{V_1^2 - V_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{v_2^2 - v_1^2}{2g} = \frac{v_2u_2 \cos \delta}{g} - \frac{v_1u_1 \cos \gamma}{g}. \quad (7)$$

The water leaves the wheel with a velocity v_2 , and carries away, in its energy of motion, viz., $\frac{v_2^2}{2g}$, an important portion of the work done on the pump by the prime mover. If the whole of this energy could be made available for increasing the pumping power, then, by Bernoulli's theorem,

$$h_1 + H_a + \frac{v_d^2}{2g} + \frac{p_1}{w} + \frac{v_1^2}{2g} = -h_2 + \frac{p_2}{w} + \frac{v_2^2}{2g}. \quad (8)$$

Also,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} = h_3 + \frac{p_2}{w} + \frac{V_2^2}{2g}, \quad (9)$$

the term $\frac{u_2^2 - u_1^2}{2g}$ being the variation of pressure-head due to

centrifugal action between the wheel inlet and outlet. Hence, by eqs. (8) and (9),

$$\begin{aligned} h_r + H_a + \frac{v_d^2}{2g} &= \frac{V_1^2 - V_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{v_2'^2 - v_1'^2}{2g} \\ &= \frac{v_2 u_2 \cos \delta}{g} - \frac{v_1 u_1 \cos \gamma}{g}, \quad \dots \quad (10) \end{aligned}$$

where $h_r = h_1 + h_2 + h_3$ = the head equivalent to the total hydraulic resistances.

If the inlet flow is radial, i.e., if $\gamma = 90^\circ$, and if the hydraulic resistances and the velocity of delivery can be diminished to such an extent that h_r and $\frac{v_d^2}{2g}$ become sufficiently small to be disregarded without much error, then eq. (10) becomes

$$H_a = \frac{v_2 u_2 \cos \delta}{g} = \frac{v_w'' u_2}{g}, \quad \dots \quad (11)$$

and the total available energy is transformed into useful work.

Volute.—The water issues from the outlet-surface into a casing, or volute, which surrounds the wheel and which should always be designed in such a manner that the disturbance in the fluid mass might be as small as possible, since the least disturbance in the stream-line motion causes a loss of energy in shock. Thus its sectional area on any normal plane, through the centre of the wheel, should be proportional to the quantity of water which flows across the section in the same given time, and the corresponding mean velocity of flow, v_s , in the volute is necessarily constant. If the width of the volute is also constant, its profile will evidently be an Archimedean spiral. By making the gradually increasing sections sufficiently large the velocity of flow, v_s , may be made very small, and a bell-mouth entrance into the discharge-pipe may become unnecessary.

Figs. 320 to 325 are of interest as showing the experimental stages through which Farcot's pump passed in the process of its gradual development.

The assumption that the total available energy may be transformed into useful work is altogether inadmissible in practice, as a large portion is consumed in overcoming frictional resistance and in the production of eddies.

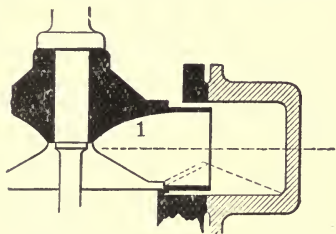


FIG. 320.

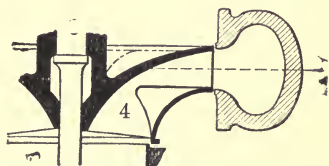


FIG. 323.

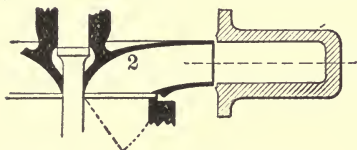


FIG. 321.

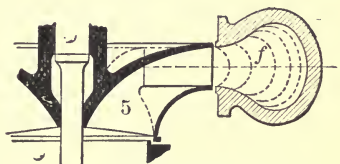


FIG. 324.

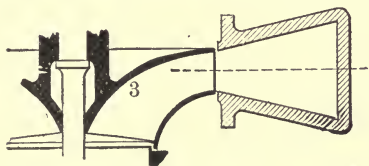


FIG. 322.

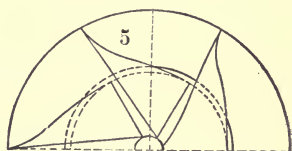


FIG. 325.

Again, even with the most perfectly designed volute, the hypothesis that the whole of the energy of motion, $\frac{v_2'^2}{2g}$, may be utilized in increasing the pumping power is untenable. The water, as it leaves the wheel, with a velocity v_2' , impinges upon the fluid mass in the volute, and the radial component v_r'' of v_2' must necessarily be almost, if not wholly, destroyed, the corresponding loss of head being $\frac{(v_r'')^2}{2g}$. The tangential component of v_2' , viz., v_w'' , is also changed into v_s , the velocity

are sufficiently small, as compared with H_a , to be disregarded without much error, and if $\gamma = 90^\circ$, then eq. (14) becomes

$$H_a = \frac{u_2^2 - V_2^2}{2g}. \quad (15)$$

But

$$V_2 = u_2 \frac{\sin \delta}{\sin (\beta + \delta)}, \quad \text{and} \quad v_w'' = v_2 \cos \delta = u_2 \frac{\sin \beta \cos \delta}{\sin (\beta + \delta)}.$$

Therefore

$$2gH_a = u_2^2 \left\{ 1 - \frac{\sin^2 \delta}{\sin^2 (\beta + \delta)} \right\} = u_2^2 \frac{\sin \beta \sin (\beta + 2\delta)}{\sin^2 (\beta + \delta)},$$

or

$$\frac{u_2^2}{2gH_a} = \frac{\sin^2 (\beta + \delta)}{\sin \beta \sin (\beta + 2\delta)}, \quad (16)$$

and the efficiency $\eta = \frac{gH_a}{v_w'' u_2} = \frac{1}{2} \frac{\sin (\beta + 2\delta)}{\cos \delta \sin (\beta + \delta)},$

or
$$\eta = \frac{1}{2} \{ 1 + \tan \delta \cot (\beta + \delta) \}.$$

The efficiency increases as β , the outlet-tip angle, diminishes, and would be unity, i.e., perfect, if β could be 0.

If the blade is radial at the outlet, i.e., if $\beta = 90^\circ$, then

$$\begin{aligned} \text{the efficiency} &= \frac{1}{2} \{ 1 + \tan \delta \cot (90^\circ + \delta) \} \\ &= \frac{1}{2} (1 - \tan^2 \delta), \end{aligned}$$

and could never exceed $\frac{1}{2}$.

For any given value of β the efficiency is a maximum when

$$\beta + 2\delta = 90^\circ.$$

This can be easily shown analytically, or geometrically as follows:

Upon any line AB as diameter describe a semicircle. Draw a chord AC making the angle β with AB . Draw any chord AD making an angle δ with AC , and join DB intersecting AC in O .

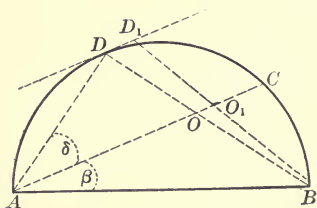


FIG. 326.

The efficiency is greatest when $\tan \delta \cot (\beta + \delta)$ has its greatest value.

But since the angle in a semicircle is a right angle,

$$\tan \delta \cot (\beta + \delta) = \frac{DO}{AD} \cdot \frac{AD}{DB} = \frac{DO}{DB},$$

and the efficiency is therefore greatest when $\frac{DO}{DB}$ is a maximum.

Now $\frac{DO}{DB}$ is nil both when D coincides with A and also with C , and must consequently be a maximum, or stationary, at some position of D between A and C . This position is at once found from the condition that if D_1 is a consecutive point and if D_1B is joined intersecting AC in O_1 , then

$$\frac{DO}{DB} = \frac{D_1O_1}{D_1B},$$

so that DD_1 must be parallel to OO_1 or AC , and is therefore a tangent to the semicircle at D , which is necessarily the middle point of the arc AC . Hence, since the arc AD = the arc CD ,

the angle ABD = the angle CAD = δ ,

and therefore

$$90^\circ - (\beta + \delta) = \delta,$$

or

$$\beta + 2\delta = 90^\circ.$$

Hence, too,

the max. efficiency = $1 + \tan \delta \cot (90^\circ - \delta)$

$$= \frac{1}{2} \sec^2 \delta = \frac{1}{2} \sec^2 \left(45^\circ - \frac{\beta}{2} \right).$$

The outlet velocities corresponding to this maximum efficiency are represented by the sides of the triangle fkh , Fig. 327. The two triangles fuh and fxh are equal in every respect.

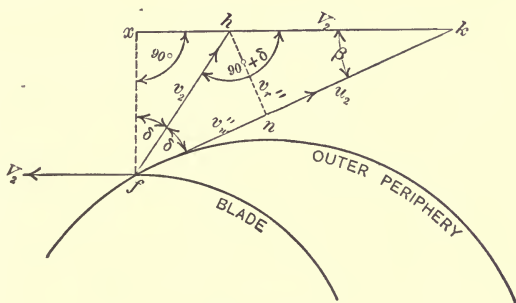


FIG. 327.

Also, $2 \sin^2 \delta = 1 - \cos 2\delta = 1 - \sin \beta$

and $2 \cos^2 \delta = 1 + \cos 2\delta = 1 + \sin \beta$.

Hence

$$V_2 = u_2 \frac{\sin \delta}{\sin (90^\circ + \delta)} = u_2 \tan \delta = u_2 \left(\frac{1 - \sin \beta}{1 + \sin \beta} \right)^{\frac{1}{2}},$$

and

$$2gH_a = u_2^2 - V_2^2 = u_2^2 \frac{2 \sin \beta}{1 + \sin \beta},$$

or

$$u_2 = \sqrt{\frac{1 + \sin \beta}{2 \sin \beta}} 2gH_a.$$

$$\text{Also, } v_2 = u_2 \frac{\sin \beta}{\sin (90 + \delta)} = u_2 \frac{\sin \beta}{\cos \delta} = \sqrt{\sin \beta \cdot 2gH_a},$$

$$v_w'' = v_2 \cos \delta = u_2 \sin \beta = \sqrt{\frac{\sin \beta (1 + \sin \beta)}{2} 2gH_a},$$

$$\text{and } v_r'' = v_2 \sin \delta = \sqrt{\frac{\sin \beta (1 - \sin \beta)}{2} 2gH_a}.$$

From these equations the following Table has been prepared:

β	δ	$\frac{u_2}{\sqrt{2gH_a}}$	$\frac{v_2}{\sqrt{2gH_a}}$	$\frac{v_r''}{\sqrt{2gH_a}}$	η
5°	42° 30'	2.497	.296	.20	.92
8 26'	40 47	1.977	.383	.25	.87
10	40	1.838	.416	.267	.80
15	37 30	1.56	.500	.30	.79
20	35	1.40	.58	.33	.74
30	30	1.22	.70	.35	.67
40	25	1.13	.80	.34	.61
45	22 30	1.09	.84	.32	.58
50	20	1.073	.87	.30	.56
58 36	15 42	1.041	.92	.25	.54
60	15	1.038	.93	.24	.53

The value of the radial component, v_r'' , is greatest when $\sin \beta(1 - \sin \beta)$ is a maximum, i.e., when $\sin \beta = \frac{1}{2}$ or $\beta = 30^\circ = \delta$.

Two values of β , the one less and the other greater than 30° , correspond to every other value of v_r'' .

Generally speaking, v_r'' lies between $\frac{1}{3}\sqrt{2gH_a}$ and $\frac{1}{4}\sqrt{2gH_a}$, and the assumption is sometimes made that the radial component of the velocity of the water as it passes through the wheel is constant and equal to $\frac{1}{4}\sqrt{2gH_a}$.

EXAMPLE.—A centrifugal pump, with an outlet-tip angle (β) of 20° , has an efficiency of 60 per cent. Assuming $v_r'' = \frac{1}{4}\sqrt{2gH_a}$, then

$$\begin{aligned}
 gH_a &= .6u_2v_w'' = .6u_2(u_2 - v_r'' \cot 20^\circ) \\
 &= .6u_2\left(u_2 - \frac{1}{4}\sqrt{2gH_a} \times 2.7475\right),
 \end{aligned}$$

and

$$u_2 = 1.32 \sqrt{2gH_a}.$$

Also,

$$\begin{aligned}
 V_2 &= v_r'' \operatorname{cosec} 20^\circ = \frac{1}{4} \sqrt{2gH_a} \times 2.9238 \\
 &= \sqrt{2gH_a} \times .73095.
 \end{aligned}$$

Therefore

$$u_2^2 - V_2^2 = 1.208 \times 2gH_a.$$

Hence, if the inlet flow is radial, equation (14) gives

$$\begin{aligned}
 h_r + \frac{v_d^2}{2g} - \frac{v_s(v_w'' - v_s)}{g} + H_a &= \frac{v_2 u_2 \cos \delta}{g} - \frac{v_2^2}{2g} \\
 &= \frac{u_2^2 - V_2^2}{2g} = 1.208H_a,
 \end{aligned}$$

and

$$h_r + \frac{v_d^2}{2g} - \frac{v_s(v_w'' - v_s)}{g} = .208H_a.$$

Certain existing experimental results give $.42H_a$ as an average value of h_r ; and taking $.03H_a$ as the average value of $\frac{v_d^2}{2g}$, then

$$\frac{v_s(v_w'' - v_s)}{g} = .42H_a + .03H_a - .208H_a = .242H_a.$$

The term $\frac{v_s(v_w'' - v_s)}{g}$ must necessarily vary considerably with the design of the pump.

3. Thomson's Vortex or Whirlpool-chamber (Figs. 328 and 329).—It has been suggested that the energy of motion inherent in the water, as it leaves the wheel, may be more completely utilized, and the pumping power therefore increased, by the addition of an exterior chamber of radius r_3 , in which the water, in virtue of its motion, is left free to

revolve, and tends to assume the condition designated by James Thomson as the vortex, or whirlpool, of free mobility. The centrifugal action of this fluid mass develops an outward force which is added to the outward force developed within the wheel and materially increases the pumping power. The outward force produced within the wheel is due to centrifugal

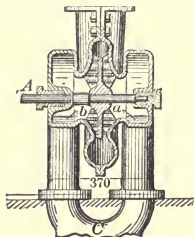


FIG. 328.

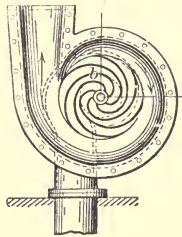


FIG. 329.

action only, if the blades are radial; but if, as is generally the case, the blades are curved, it is partly due to the radial component of the pressure between the blades and the water, and this pressure may be very great if the pump is run at a high speed.

The chief properties characterizing the fluid mass in the whirlpool-chamber are the following:

(1) Each fluid particle moves with a velocity (v) inversely proportional to its distance (r) from the axis of rotation. Thus

$$r_2 v_2 = r v = r_3 v_3,$$

v_3 being the water's velocity at the outlet-surface of the whirlpool-chamber.

(2) The angle (θ) between the radial distance (r) to any particle and its direction of motion is constant, and the stream-lines are therefore equiangular spirals.

Thus if v_r''' and v_w''' are the radial and tangential components of v_3 ,

$$\begin{aligned} v_r'' &= v_2 \cos \delta, & v_r''' &= v_3 \cos \delta, \\ v_w'' &= v_2 \sin \delta, & v_w''' &= v_3 \sin \delta, \end{aligned}$$

and therefore

$$\frac{v_r''}{v_r'''} = \frac{v_2}{v_3} = \frac{r_3}{r_2} = \frac{v_w''}{v_w'''}.$$

(3) Each particle is free to move to any position within the whirlpool without interfering with the general motion of the other particles, as, in moving towards or from the centre, it assumes of itself, subject simply to the laws of motion under a central force, the velocity due to its position in the whirlpool.

(4) For any equal particles, whatever positions they may momentarily occupy in the whirlpool, the sum of the energies corresponding to velocity, to pressure, and to height is constant.

Thus each particle gives up its velocity in accordance with the law of motion just stated, and the head available for increasing the pumping power

$$\begin{aligned} &= \frac{v_2^2}{2g} - \frac{v_3^2}{2g} = \frac{v_2^2}{2g} \left(1 - \frac{r_2^2}{r_3^2} \right) \\ &= \frac{v_2^2}{2g} \left\{ 1 - \left(\frac{v_w'''}{v_w''} \right)^2 \right\}. \end{aligned}$$

Again, the term $\frac{v_s(v_w''' - v_s)}{g}$, representing the gain of head in passing from the whirlpool-chamber into the volute, must be substituted for the term $\frac{v_s(v_w'' - v_s)}{g}$. Thus

$$\text{the efficiency} = \frac{H_a}{H_g},$$

in which $H_g = u_2 v_w'' = u_2(u_2 - V_2 \cos \beta)$, and the actual lift is

$$H_a = \frac{u_2^2 - V_2^2}{2g} + \frac{v_2^2}{2g} \left(1 - \frac{r_2^2}{r_3^2} \right) \frac{v_s(v_w''' - v_s)}{g} + - \frac{v_d^2}{2g} - h_r.$$

Ex. Assume that the four last terms in the preceding equation are sufficiently small to be disregarded. Then

$$\text{the efficiency} = \frac{u_2^2 - V_2^2 + (v_r''^2 + v_w''^2) \left(1 - \frac{r_2^2}{r_3^2} \right)}{2u_2(u_2 - V_2 \cos \beta)}.$$

First. Let $\beta = 90^\circ$, i.e., let the blade outlet-lip be radial. Then

$$v_w'' = u_2 \quad \text{and} \quad v_r'' = V_2.$$

Therefore

$$\begin{aligned} \text{the efficiency} &= \frac{u_2^2 - V_2^2 + (u_2^2 + V_2^2) \left(1 - \frac{r_2^2}{r_3^2}\right)}{2u_2^2} \\ &= 1 - \frac{1}{2} \frac{r_2^2}{r_3^2} - \frac{1}{2} \frac{V_2^2 r_2^2}{u_2^2 r_3^2}. \end{aligned}$$

Second. Let $\beta = 0$, i.e., let the blade outlet-lip be tangential. Then

$$v_w'' = u_2 - V_2 \quad \text{and} \quad v_r'' = 0.$$

Therefore

$$\begin{aligned} \text{the efficiency} &= \frac{u_2^2 - V_2^2 + (u_2 - V_2)^2 \left(1 - \frac{r_2^2}{r_3^2}\right)}{2u_2(u_2 - V_2)} \\ &= 1 - \frac{1}{2} \frac{r_2^2}{r_3^2} + \frac{1}{2} \frac{V_2 r_2^2}{u_2 r_3^2}. \end{aligned}$$

The difference between these two efficiencies is comparatively small and diminishes as the diameter of the whirlpool-chamber increases. Hence their values are not largely influenced by the angle β .

The above hypothetical theory seems to indicate that a

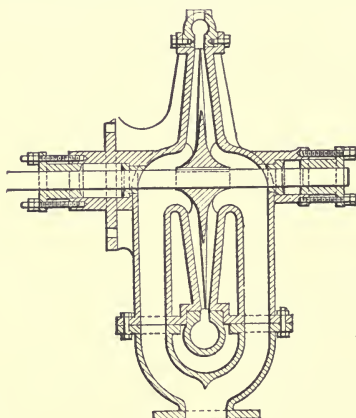


FIG. 330.

whirlpool-chamber adds to the efficiency of a centrifugal pump. Opinions, however, differ widely as to the real character of the

flow of the water within the pump, and as to the loss of energy in shock on entering the whirlpool-chamber or the volute. Some eminent authorities advocate a gradually diminishing section, Fig. 330, on the ground that it tends to produce a steadier action, while other authorities, equally eminent, claim that a flaring vortex tends to increase the efficiency, and it is urged that by widening the chamber from the depth at the wheel-outlet to a much greater depth at its exterior surface, the water will lose its energy of motion much more rapidly and will leave the chamber with a velocity more nearly equal to that in the discharge-pipe.

Experiments are urgently needed to throw light upon this important subject.

4. Practical Values. — Let d_1 , d_2 be the depths of the inlet- and outlet-surfaces.

Let t_1 , t_2 be the blade thickness at inlet and outlet.

Let n be the number of blades.

Let the inlet area = sectional area of supply-pipe.

Then, if $\gamma = 90^\circ$

$$\begin{aligned}(2\pi r_1 - nt_1 \operatorname{cosec} \alpha) d_1 v_r' &= \frac{10}{9} Q = \pi r_1^2 v_r' \\ &= (2\pi r_2 - nt_2 \operatorname{cosec} \beta) d_2 v_r'',\end{aligned}$$

the coefficient $\frac{10}{9}$ being an average value and depending upon practical considerations.

The following values are sometimes adopted in practice:

$$\begin{aligned}r_2 &= 2r_1; \\ n &= 4 \text{ to } 10; \\ t_1 &= t_2 = .2 \text{ in. to } .625 \text{ in.}; \\ 5d_1 &= 6r_1; \\ r_1^2 \sqrt{H_a} &= .661 Q; \\ d_2 &= d_1 \text{ or } = \frac{1}{2}d_1, \text{ according as the pump-} \\ &\text{faces are parallel or coned.}\end{aligned}$$

Ex. The hypothetical advantage of a whirlpool-chamber may be observed by a consideration of the comparative efficiencies of two pumps, which are precisely similar in every respect excepting that one has a whirlpool-chamber of 62 ins. diameter. Each pump delivers 20 cu. ft. of water at a speed of 225 revolutions per minute. The diameters of the suction- and discharge-pipes = 20 ins.; the diameter of the wheel = 36 ins.; the depth of the whirlpool-chamber = the depth of the wheel at outlet = $5\frac{1}{2}$ ins.; $\gamma = 90^\circ$; $\beta = 42^\circ 20' 22''.65$; number of wheel-blades = 6; thickness of blade = $\frac{1}{8}$ in.

The actual lift is given by

$$H_a = \frac{u_2^2 - V_2^2}{2g} + \frac{v_s(v_w'' - v_s)}{g} - \frac{v_d^2}{2g} - h_r,$$

$$\text{or } H_a = \frac{u_2^2 - V_2^2}{2g} + \frac{v_2^2}{2g} \left(1 - \frac{r_2^2}{r_3^2} \right) + \frac{v_s(v_w''' - v_s)}{g} - \frac{v_d^2}{2g} - h_r,$$

according as the pump has not or has a whirlpool-chamber.

$$u_2 = \frac{225 \times \pi \times 3}{60} = 35\frac{5}{8} \text{ ft. per sec.};$$

$$\operatorname{cosec} \beta = 1.484;$$

$$v_r'' = \frac{144 \times 20}{.9 \left\{ \pi \times 36 - 6 \times \frac{7}{8} \times 1.484 \right\} 5\frac{1}{2}} = 5.523 \text{ ft. per sec.};$$

$$V_2 = v_r'' \operatorname{cosec} \beta = 8.196 \text{ ft. per sec.};$$

$$\frac{u_2^2 - V_2^2}{2g} = \frac{591.473}{g};$$

$$\cot \beta = 1.097;$$

$$v_w'' = u_2 - v_r'' \cot \beta = 29.298 \text{ ft. per sec.};$$

$$v_d = v_s = \frac{20}{\pi \left(\frac{5}{6} \right)^2} = 9\frac{2}{5} \text{ ft. per sec.};$$

$$\frac{v_s(v_w'' - v_s)}{g} = \frac{184.508}{g} = \text{gain of head in passing from wheel into volute};$$

$$\frac{v_d^2}{2g} = \frac{41.986}{g};$$

$$v_2^2 = v_r''^2 + v_w''^2 = 888.876;$$

$$\frac{v_2^2}{2g} \left(1 - \frac{r_2^2}{r_3^2} \right) = \frac{888.876}{2g} \left(1 - \frac{18^2}{31^2} \right) = \frac{294.596}{g};$$

$$v_w''' = \frac{18}{31} \times 29.298 = 17.012 \text{ ft. per sec.};$$

$$\frac{v_s(v_w''' - v_s)}{g} = \frac{71.919}{g};$$

$$u_2 v_w'' = 1035.889.$$

Hence for the pump *without* a whirlpool-chamber

$$\begin{aligned} H_a &= \frac{591.473}{g} + \frac{184.508}{g} - \frac{41.986}{g} - h_r \\ &= \frac{733.995}{g} + h_r = (22.81 - h_r) \text{ ft.,} \end{aligned}$$

or $gH_a = 733.995 - gh_r,$

and

$$\text{the efficiency} = \frac{gH_a}{u_2 v_w''} = \frac{733.995 - gh_r}{1035.889} = .708 - \frac{gh_r}{1035.889}.$$

For the pump *with* a whirlpool-chamber

$$\begin{aligned} gH_a &= \frac{591.473}{g} + \frac{294.596}{g} + \frac{71.919}{g} - \frac{41.986}{g} - h_r \\ &= \frac{916.002}{g} - h_r = (28.46 - h_r) \text{ ft.,} \end{aligned}$$

or $gH_a = 916.002 - gh_r,$

$$\text{and the efficiency} = \frac{916.002 - gh_r}{1035.889} = .884 - \frac{gh_r}{1035.889},$$

which is considerably greater than the first efficiency.

EXAMPLES.

1. Find the H.P. required to drive a centrifugal pump of 14 ft. diameter, and with radial vanes, making 60 revolutions per minute and delivering 900,000 gallons of water per hour. If the lift is $30\frac{1}{2}$ ft. find the efficiency. Assume that the water on entering has no velocity of whirl. *Ans.* 27.5; .5.

2. The wheel of a centrifugal pump is .6 ft. in diameter; the turning moment on the spindle is 12 lbs.-ft. If 160 gallons of water are raised per minute, find the mean velocity with which the water leaves the wheel; assuming that on entering it has no velocity of whirl. *Ans.* 24.1 ft. per sec.

3. A centrifugal pump has a 36-in. wheel of a uniform breadth of $5\frac{1}{2}$ ins. The wheel makes 225 revolutions per minute and delivers 20 cu. ft. of water per second into a discharge-pipe of 20 ins. diameter. The angle (β) of the blades at the outer periphery is $42^\circ 20'$. Assuming the velocity of discharge to be the same as the mean velocity of flow in the volute and disregarding vane-thickness, find (a) the peripheral speed; (b) the velocity of whirl and radial velocity of flow; (c) the gain of head available for useful work on entering the volute, and (d) the efficiency.

There are six $\frac{5}{8}$ -in. blades. If a 62-in. whirlpool-chamber is added, find the gain of head available for useful work, (e) due to chamber; (f) on entering volute.

Ans. (a) 33.35 ft. per sec.; (b) 29.425 and 5.4 ft. per sec.; (c) 5.8 ft.; (d) .708; (e) 9.27 ft.; (f) 2.27 ft.

4. A centrifugal pump with a 12-in. fan delivers 1000 gallons per minute, the actual lift being 20 ft. and the *gross* lift (allowing for friction, etc.) 30 ft. Find the revolutions of the pump per minute ($v_w'' = \frac{u_2}{2}$).

Ans. 836.52.

5. In a centrifugal pump the external diameter of the fan is 2 ft., the internal 1 ft., and the depth 6 in. Determine the speed and efficiency of the pump when delivering 2000 cu. ft. per minute against a pressure head of 64 ft., the inclination of the wheel-vanes at outlet-surface being 90° , and γ being also 90° . *Ans.* 619.24 revols. per min.; .4866.

6. A centrifugal pump delivers 1500 gallons per minute. Fan, 16 in. diameter; lift, 25 ft.; inclination of vanes at outer periphery to the tangent, 30° . Find the breadth at the outer periphery, and also the revolutions per minute, assuming the *gross* lift to be $1\frac{1}{2}$ times the actual lift, and that $v_w'' = \frac{u_2}{2}$.

Also find the proper sectional area of the chamber surrounding the fan for the proposed delivery and lift. Examine the working of the pump at a lift of 15 ft. ($v_w'' = 0$).

Ans. Breadth, $\frac{4}{3}$ in.; revolutions, 700; 23.5 sq. ins.

7. For a given discharge (Q) and head (H), and considering only the losses of head due to flow and to the resistance in the wheel, show that the maximum efficiency of a centrifugal pump of diameter D is

$$1 - A \frac{D^2 H^{\frac{1}{2}}}{Q},$$

A being a constant depending on the size of the wheel.

8. A centrifugal pump with an efficiency of .75 and a radial flow at inlet, lifts 35 cu. ft of water per second a height of 20 ft. At the outer periphery the vane-angle (β) is 15° and the radial velocity is 5 ft. per second. If the wheel makes 140 revolutions per minute, find (a) its diameter. If the diameter of the outer periphery of the wheel is three times that of the inner periphery and if the radial velocity at the latter is 8 ft. per second, find (b) the vane-angle at the inner periphery and (c) the depths of the wheel at the inner and outer peripheries.

Ans. (a) 5.455 ft.; (b) $30^\circ 58'$; (c) .765 ft.; .41 ft.

9. The pump in the preceding example is supplied with a vortex-chamber of $6\frac{2}{3}$ ft. diameter. Show that the "gain of head" is a maximum when the velocity of flow in the volute is 8.46 ft. per second. Also show that the frictional loss of head is 4.18575 ft.

10. In a centrifugal pump the diameter of the fan = 12 ins., the depth = 2 ins., the lift = 25 ft., and the delivery = 300 cu. ft. per minute. Determine (a) the speed; (b) the efficiency; and (c) the power expended when the vane-angle (β) at the outer periphery is (1) 90° ; (2) 45° ; and (3) 30° ; γ being 90° .

Ans. (1) (a) 785 revs. per min.; (b) .47; (c) 30 H.P.;
(2) (a) 805.8 " " " (b) .58; (c) 24.4 H.P.;
(3) (a) 846.1 " " " (b) .68; (c) 22.9 H.P.

11. A centrifugal pump delivers 10,000 gallons per minute. The actual lift is 50 ft. The radial velocity at the outlet-surface is one eighth of that due to the actual lift and $u_2 = 2v_w''$. Find (a) the radius of the wheel; (b) the vane-angles; (c) the speed of the wheel; (d) the efficiency, taking $\gamma = 90^\circ$; and $d_1 = d_2 = \frac{v_2}{6}$.

Ans. (a) 1.9 ft.; (b) $56^\circ 16'$; $23^\circ 16'$; (c) 331 revs per min.; (d) .74.

12. The internal and external diameters of the fan of a centrifugal pump with radial flow at inlet are 9 ins. and 18 ins., respectively; the depth is 6 ins., and it passes 400 cu. ft. per minute against a pressure head of 16 ft. The inclination (β) of the discharging-lips of the fan being 30° , determine (a) the speed; (b) the efficiency; (c) the power ex-

pended; and (*d*) the inclination of the receiving-lips of the fan. Find (*e*) the efficiency when a whirlpool-chamber of 36 ins. diameter surrounds the fan.

Ans. (*a*) 413.58 revols. per min.; (*b*) .571; (*c*) 21.23 H.P.; (*d*) $19^{\circ} 12'$; (*e*) .581.

13. The lift of a centrifugal pump is $24\frac{3}{4}$ ft. The efficiency of the pump is .75, and the radial velocity of flow at outlet-surface of fan is 5 ft. per second. If $\cot \gamma = 4$, find the peripheral speed of the fan.

Also find its diameter, if the fan makes 160 revolutions per minute ($v_w' = 0$). Find the loss of head in hydraulic friction.

Ans. 44 ft. per sec.; $5\frac{1}{4}$ ft.; $3\frac{2}{3}\frac{3}{4}$ ft.

14. The reciprocal of the efficiency of a C. P. is 1.61, the peripheral (u_2) and radial (v_r'') velocities at outlet are 35 and 9 ft. per second respectively. Find the lift and the vane-angle (β) at outlet.

Ans. $15\frac{5}{8}$ ft.; $\tan^{-1} \frac{3}{4}$.

15. A centrifugal pump with a gross lift of 17 ft. delivers 25 cu. ft. of water per second. At the outer periphery the vane-angle is 80° and the radial velocity is 5 ft. per second. The diameters of the outer and inner peripheries of the disc are 54 ins. and 18 ins. respectively, and the hydraulic efficiency is .75. Find (*a*) the speed of the fan; (*b*) the vane-angle at the inlet periphery; (*c*) the velocity of whirl at the outlet; (*d*) the diameter of the volute; (*e*) the diameter of the suction-pipe.

If there are six $\frac{1}{2}$ -in. vanes, find (*f*) the width of the disc at the outer and inner peripheries.

Assuming the velocity of flow in the discharge-pipe to be 4 ft. per second, show that there is a loss of 5.026 ft. of head due to hydraulic friction.

Ans. (*a*) 116 revolutions per minute; (*b*) $41^{\circ} 14'$; (*c*) 26.49 ft. per second; (*d*) 1.094 ft.; (*e*) 33.8 in.; (*f*) 9.64 ins.; 4.8 ins.

16. The vane of a centrifugal pump or turbine is the involute of a circle concentric with the pump circumference. Show that $V_1 = V_2$ in an I. F. or O. F.; and $\frac{V_1}{V_2} = \frac{r_1}{r_2}$ in an A. F.

17. If the lips of the pump-vanes are radial, show that the efficiency cannot exceed .5, but that it might be increased to .875 by the addition of a whirlpool-chamber.

18. A centrifugal pump with a 21-in. fan pumps $110\sqrt{3}$ cu. ft. per second to a height of $31\frac{1}{4}$ ft. The outlet-lip makes an angle of 60° with the periphery. The depth of the fan is 6 ins. Find the peripheral speed, the H.P. and the speed of the pump in revols. per minute.

Also find the loss of head due to frictional resistance.

Ans. 60 ft. per second; $1623\frac{3}{4}$ H.P.; $654\frac{6}{11}$; $31\frac{1}{4}$ ft.

19. A centrifugal pump, with six $\frac{1}{2}$ -in. blades, makes 140 revolutions per minute and raises $5062\frac{1}{2}$ tons of water per hour to the height of 20 feet. The blade-angle and radial velocity of flow at outlet are $\cot^{-1} 4$ and

5 ft. per second, respectively, and the hydraulic efficiency of the pump is a little more than 60 per cent ($= \frac{2}{3}$). The wheel is surrounded by a vortex-chamber having a diameter 20 per cent greater than that of the wheel. Assuming that the inlet-flow is radial, and that $2v_s = v_w'''$, and disregarding frictional resistances, determine the peripheral speed, diameter and breadth of the wheel, and the gains of energy in ft.-lbs. in the vortex-chamber and in the volute.

Ans. 44 ft. per sec.; 6 ft.; 8.17 ins; 8070; 8789.

20. Compare the efficiencies of two centrifugal pumps, which are precisely similar in every respect, excepting that one has a whirlpool chamber of 48 ins. diameter. Each pump delivers 20 cu. ft. per second at a speed of 225 revolutions per minute. The diameters of the discharge- and suction-pipes = 20 ins.; the diameter of the wheel = 36 ins.; the depth of the wheel and the whirlpool-chamber at outlet = $3\frac{1}{2}$ ins.; $\gamma = 90^\circ$; $\beta = 22^\circ 38'$; the number of wheel-blades = 6; the blade-thickness = $\frac{5}{8}$ in.; $h_r = .3H_a$.

Ans. .73 — A and .79 — A where $A = \frac{gh_r}{503.6}$.

21. In a centrifugal pump the diameters of the suction- and discharge-pipes = 48 ins.; the number of wheel-blades = 6; the blade thickness = $\frac{7}{8}$ in.; the radial velocity of flow at outlet = 2.877 ft. per second; the velocity of flow in the volute and discharge-pipe = 5.817 ft. per second; the peripheral speed of the wheel outlet-surface = 34.6276 ft. per second. Disregarding the frictional losses in the suction- and discharge-pipes and in the wheel-passages, determine the velocity of whirl at outlet, the blade-tip angles at outlet, the delivery in cubic feet per second, the speed in revolutions per minute and the actual lift, the efficiency being .759.

Ans. 23.695 ft. per second; $\beta = 14^\circ 44'$; 73.13 cu. ft.; 80.13; 19.34 ft.

22. A centrifugal pump, with an actual lift of 10 ft., delivers 37.85 cu. ft. of water per second at a speed of 68 revolutions per minute. The number of blades = 6; the blade-thickness = $\frac{7}{8}$ in.; the wheel-depth at outlet = 9 ins.; the diameters of the suction- and discharge-pipes = 36 ins.; the diameter of the wheel = 90 ins.; $\beta = 19^\circ 7' 26.67''$; $\gamma = 90^\circ$. Find the gain of head in passing from the wheel into the volute and the frictional loss (h_r) in the discharge- and suction-pipes and in the wheel-passages. Also find the efficiency.

Ans. 2.193 ft.; 1.911 ft.; .65.

23. In the centrifugal pumps for two torpedo-boat destroyers the diameter of eye = 7 ins.; the diameter of wheel = 20 ins.; the number of blades = 6; the thickness of blades = $\frac{3}{16}$ in.; the width of the wheel at outlet = $1\frac{9}{16}$ ins.; the actual lift = $63\frac{1}{2}$ ins.; $\cot \beta = 5.167$. The pumps are driven by a vertical non-condensing engine with a $4\frac{1}{2}$ -in. cylinder, a 4-in. stroke, and a $\frac{5}{8}$ -in. piston-rod. With a boiler-pressure

of 220 lbs. per square inch above the atmosphere and a cut-off at $\frac{5}{8}$, the delivery was found to be 1113 gallons (U. S.) at 420 revolutions per minute. The frictional losses, due to one upper bend, two 7-in. bends, one bad check-valve, one gate-valve, and about 8 ft. of 7-in. pipe, were respectively estimated at $.3h_d$, $.4h_d$, h_d , $.1h_d$, and $.4185$ ft., h_d being the head corresponding to the velocity of discharge (= velocity of flow in volute). Find (a) the mechanical efficiency; and also find, on the ordinary hypotheses and assuming $\gamma = 90^\circ$, (b) the radial velocity of flow; (c) the loss in shock on entering the volute; (d) the hydraulic efficiency.

Ans. (a) 6.02 per cent; (b) 4.014 ft. per sec.; (c) 61.7 ft.-lbs.; (d) .434.

24. Show how the results in the preceding example will be affected with a delivery of 2000 U. S. gallons at an assumed speed of 700 revolutions per minute.

Ans. (a) 6.67 per cent; (b) 7.214 ft. per sec.; (c) 120 ft.-lbs.; (d) .409.

25. Determine the hypothetically best speeds in revolutions per minute for the pumps in Examples 23 and 24, and calculate the corresponding maximum hydraulic efficiencies.

Ans. In Ex. 23 best speed = 292.7 rev. per min.

" " 24 " " = 526 " " "

26. A centrifugal pump delivers 20 cu. ft. of water per second at a speed of 225 revolutions per minute; the diameter of the discharge-pipe is 20 ins., the diameter of the wheel is 36 ins.; the width of the wheel at outlet is $5\frac{1}{2}$ ins.; the number of blades = 6; the blade thickness = $\frac{5}{8}$ in.; $\gamma = 90^\circ$; $\operatorname{cosec} \beta = 1.484$. Find the hydraulic efficiency, and also find the diameter of the whirlpool-chamber which will increase this efficiency

by .1234.

Ans. $.708 - \frac{gh_r}{1035.89}$; 48 ins.

27. A centrifugal pump making $229\frac{1}{2}$ revolutions per minute delivers $23\frac{1}{2}$ cu. ft. of water per second. The diameter of the discharge-pipe = 18 ins., of the wheel = 42 ins., and of its whirlpool-chamber = 48 ins. The width of the wheel at outlet = 3.452 ins., and of the whirlpool-chamber at its outer circumference = 2.5 ins. The tip angle β at outlet = $\cot^{-1} 3.6$. Assuming the ordinary whirlpool theory and disregarding hydraulic resistances, determine (a) the radial velocity of flow (v_r''); (b) the actual velocity, v_s , with which the water leaves the wheel; (c) the loss in entering the whirlpool-chamber; (d) the hydraulic efficiency. There are six blades each $\frac{3}{4}$ in. thick.

Ans. (a) 9.3569 ft. per sec.; (b) 12.567 ft. per sec.;
(c) 76.4142 ft.-lbs.; (d) .49.



INDEX.

- Abbot, 252, 253, 257
 Abrupt changes of section, loss of head due to, 164
 Accumulators, 339
 Accumulator, Brown's steam, 344
 differential, 342
 Air in a pipe, 183
 Air, retarding effect of, 224
 Applications of Bernouilli's Theorem, 12
 Aqueducts, circular, 242
 egg-shaped, 244
 flow-in, 240
 square, 243
 Arc of discharge in overshot wheel, 452
 Aspirator, 16
 Axial-flow turbine, 490

 Balancing of hoists, 345
 Barker's mill, 375
 Barlow's curve, 73
 Barnes, 130
 Barometer, water, 7
 Bazin, 230, 246, 248, 249, 250, 252, 257, 258, 260, 266
 Bazin's velocity curve and formula, 265, 266
 Bazin's weirs, 99
 Bear, punching, 339
 Beardmore, 247
 Beaufoy, 122
 Belgrand, 226
 Belgrand's sewer formula, 246
 Belidor, 386
 Bellmouth, 36
 Bends in pipe, 168
 Bends, river, 269
 Bernouilli's Theorem, 8
 applications of, 12
 Bidone, 60, 284

 Binding-press, 338
 Boileau, 268
 Boileau's velocity curve and formula, 268
 Borda, 60
 Borda's mouthpiece, 58
 Borda's turbine, 382
 Bordered vane, 368
 Bossut, 418
 Bourgogne canal, experiments on, 249, 257
 Bovey's tables of coefficients of discharge, 39, 40
 Boyden's hook gauge, 298
 Boyden's diffuser, 492
 Brakes, hydraulic, 353
 Bramah's press, 336
 Branched pipe connecting three reservoirs, 191
 Branch main of uniform diameter, 188
 Breadth of water-wheels, 438
 Breast-wheels, 440
 Breast-wheel, efficiency of, 441
 losses of effect in, 442
 mechanical effect of, 442
 speed of, 441
 Bresse, 291, 292, 296, 309
 Broad-crested weir, 94
 Brotherhood hydraulic engine, 345.
 Brown's steam-accumulator, 344
 Brumings, 247
 Bucket, capacity of, 458
 form of, 435, 458
 Buckets, number of, 458
 Burdin's wheel, 385

 Canal-lock, time of emptying and filling a; 59
 Capacity of water-wheel buckets, 458

- Capillary phenomenon, 130
 Capillary tubes, flow in, 130
 Castel's table of mouthpiece coefficients, 69
 Centre of pressure, xiv
 Centrifugal force, effect of, 451
 Centrifugal pump, 76
 analysis of, 553
 efficiency of, 555
 height of suction in, 554
 losses due to hydraulic resistance in, 554
 values of α , β , and γ in, 556
 vortex chamber in, 565
 work of, 394, 555
 Centrifugal turbine, 393
 Chamber, whirlpool, 76, 565
 Channel-flow assumptions, 220
 Channel, bottom velocity of flow in a, 266
 flow between bridge piers in a, 296
 flow in an open, 221
 flow through contracted portion of a, 293
 form of, 228
 maximum velocity of flow in a, 236, 258, 265
 mean velocity of flow in a, 268
 mid-depth velocity of flow in a, 265
 of great width as compared with the depth, 288
 of rectangular section and small slope, 287
 steady flow in a, 221
 surface velocity of flow in a, 265
 value of α and β in a, 249; of γ in a, 250; of n in a, 251
 variation of velocity in a section of a, 257
 Channels, cycloidal, 239
 differential equation of flow in, 275
 examples of, 228
 longitudinal profile of, 285
 of constant section, steady flow in, 271
 of varying section, flow in, 271
 rectangular, 229
 semi-circular, 238
 semi-elliptic, 239
 surface-slope in, 227
 trapezoidal, 231
 with change of section, 293
 with constant mean velocity of flow, 235
 Chezy's formula, 163
 Chezy's experiments on Courparlet channel, 247
 Circular orifices, 81
 Cock in cylindrical pipe, 169
 Cocks, loss of head due to, 169
 Coefficients, hydraulic, 29
 Coefficients for turbines, 519
 Coefficient of contraction, 34
 discharge, 38
 friction, 124
 resistance, 34
 velocity, 30
 viscosity, 269
 Coker, 130
 Combined-flow turbines, 495
 Compressibility, 25
 Constants, useful, xvii
 Continuity, 27
 Contraction, imperfect, 34
 incomplete, 35
 loss of head due to abrupt, 165
 Coulomb, 122
 Courparlet channel, experiments on, 247
 Critical velocity, 129
 Cunningham, 257
 Current-meters, 306
 Cylinders, thickness of, 337, 344
 Cylindrical body in pipe, pressure on, 406
 Cylindrical mouthpiece, 63
 Danaïdes, 386
 Darcy, 126, 139, 249, 260, 303
 Darcy gauge, 302
 D'Aubuisson, 126
 Defontaine's velocity-curve formula, 262
 Density, 2
 Diagrams of pipe-flow experiments, 146
 Didion, 403
 Differential accumulator, 342
 equation of steady varied motion, 275
 Diffusor, Boyden's, 492
 Divergent mouthpiece, 66
 Downward-flow turbine, 490, 494
 Draft-tube, theory of, 529
 Drummond on Miner's Inch, 44
 Dubiat, 247, 258, 403
 Dupuit, 293
 Efficiency of centrifugal pumps, 555
 Efficiency of turbines, conditions governing, 510
 remarks on, 519
 effect of centrifugal force on, 508

- Elasticity of volume, 6
- Elbows, loss of head due to, 167
- Ellis, 177
- Energy, losses of energy in hydraulic machines, 351
 - lost in shock, 55
 - of jet of water, 69
 - of water-fall, 7
 - transmission of, 156
 - of fluid, kinetic, 11; pressure, 11; weight, 11
- Engine, hydraulic, 347
 - speed of steady motion in, 351
- Enlargement of section, loss of head due to, 167
- Equations, general, 53
- Equipotential surface, 20
- Equivalent uniform main, 186
- Erosion caused by watercourses, 227
 - effect of, 226, 227
 - table of, 269
- Examples, 109, 210, 328, 355, 408, 539, 572
- Exner, 308
- Expansion, cubical, 6
- Experimental tank, 29
- Eytelwein, 247, 248

- Farmer, 49, 81
- Flamant, 144
- Float adjustment in experimental tank, 41
- Floats, sub-surface, 300
 - surface, 300
 - twin, 301
- Flow from vessel in motion, 26
 - in a frictionless pipe, 27
 - in aqueducts, 240
 - influence of pipe's inclination and position upon the, 138
 - in pipes, 133
 - in pipe of uniform section, 133
 - varying diameter, 184
- Fluid, definition of, xiii
 - friction, 121
 - motion, 1
 - pressure, xiii
 - rotation, 17
 - whirling of, 19
- Foss, 143
- Fourneyron's turbine, 491
- Fournie, 142
- Francis, 86, 89, 301
- Freeman, 178
- Free surface, 20
- Friction, coefficient of, 124
 - in pipes, surface, 125
- Friction, laws of fluid, 123
- Frictionless pipe, flow in, 27
- Froude, 131
- Froude's table of frictional resistances, 121
- Fteley, 89
- Funk, 247

- Ganguillet & Kutter's formula, 250
- Gas, definition of, xiii
- Ganges, experiments on, 257, 278
- Gaukler, 253
- Garonne, experiments on, 266
- Gauge, Darcy, 303
 - Hook, 298
- Gauging, methods of, 297
 - of pipe-flow, 207
- Gaugings on the Ganges, 278; Mississippi, 253
- General equations, 53
- Gerstner's formula, 421
- Graphical representation of losses of head, 170
- Grashof, 431
- Grassi, 6

- Hagen, 139, 253
- Head, 27
- Hele Shaw, 129
- Herschel, 208
- Hoists, hydraulic freight, 345
- Hook-gauge, Boyden's, 298
- Humphreys, 252, 253, 257
- Hurdy-gurdy, 485
- Hydraulic coefficients, 29
 - engine, 344; analysis of, 347
 - gradient, 13
 - intensifier, 342
 - jack, 338
 - mean depth, 222
 - mean radius, 135
 - press, 335
 - ram, 334
- Hydraulic transmission, 156
- Hydraulics, definition of, 1
- Hydrodynamometer, Perrodil's, 308
- Hydrometric pendulum, 308
- Hydrostatics, fundamental principles of, xiv

- Ice, weight of, 3
- Impact, 359
 - apparatus, 369
 - coefficient of, 371
 - on a flat vane, 359
 - on a curved vane, 388
 - on a hemispherical vane, 367
 - on a surface of revolution, 364

- Impact on a vane with borders, 368
- Impact on a wheel, 378
- Imperfect contraction, 34
- Inclination, influence of pipe's, 138
- Injector, 15
- Intensifier, 341
- Inversion of the jet, 48
- Inverted siphon, 182
- Inward-flow turbine, 490, 493
- Jack, hydraulic, 338
- Jackson, 251
- Jet, energy of, 69
 - inversion of, 48
 - measurer, 37
 - momentum of, 69
 - propeller, 373
- Jet reaction wheel, 375; efficiency of, 376; useful effect of, 376
- Jet turbine, 400
- Knibbs, 142
- Kutter, 142, 230, 253
- Laminar motion, 2
- Lampe, 143
- Lesbros, 48
- Level surface, 20
- Levy, 143
- Lift, balanced ram, 345
 - hydraulic ram, 346
- Limit turbine, 494
- Lines of force, 20
- Liquid, definition of, xiii
- Lock, time of filling a, 50
- Longitudinal profile of open channel, 285
- Loss of energy in shock, 55
- Loss of head due to abrupt change of section, 164; bends, 168; cocks, 169; contraction of section, 169; elbows, 167; enlargement of section, 167; orifice in diaphragm, 166; sluices, 169; valves, 169
- Losses of head, graphical representation of, 170
- Losses in centrifugal pumps, 559
 - in turbines, 531
- Magnus, 48
- Main, equivalent uniform, 186
 - of uniform diameter, branch, 188
 - with several branches, 201
- Manning, 230, 252
- Mariotte, 403,
- Metacentre, xv
- Meters, 207
 - inferential, 209
 - piston, 209
 - rotary, 209
 - Schonheyder's, 208
 - Venturi, 207
- Meyer, 269
- Miner's Inch, 44
- Mississippi, experiments on, 253, 267
- Mixed-flow turbines, 495
- Momentum of jet, 69
- Morin, 403, 431
- Motion, fluid, 1
 - in plane layers, 2
 - in stream-lines, 2
 - laminar, 2
 - permanent, 1
 - steady, 1
- Motor driven by water flowing along a pipe, 179
- Mouthpiece, Borda's, 58
 - convergent, 66
 - cylindrical, 63
 - divergent, 66
 - ring-nozzle, 61
- Navier's hypothesis, 203, 264
- Notch, 83
 - rectangular, 83
 - triangular, 92
- Nozzles, 174
 - Ellis' experiments on, 177
 - Freeman's experiments on, 178
- Open channels, 220
- Orifice fed by two reservoirs, 195
 - flow through an, 23
 - in a diaphragm, loss of head due to, 166
 - in a thin plate, 22
 - in vertical plane surfaces, 78
 - in vessel in motion, 26
 - with a sharp edge, 22
- Orifices, circular, 81
 - large, 78
 - rectangular, 78
 - semi-circular, 49
 - triangular, 92
- Orleans canal, experiments on, 247
- Outward-flow turbine, 491
- Overshot-wheel, 450
 - arc of discharge in, 452
 - bucket angle of, 456
 - division angle in, 456
 - effect of centrifugal force in, 451; impact on, 469; weight on, 467

- Overshot-wheel, 450
 number of buckets in, 456, 458
 pitch-angle in, 457
 speed of, 450
 useful effect of, 467
 weight of water on, 452
- Packing, cup-leather, 336
 hemp, 336
- Parabolic path of jet, 25
- Paraboloidal surface, 20
- Paris sewer formula, 246
- Pastal's press, 336
- Path of fluid particle in turbine, 486
- Pelton wheel, 486
- Pendulum, hydrometric, 308
- Permanent régime, 1
- Perrodil's hydrodynamometer, 308
- Piezometer, 12
- Piobert, 403
- Pipe connecting three reservoirs,
 branched, 191, 200; two reser-
 voirs, 162
 equivalent uniform, 186
 flow assumptions, 133
 flow diagrams, 144
 flow in frictionless, 27
 Williams' experiments on flow in,
 206
 of uniform section, flow in, 133
 of varying section, 184
 thickness of, 158, 159
 variation of velocity in transverse
 section of, 202
- Pipe-flow, effect of inclination on,
 138
- Pitch-back wheel, 472
- Pitot tube, 302
- Plane layers, motion in, 2
- Poiseuille, 128, 131
- Poncelet, 48, 418
- Poncelet wheel, 424; design of, 433
- Position, influence of pipe's, 137
- Practical coefficients in centrifugal
 pumps, 569; turbines, 519
- Piess, Bramah's, 336
 Baling, 337
 hydraulic, 336
- Pressure, centre of, xiv
 due to shock, 160
- Pressure-head, 11
 on cylindrical body in pipe, 406
- Pressure on thin plate in pipe, 404
 of fluids, xiii
- Prony, 246, 248, 259
- Propeller, jet, 373
- Pumps, centrifugal, 547; analysis
 of, 553; vortex-chamber in, 565
- Punching bear, 339
- Radiating current, 72
- Ram, hydraulic, 335
- Rayleigh, Lord, 48
- Reaction, 373
- Reaction wheel, efficiency of, 376
- Rectangular orifices, 78
- Régime, permanent, 1
- Reservoir sluices, 97
- Reservoirs, branched pipe connect-
 ing three, 191, 200
 orifice fed by two, 195
 pipe connecting two, 162
- Resistance of ships, 131
 of motion of solids, 402
- Retarding effect of air, etc., in chan-
 nel flow, 224
- Revy's meter, 306
- Reynolds, 129, 130, 139, 141
- Rhine, experiments on, 247, 262, 266
- Ring-nozzle, 61
- River-bends, 269
- Riveter, portable, 338
- Rotation of fluids, 17
- Rühlmann, 285, 286, 293
- Sagebien wheels, 449
- Saone, experiments on, 257
- Schiele turbine, 208
- Schonheyder's meter, 257, 266
- Segner, 375
- Seine, experiments on, 257, 266
- Sharp-edge orifices, 22
- Ships, resistance of, 131
- Shock, energy due to, 55
 loss of energy in, 55
 pressure due to, 160
- Simpson's rule, 309
- Siphon, 181
 inverted, 182
- Slotte, 269
- Sluice in cylindrical pipe, 169
 in rectangular pipe, 169
 loss of head due to a, 169
- Sluices, 437
 reservoir, 97
- Smith, Hamilton, Jun., 87
- Snow, weight of, 3
- Sonnet, 260
- Specific gravity, xiii
- Spiral flow of water, 75
- Standing wave, 281
- Steady flow in channels of constant
 section, 221

- Steady motion, 1; in pipe of uniform section, 133
- Steady varied motion, differential equation of, 202
- Stearns, 89
- Storage of energy, 340
- Stream line, 2
- Strickland, 49
- St. Venant, 248
- Suction-tube, theory of, 529
- Surface-floats, 300
- Surface-friction in pipes, 126
slope in channels, 226
tension, 49
velocity, 258-265
- Tables of backwater function, 290,
291, 292
bottom velocities 269
Castel's results, 69
coefficients of discharge, 39, 40
coefficients of weir discharge by
Fteley & Stearns, 89
density of water, 4
discharge through Miner's Inch, 46
discharge through nozzles, 177,
178
elasticity of volume of water, 6
erosion and viscosity, 269
expansion of volume of water, 6
expansion of water, 4
frictional losses in hose, 178
maximum velocities, 269
 c and y in $v = cm^2i^y$, 153
showing best relative dimensions
for trapezoidal section, 233
slopes and mean velocities, 227
slopes of trapezoidal section, 231
values of c and b in Bazin's formula, 311 to 322
values of $\frac{v'r}{\sqrt{gH_g}}$ for centrifugal
pumps, 556
values of γ in Bazin's formula, 250
values of $\frac{v'r}{\sqrt{gH}}$ for turbines, 503
values of m and n in $Q = m(h+n)$,
316
values of n in Ganguillet & Kutter's formula, 251
viscosity of water and mercury,
269
values of c and b in channel formula, Ganguillet & Kutter, 323-
326
- Tables, values of c and b in Manning's formula, 327
- Tachometer, 308
- Tadini, 248
- Tank, experimental, 29
- Tension, surface, 49
- Theory of suction or draft tube, 529
of turbines, 497
- Thijbault, 403
- Thickness of hydraulic pipes and
cylinders, 337-344
- Thomson, James, 77, 93, 269
- Thomson's turbine, 565
- Throttle-valve, loss of head due to,
169
- Thrupp, 139
- Time of emptying and filling a
canal-lock, 50
- Toricelli's theorem, 24
- Transmission of energy by hydraulic pressure, 136
- Trautwine, 89
- Triangular notch, 92
- Tub-wheel, 387
- Turbine, axial-flow, 490, 494
Borda's, 382
Boydén's, 491
centrifugal, 393
combined, 495
efficiency of, 501, 510, 519
Fontaine's, 494
Fourneyron, 491
impulse or Girard, 482, 507, 513,
517
inward-flow, 491, 493
jet, 400
Jonval, 494
limit, 494
losses of effect in, 531
mixed-flow, 490, 495
outward-flow, 491
parallel-flow, 494
practical values of velocities in,
519
radial-flow, 490
reaction, 482, 516
Schiele, 495
Scotch, 375
Segner, 376
Swain's, 495
tangential, 393
theory of, 497
Thomson, 491, 493
useful work of, 501
ventilated 483
vortex, 491, 493
Whitelaw, 375

- Tutton, 146, 253, 289, 291, 292
 Tweddell's differential accumulator, 342
- Undershot-wheel, 416
 Undershot wheel, actual delivery in ft.-lbs. of, 423
 depth of crown of, 431
 efficiency of, 417, 420; Poncelet, 428
 form of course of, 429
 in a straight race, 418
 losses of effect with, 421
 modifications to increase efficiency of, 423
 number of buckets in, 419
 Poncelet's, 424; efficiency of, 428
 useful work of, 417, 420
 with flat vanes, 417
- Uniform main, equivalent, 186
- Unwin, 403
- Useful constants, xvii
- Vallot, 143
- Values of c , x , and y in $v = cm^x i^y$, 153
- Valve, loss of head due to a, 169
- Vane, best form of, 388
 cup, 367
- Velocity, bottom, 260, 266
 critical, 129
 curve in a channel, 257
 formulæ, Bazin's, 266
 formulæ, Boileau's, 268
 maximum, 260, 267
 mean, 258, 265
 mid-depth, 265
 of whirl, 498
 rod, 301
 surface, 258, 265
 variation of, 257
- Velocities in turbines, practical values of, 519
- Vena contracta, 23
- Venant, St., 248
- Ventilated buckets, 472
- Venturi, water-meter, 16
- Vessels in motion, orifice in, 26
- Virtual fall, 13
 slope, 13
- Viscosities, table of, 269
- Viscosity, 264
 Meyer's formula for, 269
 Slotte's formula for, 269
- Volute of centrifugal pump, 558
- Vortex, circular, 74
 compound, 76
 free, 74
 free-spiral, 75
 forced, 75
 motion, 74
- Water, pressure of, 6)
 weight of, 2
- Water-barometer, 7
- Water-meter, 207
- Water-pressure engine, 347
- Water-wheels, classification of vertical, 416
- Wave propagation, velocity of, 161
- Weight of fresh water, 3
 of ice, 3
 of salt water, 3
- Weir, 83
 Bazin's flow-over, 99
 Beam, 104, 107
 broad-crested, 94, 106
 drowned, 88, 106
 inclined, 89
 rectangular, with end contractions, 86; without end contractions, 85
 sharp-crested, 99, 107
 submerged, 88
- Weisbach, 36, 60, 166
- Weser, experiments on, 247
- Wheel, breast, 440
 hurdy-gurdy, 485
 in straight race, 418
 jet reaction, 375
 overshot, 450
 Pelton, 486
 pitch-back, 472
 Poncelet, 424
 Sagebien, 449
 undershot, 416
- Whirling fluids, 19
- Whirlpool-chamber, 76
- Whirl, velocity of, 519 498
- Whitelaw, 375
- Williams, 206
- Woltmann, 247

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